The Rise of 15%:

Emergence and Persistence of Commissions in Advertising

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Abstract: This paper models the early history of advertising agencies, focusing specifically on the evolution of their compensation. The early history of agency compensation is characterized by a percentage commission on advertiser's media budget, which aligned the incentives of the agent with the incentives of the publisher for whom the agents initially worked for. But the commissions persisted even through the later history when agencies switched principals and started working for advertisers instead. The proposed model explains this persistence by heterogeneity of agency types co-existing in the market after the switch, and price competition thus setting the compensation of agencies that switched by bids of agencies that did not. The model also explains why the switch of principals coincided with the emergence of national brands, and why the competition to represent these national brands eventually eroded agency profits to a point when collusion to maintain the pre-switch commission structure became desirable.

Key words: Advertising, Agency, Compensation, Commission, Media, Principal-Agent

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Introduction

This paper tries to explain the origins and surprising persistence of commissions in the compensation structure of advertising agencies. The origins of the commissions lie in the early 19th century when agents worked as employees of individual publishers, selling advertising space in their newspaper to local businesses. It was natural for the newspaper publisher (the principal) to provide a commission as an incentive to the agent following the classic principal-agent argument in Holmstrom (1979). The commission compensation structure also continued to make sense around the middle of the 19th century when the early agents morphed into independent middlemen selling space in multiple newspapers. Such middlemen competed only on price because a mid-19th century advertiser provided both the ad copy and the media buying plan. With every agent taking orders for space in most newspapers, the switch from exclusive agency to the middleman model intensified price competition between the agents. Trying to escape this competition, some agents "switched sides" in the 1880s, and started to work for the advertisers. Instead of competing on only price, the agents who switched sides also started providing their advertiser clients a range of services ranging from market research to developing copy to assisting with media planning. Surprisingly, such "full-service" advertising agencies continued to charge their clients a percentage commission on the media space they bought on their behalf. At they continued charging a percentage commission for at least another hundred years afterwards.

A percentage commission gives the agent neither an incentive to be frugal with his principal advertiser's budget nor an incentive to be as creative as possible, so it seems like a sub-optimal compensation structure. The main question of our paper is how did commissions persist through the time period during which the agents switched principals from publishers to advertisers, and how did it then survive for almost one hundred years as the standard agency compensation. At the same time, we want to capture another pattern in the evolution of full-service agencies: the late 19th century switch coincided with the emergence of national brands of consumer packaged goods and national agencies that served the manufacturers of these goods.

Our proposed explanation is presented using a stylized model of the evolution of the advertising agency industry. The model's explanation centers on the heterogeneity of agency business models present in the market during the switch: full-service agencies continued to also engage in purely transactional business well into the 20th century, and old-style space-selling middlemen continued to compete with them based on price alone. The reason for this co-existence of heterogeneous business models within the market and even within a single agency was the scale heterogeneity of advertisers: large national advertisers warranted the investment in ad quality

while small local advertisers did not, and the agency wanted to serve all of them. Our model shows how the pricing of full-service agencies was thus set in competition with space middlemen who still charged a commission. In this way, the commission compensation that made sense for the middlemen was perpetuated through competitive bidding (i.e. "pitching" in advertising agency language) from exclusive salesmen to space middlemen to full-service agencies. Regarding the co-incidence of the switch and the emergence of national brands, our model demonstrates the advantage pioneering national agencies had in attracting the business of the emerging national brands over space-dealing middlemen, but it also highlights the intensification of price competition that bidding for national-level contracts brought.

It is well known that the commission compensation structure, along with its non-negotiable 15% level, was eventually sustained through collusion between agencies and publishers (e.g. Haase 1934). Why did the agencies first switch away from the publishers, and then colluded to essentially get their old compensation back? Our model establishes the incentives for this collusion by showing that once enough national full-service agencies appear, they compete away their quality advantage over the space middlemen, and their profits fall even lower than the profits they would collect it they only completed locally on price. We show that shifting the locus of competition for advertiser business to the national level intensifies price competition even without increasing the number of firms, and the participating agencies tend to earn less than they did when they merely sold space in local markets.

We contribute to the literature on agent compensation in general and advertising agency compensation specifically. In general, we show how *buyer's* agents can end up with a compensation structure theoretically more suitable to *seller's* agents (e.g. as described in Jensen and Meckling 1976; Holmstrom 1979; Basu et al. 1985, and Holmstrom and Milgrom 1987, 1991). Regarding the relatively smaller advertising agency compensation literature we are aware of only two papers that address the choice of compensation scheme: Spake et al (1999) who advocate for basing the agency compensation on its added value to advertiser profits instead of using the commission on media billings, and Hao (2005) who focuses on the design of an incentive-based scheme in a standard principal-agent framework modified with an exogenous budget constraint of the principal. In this paper we provide a model of pricing in a market consisting of heterogeneous players. Our model of competition between agents relies on a scoring auction (Che 1993) first applied as a model of an advertising agency contest by Horsky, Horsky and Zeithammer (2016). We extend the scoring auction model to an asymmetric-bidder situation in which a national agency competes with local agencies, both before and after it switches sides. Our model contributes to the literature on

incentives of companies to invest in quality in a competitive setting (e.g. Shaked and Sutton 1982, Sutton 2007), and we show how a pioneering investor may be worse off because being such a pioneer requires national scale, and the asymmetric competition that arises when one firm goes national and invests in quality is very intense.

We further contribute to the academically under-researched area concerning the relationship between firms and their advertising agencies. The quantitative marketing literature in this area is sparse and includes the following: Gross (1972) who provides the first model of the pitch contest based on creativity alone and Horsky, Horsky and Zeithammer (2016), who address the question of participation stipends in the pitch contests; Villas-Boas (1994), who examines the potential benefits had agencies been allowed to service competitive accounts; Schmalensee, Silk and Bojanek (1983) and Silk and Berndt (1993, 1994), who find scale and scope economies in advertising agency costs; Horsky (2006), who examines the conditions under which an advertiser should unbundle the creative and media services to two separate providers; and Wernerfelt, Silk and Yu (2021) who use the fact that some modern firms bring the advertising function in house to test theory of the firm. All of this previous work takes the type of the compensation scheme as exogenously given.

Since we model the co-emergence of national brands and full-service advertising agencies, we contribute to the literature that studies the history of branding, e.g. Bronnenberg, Dhar, and Dube (2007). Since the explanation for the long-run persistence of the fixed 15% commission centers on collusion between the agencies and the publishers, we also contribute to the literature on cartel stability, e.g. Ellison, Glenn (1994) and Levitt, Syverson, and Ferreira (2008).

More generally, we identify an important general research question that transcends the advertising agency industry: why are *buyer*'s agents often compensated with a percentage commission? While our paper is laser-focused on the compensation of historical advertising agencies, several other industries also involve buyer's agents receiving a commission: real estate, life insurance, and commercial trash hauling. In the Discussion section of our paper, we explain how our findings may translate to these industries. And last but not least, we also discuss the implications of our findings for digital advertising, where emerging "media agencies" are again demanding percentage commissions from advertisers they work for just like the old agencies we study.

The paper is structured as follows: We start with detailing the genesis of mass advertising industry and the emergence of ad agents. Then, we present our model that rationalizes the historical patterns. A discussion of the implications of our finding both for the analysis of advertising agencies specifically and for the analysis of agency compensation in general concludes the paper.

The Genesis of the Mass Advertising Industry

To understand the genesis of the commission compensation in advertising we need to go back to when mass media advertising and advertising agents first emerged. Our historic review of the U.S. mass advertising industry is based on books and articles that examine this time frame: Fowler (1900), Sheldon (1925), Gundlach (1931), Young (1933), Goode (1933), Haase et al. (1934), Hower (1939), Klaw (1956), Gamble (1959), Holland (1974), Holland (1981), Pope (1983), and the *New York Times* (1926,1927,1956,1958, 1960).

Agents emerge in early 19th century as salesmen of advertising space

With the spread of mass-circulation newspapers[1] in the early 19th century, their publishers understood they could augment their subscription-based income by the sale of print space to potential advertisers. Potential advertisers had to be persuaded to spend money on advertising, and they lacked information about the rates, availability, and circulation of newspapers. Yet, the publishers of the day did not have the skills or bandwidth to solicit paid advertisements: *"Newspaper publishers were, for the most part, editors, not business men. They depended primarily on subscriptions for income."* (Young 1933, p. 21).

To recruit advertisers, the publishers contracted the sale of print space for advertisements to agents whose "*chief service in this early period was to promote general use of advertising*" (Hower 1939, p. 24). Much like other salesmen, the agents were compensated by a percentage commission on their sales of space. Thus, the agents' incentives were aligned with those of the publishers. In terms of economic theory, the newspaper publishers converged to a compensation plan for their agents, consistent with agency theory (e.g., Jensen and Meckling 1976; Holmstrom 1979; Grossman and Hart 1983; Holmstrom and Milgrom 1987, 1991) and the one advocated in marketing for salesperson compensation starting with Basu et al. (1985).

The first agents started as employees of individual publishers, selling advertising space in their newspaper to local businesses. Unlike today, the advertiser provided the advertisement, and its copy and layout were bold presentation of facts (i.e., mention of available goods). As seasoned advertisers accumulated and the number of newspapers exploded in the early 19th century[2], the agents morphed from exclusively representing one newspaper into "one-stop-shop" middlemen

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[1] Today, except for the weekly *The Economist*, the "newspaper" evokes a daily publication. In the 19th century, most (74%) "newspapers" were published weekly, and they were the "favorite advertising mediums" (Rowell 1872, p. 6).

[2] The number of newspapers in the U.S. increased from 75 in 1790 to over 360 in 1820, to over 1400 in 1840 (Hower 1939).

selling space in multiple papers, thus allowing advertisers to place the same advertisement in multiple papers simultaneously. This arrangement involved a clear economy of scope in contracting, the importance of which grew with the increasing number of newspapers. The space-selling middlemen, sometimes called "space jobbers" continued to receive a commission on the sales they generated, i.e. a wholesale discount, and this compensation structure continued to align their incentives with those of the publishers. The first such agent was Volney B. Palmer in Philadelphia in 1842 (Oswega Palladium, 1846). Palmer considered himself a 'newspaper agent' working on behalf of many newspapers that generally compensated him with 25 percent of the revenue he generated.

The switch from exclusive agency to the middleman model intensified price competition between the agents. By the 1850s, "New York, Philadelphia, and Boston each had a number of advertising agents clamoring for business, and each agent claimed authority to represent every paper of importance in the country" (Hower 1939, p. 16). Differentiation was clearly difficult to sustain, with newspapers willing to accept advertising revenue from just about any agent. The business was transactional, with advertisers providing copy and soliciting bids on a specific media plan in order to find the cheapest way of serving their ads. By 1860s, some agents followed the lead of Carlton & Smith Agency (which eventually became the juggernaut J. Walter Thompson Company), and took the middleman business model to its logical conclusion by purchasing advertising space upfront in bulk, and collecting the profit from the differential between the rates they negotiated with publishers and the prices they charged advertisers. However, price competition continued to be fierce, putting downward pressure on agency profits. "*Each agent sought lower rates solely in order to underbid his rivals, and advertisers naturally encouraged such competition by demanding bids from many agents on every contemplated order for advertising*" (Hower 1939, p. 63).

Some agents switch sides to start working for advertisers

As early as the mid-19th century, advertising agents searched for differentiation strategies to escape the price competition in space jobbing. The rapidly maturing industry also exposed a conflict of interest involved in the agents' attempt to serve both sides of the market: although the agents significantly increased the volume of trade that publishers and advertisers transacted with each other,[3] they were eventually held in contempt by both publishers and advertisers. The problem that arose from the advertiser perspective was the secretive nature of the discounts different agents were able to obtain from different publishers (the commissions varied between 10% and 50%). The

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[3] In 1879 56% of newspaper revenue came from circulation sales. By 1919 this percentage decreased to 34%, and the remainder two thirds of newspaper revenues was generated from advertising (Pope 1983).

agents were supposed to provide advertisers with impartial advice on which publications best suited their advertising needs. However, advertisers suspected agents were directing them to place ads in publications that paid the highest commissions, rather than those that represented the best advertising value. The publishers, in turn, "*never held to their stated prices...and preferred to accept almost any price for space rather than the risk of letting it go unused*" (Hower 1939, p. 17), effectively letting the competition among the agents drive their advertising revenue down.

The solution to the conflict of interest described in the previous paragraph was the "open contract" innovation by the N.W. Ayer & Son agency (hereafter referred to as "Ayer") dating back to 1875.[4] Ayer's idea was to move from a short-term transactional relationship with advertisers towards a more long-term exclusive contract with them based on service ranging from market research to developing copy to assisting with media planning.[5] The contract he proposed was "open" in the sense that the advertiser no longer specified the exact frequency and placement of their ads, but instead provided a budget and allowed the agent to buy whichever space he deemed to be the most advantageous for the goals of the advertising campaign. The media planning and buying expertise supported by market research naturally came first in the 1880s, followed by the creative work starting in the 1890s.

To win the client's business under the open contract structure, the agent was effectively marketing his advertising consulting services While the agent's reputation for high-quality service obviously went a long way towards securing the deal, it became standard agency practice to perform much of the work upfront, and "pitch" the ideas to prospective clients. The first recorded upfront market research was performed in 1880 by Ayer to secure an open contract with the Nichols-Shepard Company – a manufacturer of threshing machines form Michigan. Ahead of the pitch, Ayer's agents performed extensive nation-wide research (an extremely arduous undertaking given the times) of the amounts and types of grain produced in different places, as well as the identity of circulation of local newspapers read by the farmers. The research findings were collated in a large book presented to the managers of Nichols-Shepard who immediately offered to purchase the book. Ayer's agents famously replied: "*We are not in the book business, we are in the advertising business. If you are not our customer, it is not for sale at any price. If you are our customer, it is your for the asking*" (Hower 1939, p. 90). By 1890, major agencies employed full-time artists engaged in copy development. *"It was a nuisance to the agent..., but if "copy" would help sell space then copy he would produce. And if a package,*

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[4] Ayer's first client who agreed to the open contract was Dingee & Conard – a rose grower from Pennsylvania who signed up for a year in 1875 and remained with the agency for many years to follow.
[5] "I will not be an order taker any longer...I will have clients rather than people who just give me orders" (Ayer 1921, p. 5).

a name, and a trade-mark were also needed to get a new advertiser started, then these, too, the agent would assist with or suggest" (Young 1933, page 29). With advertisers increasing demand for persuasive type advertising, the agencies' emphasis shifted to ad creation instead of media selection: *"Ninety per cent of the thought, energy and cost of running our agency goes into copy"* (Albert Lasker, ad agency partner in *Lord and Thomas,* 1906). Thus began the creation of the "full-service" advertising agency as we know it today that performed both creative planning and production and media brokering.[6]

Having described the essence of the open contract, we now turn to its compensation structure. The open contract guaranteed that the agent would no longer make secret deals with publishers for placing ads with them. Instead, it would accept payment only in the form of a set rate of commission of the list price, a rate that was agreed upon between the publisher, agent, and advertiser. This rate varied, experimentally in the beginning, but eventually stabilized at 15% by early 1890s. While Ayer and other agencies who followed his lead thus "switched sides" from working for newspapers on selling advertising space to working for advertisers to buy advertising space, the agency compensation structure remained a commission on the money spent. The main question of our paper is how did this seemingly outdated and mis-aligned compensation structure persist. Our explanation centers on the heterogeneity of advertisers present in the market, to which we turn next.

By the 1890s, the open contract with 15% commission became standard practice in the advertising industry. However, full-service agencies using the open contract continued to engage in purely transactional business well into the 20th century, and old-style space-selling middlemen continued to compete with them based on price alone. The reason for this co-existence of heterogeneous business models within the market and even within a single agency was the scale heterogeneity of advertisers: large national advertisers warranted the investment in ad quality while small local advertisers did not, and the agency wanted to serve all of them. Our model shows how the pricing of full-service agencies was thus set in competition with space middlemen who charged a commission. In this way, the commission compensation that made sense for the middlemen was perpetuated through competitive bidding (i.e. "pitching" in advertising agency language) from exclusive salesmen to space middlemen to full-service agencies.

National advertising follows the emergence of consumer brands in the late 19th century

The emergence of full-service agencies coincided with the emergence of large national advertisers. The case of Nichols-Shepard Company described above shows how a national B2B brand can benefit

from market research and media expertise of a full-service agency with national reach. In the B2C market, the 1870s and 1880s marked the emergence of national brands that remain household names till today: (e.g., Heinz Ketchup in 1876, Burpee Seeds in 1876, or Ivory soap in 1879). Manufacturing industries ranging from soap to canned food to cigarettes introduced new production techniques, created standardized products in large quantities, and sought to find and persuade buyers. Along with the national manufacturers, large department stores appeared (Macy's in New York in 1858, Wanamaker's in Philadelphia in 1876, and Marshall Field's in Chicago in 1887) and soon expanded beyond their home cities. For rural markets, the Sears Roebuck (established in 1886) and Montgomery Ward (established in 1872) served Americans who lived in the countryside–a majority of the U.S. population until about 1920.

Full-service advertising agencies with national reach were ideally suited to take on the multi-faceted job of building these national brands. A classic example of an advertising agency assisting with all elements of branding is Ayer's 1898 work with the National Biscuit Company: not only did Ayer come up with the "Nabisco" and the "Uneeda biscuit" brand names, it also developed a "*coordinated plan for reaching the public through newspapers, magazines, street-car advertisements, posters, and painted signs*" in "*probably the largest campaign in the country up to this time*" (Hower 1939, p. 115). Our model demonstrates the advantage pioneering national agencies had in attracting the business of the emerging national brands over space-dealing middlemen, but it also highlights the intensification of price competition that bidding for national-level contracts brought.

Collusion: The Recognition System

Once enough full-service agencies entered the market, their quality-production advantage over the space middlemen was competed away, and agency profits once again slumped. To solve this problem, the full-service agencies turned to collusion. For over a century, the 15% commission rule was considered status quo. Supported by the media and large agencies, this commission-based compensation was the core element of an interrelated set of trade practices, collectively referred to as the *recognition system*. The publishers' association "recognized" prospective agencies that met the association's standards. Agencies were screened in areas of financial resources, advertising experience, types of accounts, moral standing, and bill paying reputation. Agencies which passed the screening had their names disseminated to the association's members.

The *recognition system* fostered standards relating to the granting of credit and commissions by publishers to only "recognized" agencies and discouraged splitting commissions directly with advertisers. While aiding in further stabilizing and legitimating the advertising business, these

practices inhibited bypassing the use of independent advertising agencies by direct contact of advertisers with the media. As early as 1893 the *American Newspaper Association* adopted a resolution stating that commissions would be paid only to recognized advertising agents and not to advertisers. Similarly, in 1902 the national magazines' trade association, the *"Quoin Club,"* advocated the use of the commission system[7]. The *recognition system* limited price competition among agencies and enabled the bundling of the increasing range of services offered by independent full-service agencies. Freed from the threat of under-cutting, agencies competed for business on the basis of service rather than price.

In 1918, a year after its foundation, the *American Association of Advertising Agencies (4As)* was instrumental in further stabilizing the compensation practice at the standardized commission rate of *15% of the* gross space rate. *"It can be said that since 1918, on the testimony of agency men, agents have been a major factor in setting the rate of discount and have endeavored to make the rate they desired uniform for various classes of mediums. The figure set by agents as the desirable uniform rate, namely 15%, is one which we said to reflect costs of doing business in 1918"* (Haase 1934, page 3). *"The remuneration of the agent was not a payment from the publisher, or for services rendered to the publisher, but was a peculiar billing device under which the agent's rate of payment was determined by agents according to their current (1918) costs of doing business"* (Haase 1934, page 27).

Essentially the *recognition system* was a cartel agreement between full-service agencies and publishers to prevent direct dealings between advertisers and the media and to stop small advertising shops and media independents from undercutting prices. Large advertisers (mostly national advertisers of branded products) and smaller non-member agencies continuously criticized the commission arrangement. The longstanding survival of the commission system indicates that the power of the large advertisers to change the compensation system was limited. Our model establishes the incentives for collusion by showing that shifting the competition to the national level intensifies price competition even without increasing the number of firms.

As early as 1924, the Federal antitrust authorities undertook investigations of the *recognition system*, alleging that the system constituted a conspiracy in restraint of trade.[8] The lawsuit was

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[7] Advertising in magazines started as early as 1867 in religious magazines, in the 1870s in mail-order magazines, and in the 1880s in national magazines.
[8] The first complaint alleged discrimination against national advertisers by the refusal of some newspapers to allow the *15%* commission to them directly without the use of an agency. In court testifying on the case, Vick Chemical Company claimed that the company was blacklisted from a list furnished to newspapers by advertising agencies, because of its policy to place its own advertising (which could collect the *15%* commission). Other advertisers claimed to have established an agency under a separate name so that the discount would not be refused. For example, Carter Medical Company as early as 1895 set up their internal advertising agency (under a separate name), because they foresaw the refusal of *15%* to direct advertisers (New York Times 1927).

eventually dismissed, and the recognition system survived another sixty years despite additional antitrust actions in the 1950s, the emergence of multiple forms of new media, and a multitude of other shocks. Please see Horsky and Zeithammer (2022) for a detailed empirical analysis of the cartel and its eventual demise in the 1990s.

Modeling challenges

The early history of advertising agencies outlined in the previous section documents a coincidence between the agencies switching sides and their growth to represent national advertisers at the national level. It is obvious that the service an agency provides to its advertiser client involves an economy of scale: spreading the fixed cost of ad copy improvement (in the form of creative work and market research) over a larger volume of advertising increases the chance the investment will break even. Therefore, we hypothesize that the emergence of national advertisers who demand the larger volume (represented in our model by a greater number of publishers where the ad appears) caused the advertising agents with national reach to switch sides and become full-service agencies for these new large advertisers. But the national agency profit-lift from providing better ads to its advertiser client is not fixed - it depends on the competition's quality investment decisions, as well as on the structure (national vs. local) of the market. History tells us that price competition intensified when national middlemen selling space emerged, so the profit lift from ad improvement may have become smaller at the same time. Moreover, pioneering national agencies need to compete with local agencies remaining in individual markets / cities comprising the nation, so a model of asymmetric competition on both price and quality is needed to capture the situation.

In addition to qualifying the basic economy-of-scale intuition with a formal model of agencies switching sides in equilibrium of a pricing game, we also need to explain why the pricing of advertising continued to follow the commission structure even after the national agents switched sides. History suggests that only some agents switched, and others continued to operate as middlemen. What type of pricing model can both predict this co-existence as an asymmetric equilibrium in the game to become a full-service agency, and explain why the pricing of full-service agents was effectively driven by their competitors working on a commission basis?

Finally, our model needs to clearly show the incentives for collusion after the emergence of national full-service advertisers. If becoming full-service escaped competitive price pressures in the short run, why did this softening not last? One obvious explanation is an explosion in the number of competing agencies. But is it possible that something about national-level competition inherently reduces profits after a fixed set of existing agencies switch sides to become full service? What if the

number of competitors did not change, but their switch from competing at the local level to competing at the national level was enough to reduce profits? Our model addresses all of the above challenges in as parsimonious a way as we were able to specify.

Model assumptions

Publishers: Two cities exist, each with one newspaper publisher. The publishers rely on agents to price and sell the advertising space in their papers. Together, the two cities are a nation.

Discrete time: Time t starts in the early 19th century and flows discretely and slowly, for example, in decades or other epochs. Because of the long calendar time between periods, we assume all actors are acting myopically, maximizing their payoffs within each period. Our model has four important periods characterized by the evolving nature of competition in the market, as explained next.

Agents over time:

Period 1: In the beginning of time, each city has one local advertising agent. These agents can be either commissioned salesmen of newspapers or middlemen selling space.

Period 2: A new type of agent – one with national potential – arrives. National potential is the ability to place ads for a single advertiser in both cities – an economy of scope useful for serving national advertisers. However, this potential is not realized right away – the agent starts as a local agent in both cities for a period, allowing us to analyze the impact of local competition on the agency incentives for switching sides and becoming full-service. This assumption is realistic given the novelty of national advertising.

Period 3: The agent who entered in the second period realizes its national potential. This development allows us to analyze the impact of economies of scope on the incentives for switching sides in the absence of competition at the national level.

Period 4. After seeing the success of the first national agent, the local agents either consolidate to a single national-level entity or get bought out by a new national entrant. This assumption allows us to analyze the impact of agency competition at the national level on the incentives for switching sides without increasing the overall number of players in the industry.

Advertisers: In each period, both local and national advertisers arrive, and buy one unit of advertising each. National advertisers run businesses, such as manufacturing consumer packaged goods or running a mail-order catalog, that benefit from advertising in both cities. In contrast, local advertisers are only interested in local advertising because their business, such as a retail store, only operates in the local city.

Creative technology: The quality of advertising copy is measured in the extra advertiser profit generated by one unit of advertising. When advertisers provide their own copy, the ad quality is normalized to q_0. Advertising agents can invest a fixed cost K to increase the effectiveness of the copy to a certainty-equivalent quality $q_1 > q_0$, for example by hiring artists or by performing market research. The cost K is sunk at the time of pitching the idea to the advertiser, and the quality-improvement technology exhibits an economy of scale in that the single investment improves ad quality in both cities for a national advertiser. Let $\Delta \equiv q_1 - q_0$ be the difference in quality.

Pricing: New advertisers do not bargain, they accept the price quoted by their agent. So in the first epoch, the list price of one unit of advertising is fixed at the q_0 level. We assume the list price stays at that level afterwards.

We model the wholesale (to agents) cost of advertising as follows: Each agent a in city i can place the advertising in the local paper for a privately known percentage discount $\beta_{a,i}$ off the list price. In the exclusive agency relationship, $\beta_{a,i}$ represents the agent's percentage commission, and in the middleman model $\beta_{a,i}$ represents a wholesale discount. Let p be the list price of advertising in both cities. The agent's a's cost of placing the ad in city i is $(1 - \beta_{a,i})p$. If he wins the auction and charges the advertiser a gross revenue of R, the agent's payoff thus becomes $R - (1 - \beta_{a,i})p$.

Seasoned advertisers bargain with all agents they can (all local in their city if local, all local and national if national) to get the best deal. Following Horsky, Horsky and Zeithammer (2016), we model the process of selecting an agent and determining the price of advertising as each advertiser running a second-score auction (which simplifies to the second-price auctions when the copy does not differ across agents) as outlined by Che (1993). The second-score auction remains tractable even with asymmetric competitors – a key advantage in our setting. Moreover, it is a useful sealed-bid device for approximating the prices that would prevail if agents engaged in back-and-forth open bidding (Horsky, Horsky and Zeithammer 2016).

Formally, when one of the agents has a higher quality $q_1 > q_0$ and cost β, his score is $q_1 - (1 - \beta)$ while the competitor's score is $q_0 - (1 - \gamma)$ where γ is the competitor's cost. The higher-quality agent wins when $q_1 - (1 - \beta) > q_0 - (1 - \gamma)$, i.e. when $\Delta + \beta > \gamma$. To ensure the winner's payoff required by the second-score auction rules, namely $[q_1 - (1 - \beta)] - [q_0 - (1 - \gamma)]$, the advertiser pays the winning full-service agency a gross revenue $R = \Delta + (1 - \gamma)$, and the agency thus receives $\Delta + \beta - \gamma$ as its compensation.

Note that we assume participation by all agents, and thus abstract from entry games.

Distributional assumptions for tractability: We assume $\beta_{a,i}$ is *iid* across agents, cities, and periods to reflect all kinds of differences across agents and circumstances.

Since the list price $p = q_0$ does not change throughout, we normalize it to 1. If we also assume for tractability that $\beta_{a,i}$ is Uniform[0,1], then together with p=1, the agent's wholesale cost of advertising also becomes Uniform[0,1]. Given that costs range between 0 and 1, we also assume that $0 \le \Delta < \frac{1}{2}$ so that the national full-service agency's quality advantage does not completely overwhelm the price competition with a commissioned middleman.

To summarize, everyone talks about the "list price" of advertising space being 1, but the agent's actual cost of placing the ad with publisher is $(1 - \beta)$. When the agent collects gross revenue R from the advertiser, his net earnings are $R - (1 - \beta)$. When the revenue itself is a percentage of list price, e.g. $R = (1 - \delta)$, the net earnings of the agent become a "reduced discount/commission" of $\beta - \delta$.

Model solution

Our analysis of the model proceeds chronologically, and examines the impact of new agent entry on the pricing of advertising and the resulting agent incentives to become full-service agencies.

First period: monopoly agents

In the first period, each city has only one local agent who exclusively represents the local paper or acts as a middleman. When the agent does not invest in advertising quality, the price of advertising is the list price $q_0 = 1$ because the agent has all the bargaining power. The agent gets commission (or discount if the agent is an independent middleman) $\beta_{a,i}$, and the publisher gets $(1 - \beta_{a,i})$. The expected profit of each agent is thus the expected value of $\beta_{a,i}$, namely ½. When the agent does invest and switches sides to work for the advertiser, he captures the quality improvement Δ as his

additional profit (note this is formally equivalent to running the scoring auction with one bidder and no opponent). This profit increase is worth the fixed costs whenever $\Delta > K$.

Second period: duopoly of local agents

From the perspective of advertisers, each city effectively has two local agents. Therefore, both local and national (if any) advertisers need to deal with each city separately, and it is sufficient to analyze what happens in one of the cities.

Suppose first that neither of the two local agents invests in becoming a full-service agent of the advertiser who arrives. Then, the two agents compete on price for the contract to place the advertising with the local publisher. The lower-cost (higher-discount) agent wins and charges the advertiser the higher cost (lower discount), pocketing the difference between the two discounts. Each agent thus wins half the time, and the expected payoff is $\int_0^1 \int_0^\beta (\beta - \gamma)\, d\gamma d\beta = \frac{1}{6} \approx 0.166$.

Given the symmetric assumption about the possible quality improvement, the same intense price competition occurs if both advertisers do invest in becoming full-service. Not surprisingly, price competition in both symmetric-quality settings takes a lot of profit from the agent market compared to the first period: the total profit of all agents declines from 1 to $\frac{2}{3}$. Figure 3 illustrates the evolution of expected profits, both at the market level and at the individual agency level.

Now suppose that only one of the local agents invests K to increase quality, and becomes the full-service agent of the advertiser. As explained above in the description of the scoring auction, the full-service agent wins more than half of the contests thanks to the higher quality of his ads, and receives a payoff of $\Delta + \beta - \gamma$ as his compensation whenever he does. His expected profit is thus: $\int_0^1 \int_0^{\min(\beta+\Delta,1)} (\beta + \Delta - \gamma)\, d\gamma d\beta - K = \frac{1}{6} + \frac{\Delta}{6}[3 + \Delta(3 - \Delta)] - K$. The competing local agent who did not invest in quality obviously receives less, specifically $\frac{(1-\Delta)^3}{6}$. More importantly for our first result, his gross profit gain from following the full-service competitor and also investing to match the higher quality is $\frac{1}{6} - \frac{(1-\Delta)^3}{6} = \frac{\Delta}{6}[3 - \Delta(3 - \Delta)]$. Comparing the fixed cost K to the net benefit received by the first and second agent to go full-service thus yields our first lemma:

Lemma 1: When $K > \frac{\Delta}{6}[3 + \Delta(3 - \Delta)]$, no agent in the local market becomes a full-service agent. When K is lower such that $\frac{\Delta}{6}[3 - \Delta(3 - \Delta)] < K < \frac{\Delta}{6}[3 + \Delta(3 - \Delta)]$, only one agent becomes full-service. Finally, with even lower costs, both agents invest to become full-service.

It is interesting to note that vertical differentiation emerges for a range of parameter values. This feature of the solution arises from price competition erasing any profit benefit of increasing ad quality whenever both agents invest into becoming full-service. When it happens, the situation resembles the prisoner's dilemma: both agents make exactly the same gross profit as if neither invested, and they both spend an additional K of fixed cost. Both would be better off if they could only agree to remain space middlemen.

Third period: One national agent facing local competition in each market

Local advertisers continue to rely on competition between their local agents as all advertisers did in the second period. As long as K is low enough as in Lemma 1, local advertisers do not benefit from the help of a full-service agency.

National agent as "one-stop shop" national space middleman

Now consider a national advertiser, and suppose first that the national agent bidding for the advertiser's business does not invest in becoming full-service. The national advertiser's purchase decision thus comes down to price. Because of the economy of scope in contracting, the advertiser first approaches the national agent for a quote, and compares the cost to buying the ad space locally. When the national agency loses the chance to represent the advertiser nationally, i.e. when its quote is higher than the sum of the quotes from the local agents, it still participates in the local contests. It obviously cannot win both of them, but can win some of them.

While the above analysis of the separate local markets in period two applied essentially standard techniques from the auction literature, the third period's asymmetric national competition followed by a potential aftermarket is new to the literature as far as we know, so we explain our solution in the main text of the paper.

Let the national agency discounts in the two cities be $\{\beta_1, \beta_2\}$ and sum to $B \equiv \beta_1 + \beta_2$, and denote the local agency discounts in the same cities as $\{\gamma_1, \gamma_2\}$ which sum to $\Gamma \equiv \gamma_1, \gamma_2$. In the first (national) stage of the competition for the advertiser, the cheaper of $(1 - \beta_1) + (1 - \beta_2)$ and $(1 - \gamma_1) + (1 - \gamma_2)$ wins the contract. If $B > \Gamma$, the national agency thus wins the contract. Its expected first-stage profit conditional on B, denoted $\pi_1(B)$, is tractable be because the sum of two uniformly distributed variables on [0,1] has a tractable triangle distribution on [0,2]: $pdf(\Gamma) = \begin{cases} \Gamma < 1 : \Gamma \\ \Gamma > 1 : 2 - \Gamma \end{cases}$. Specifically, the expected profit $\pi_1(B)$ is:

$$\pi_1(B) = \begin{cases} B < 1: \int_0^B (B - \Gamma)\Gamma d\Gamma = \dfrac{B^3}{6} \\[2mm] B > 1: \int_0^1 (B - \Gamma)\Gamma d\Gamma + \int_1^B (B - \Gamma)(2 - \Gamma)\, d\Gamma = \dfrac{1}{3} + B(B - 1) - \dfrac{B^3}{6} \end{cases}$$

If the national agency wins the contract in the first stage, the pricing game is over. But if it loses, it competes in the second stage against each of the local agents separately. The expected second-stage profit in the first city $\pi_2(B)$ is tractable because of two simplifications afforded by our uniform assumptions: first, given a fixed B, the distribution of β_1 is uniform with easily characterizable support. Second, the joint pdf of (γ_1, γ_2) is 1, so conditioning on the loss in the first stage only influences the range of $\beta_1, \gamma_1, \gamma_2$ over which we need to integrate the gain of $\beta_1 - \gamma_1$ whenever $\beta_1 > \gamma_1$.

There are two cases: when $B<1$, β_1 is uniform on [0,B], so

$$\pi_2(B|B < 1) = \int_0^B \frac{1}{B} \int_0^{\beta_1} \int_{B-\gamma_1}^1 (\beta_1 - \gamma_1)\, d\gamma_2\, d\gamma_1 d\beta_1 = \frac{(4 - 3B)B^2}{24}$$

When $B \geq 1$, β_1 is uniform on [B-1,1], so an analogous calculation yields:

$$\pi_2(B|B \geq 1) = \int_{B-1}^1 \frac{1}{2 - B} \int_{B-1}^{\beta_1} \int_{B-\gamma_1}^1 (\beta_1 - \gamma_1)\, d\gamma_2\, d\gamma_1 d\beta_1 = \frac{(2 - B)^3}{24}$$

Since there are two symmetric local markets, the total expected earnings of a national agency are simply $\pi_1(B) + 2\pi_2(B)$. Taking an expectation of $\pi_1(B) + 2\pi_2(B)$ over the triangular distribution of B finally yields the ex-ante expected profit of $\frac{17}{60} \approx 0.283$. It can be shown that each local agent earns exactly half this amount, namely $\frac{17}{120} \approx 0.142$.

Note that while capitalizing on his unique economy of scope and earning more than the local agents (twice more, in fact), the national agent actually earns less than his period-2 precursor did as two separate agents: $\frac{17}{60} < 2 \times \frac{1}{6} = \frac{1}{3}$. The lower earnings arise from intense price competition in the first stage – when If $B < \Gamma$ and the national agent loses the first stage and advances to the second stage, his earnings conditional on B and Γ are the same as in the local-only second period of the market. But when If $B > \Gamma$, he wins too often compared to the local-only counterfactual. The combined profits of all agents decline as well, from $\frac{2}{3}$ to $\frac{17}{30}$. See Figure3 for an illustration.

Given the intense price pressure competing to place advertising nationally causes, the national agent would prefer to rewind the clock to the previous period, and compete only locally.

Alas, he is forced to provide a national quote for national advertisers because of the economies of scope this allows them – having to write and monitor only one contract instead of two. Luckily for the national agent, national advertising can also involve economies of scale in ad quality production, as we discuss next.

National agent as a full-service agent

When the national agent invests K to improve the ad effectiveness of the ads for a national advertiser, the scoring auction determines the winner of both stages. In the first stage, the higher of $2q_0 - (1 - \gamma_1) - (1 - \gamma_2)$ and $2q_1 - (1 - \beta_1) - (1 - \beta_2)$ scores wins the contract and receives the difference between the total "quality-adjusted" discounts. The second stage proceeds exactly like the asymmetric local competition in the second period.

The expected profit of the agent is derived analogously to that for the national space middleman, adjusting for the asymmetric winning chances and payoffs. We relegate the details to the Appendix, and summarize the ex-ante expected profit as $\frac{17}{60} + \frac{\Delta}{60}(55 + 70\Delta + 20\Delta^2 + 8\Delta^4)$. The local agents each make $\frac{17}{120} - \frac{\Delta}{120}(65 - 70\Delta - 10\Delta^2 + 50\Delta^4 - 18\Delta^5)$. Since their gross profit gain from also investing to become full-service is lower than the national agent's gain from being the only one to invest, we get the following result:

Lemma 2: When K is low enough that the local agents would rather match the full-service agent's quality than compete asymmetrically, namely when $K < \frac{\Delta}{120}(65 - 70\Delta - 10\Delta^2 + 50\Delta^3 - 18\Delta^4)$, all agents become full-service. When K is higher but not so high as to prevent the national agent to invest against lower-quality local agents, namely when

$$\frac{\Delta}{120}(65 - 70\Delta - 10\Delta^2 + 50\Delta^3 - 18\Delta^4) < K < \frac{\Delta}{60}(55 + 70\Delta + 20\Delta^2 + 8\Delta^4)$$

, only the national agency invests. For even higher K, no agency becomes full-service.

As in the case of local agents, vertical differentiation often occurs. Figure 1 combines the four indifference lines from Lemmas 1 and 2, and illustrates how the realization of the national agency's potential increases the amount of the parameter space for which a full-service agency emerges in equilibrium. The labels in the figure describe the types of agencies who become full-service in equilibrium of the respective markets (periods 1-2 in black, period 3 in red).

Figure 1: First national agency is more likely to become full-service than a local agency
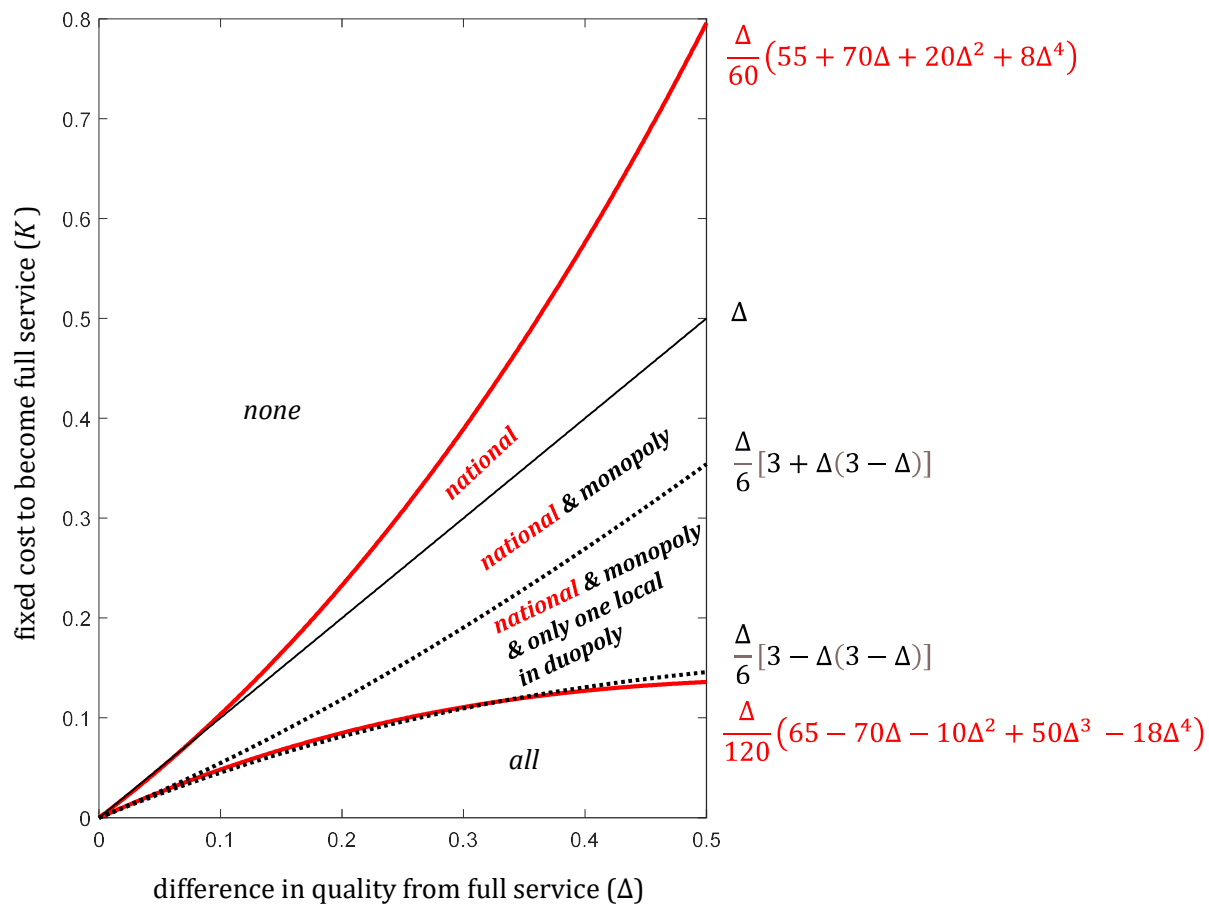
We formally summarize the message of Figure 1 in our first proposition:

Proposition 1: For every $\Delta > 0$, there is a set of K, namely $\frac{\Delta}{6}[3 + \Delta(3 - \Delta)] < K <$

$\frac{\Delta}{60}(55 + 70\Delta + 20\Delta^2 + 8\Delta^4)$, such that the first national agency in the market switches sides to work as a full-service agency of the advertiser whereas a local agency does not switch. Whenever K is lower such that one of the local agencies would switch sides, the first national agency would also switch sides for most K.

The vague "mostly" in the second part of Proposition 1 is clear from Figure 1 but cumbersome to express mathematically: it turns out that the bottom boundaries of the "one invests" regions nearly coincide, and are within 0.01 of each other. We do not believe any robust insights are available for dissecting the minute differences between the two curves. If anything, it is surprising they are so close to each other – our initial intuition was that it would take an even lower K to justify a local

agent facing a full-service opponent to also invest in the third period than in the second period: the competition field is level when all agents are local, but the local agents facing a national competitor are clearly disadvantaged, and make a lot less profit.

In addition to showing that, with a national agency, there is a substantial additional part of the parameter space in which one agency switches sides, Proposition 1 also highlights that it will be the national agency that switches while its local competitors do not. This is an important feature of the price determination in our model: the price charged by the full-service agency is effectively set by the commissions charged by local agencies.

Having provided one possible explanation why national agents switched sides to work for national advertisers as full-service agencies, we now examine the increased competition invited by this innovation.

Fourth period: Two national agencies

Suppose both national agencies offer the same quality, and focus on the bidding to serve a national advertiser (bidding for local advertisers continues to follow the model developed in period two). Then, the competition faced by each of them is exactly like the first stage of the third-period national competition without quality differentiation, and $\pi_1(B)$ captures the expected profit of each agency given its total discount B. Taking the expected value of $\pi_1(B)$ over B drawn from the triangle distribution results in the ex-ante expected gross profit of $\frac{7}{30} \approx 0.233$.

Since there are no local agencies at this point of evolution (here is where the assumed lack of local agencies at this stage simplifies analysis), the total agency gross profits in the whole markets thus decline further from period three to $\frac{7}{15} < \frac{17}{30}$ (see Figure3 for an illustration). In other words, two national agencies imply the ost intense price competition we have encountered so far.

One national agency switches sides: competition between a full-service agency and a national space-dealing middleman

The competition faced by the agency that switches sides to provide full service is exactly like the one faced by the single first national full-service in the first stage of the game analyzed in period three. We thus do not analyze it in detail here (see appendix for the details), and proceed directly to the ex-ante expected gross profit of the full-service agency of $\frac{7}{30} + \frac{\Delta}{15}(15 + 20\Delta - 20\Delta^3 + 12\Delta^4)$ and the ex-ante expected gross profit of its space middleman competitor is $\frac{7}{30} - \frac{\Delta}{15}(15 - 20\Delta + 20\Delta^3 -$

$12\Delta^4$). Given that $\frac{7}{30}$ is the expected gross profit of equal-quality agencies, we again get a result that characterizes equilibrium behavior as a function of the location in (Δ, K) parameter space:

Lemma 3: When $K < \frac{\Delta}{15}(15 - 20\Delta + 20\Delta^3 - 12\Delta^4)$, both national agencies agents become full-service. When K is intermediate, namely when

$$\frac{\Delta}{15}(15 - 20\Delta + 20\Delta^3 - 12\Delta^4) < K < \frac{\Delta}{15}(15 + 20\Delta - 20\Delta^3 + 12\Delta^4)$$

only one of the national agencies switches sides to become full-service. Otherwise, both national agencies remain space middlemen.

Figure 2 illustrates the curves defined in Lemma 3, as well as interesting regions of the parameter space defined in Proposition 2.

Proposition 2: The following relationships hold between the prevalence of full-service agencies in period four as compared to previous periods.
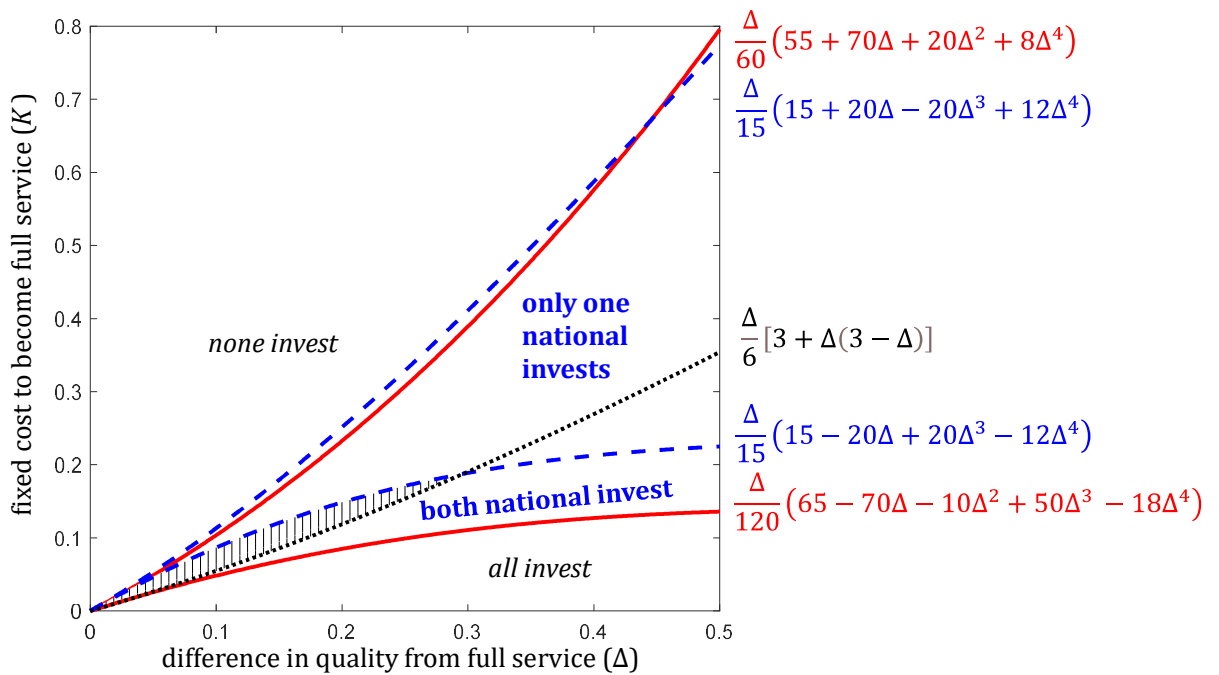
a) Both national agencies become full-service more often than local agencies facing a single full-service national competitor do.

b) For Δ low enough, there is a set of K, namely $\frac{\Delta}{6}[3 + \Delta(3 - \Delta)] < K < \frac{\Delta}{15}(15 - 20\Delta + 20\Delta^3 - 12\Delta^4)$ such that no local agency invests to become full-service in a market with only local agencies, but both national agencies do invest. This area is hatched in Figure 2.

c) Whenever K is intermediate per Lemma 2 such that only one national agency invests to become full-service in period four, the first national agency would most of the time also become full-service in period three.

One way to understand part a) is as an illustration of the importance of the economies of scale in improving advertising quality: national agencies facing a full-service national competitor invest the fixed cost more often than local agencies facing the same competitor because the same investment by national agencies generates double the benefit. The proposition confirms that this effect can outweigh the increased price competition that arises when a second national competitor arrives on the scene.

Part b) of Proposition 2 compares two duopolies - local and national - and documents that the economy of scale can be so large that no investment occurs in the local duopoly while everyone invests in the national duopoly.

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Finally, part c) documents that the upper boundary of investment by a single national agency is approximately the same as the upper boundary of only one of two national agencies investing in equilibrium. Together with part a), this means that the differentiated outcome (whereby only one agency becomes full-service) happens less often in the national agency duopoly than when a singla national agency faced local competitors. As in the previous proposition, this last statement is a bit vague because the upper boundaries of the regions in which only one national agency invests in periods three and four nearly but not exactly coincide.

Figure 2: Two national agencies both invest more often to become full service than local agencies, but a differentiated outcome with only one agency investing remains dominant

$$\frac{\Delta}{60}(55 + 70\Delta + 20\Delta^2 + 8\Delta^4)$$

$$\frac{\Delta}{15}(15 + 20\Delta - 20\Delta^3 + 12\Delta^4)$$

$$\frac{\Delta}{6}[3 + \Delta(3 - \Delta)]$$

$$\frac{\Delta}{15}(15 - 20\Delta + 20\Delta^3 - 12\Delta^4)$$

$$\frac{\Delta}{120}(65 - 70\Delta - 10\Delta^2 + 50\Delta^3 - 18\Delta^4)$$

Figure 3: Expected agency profits over time
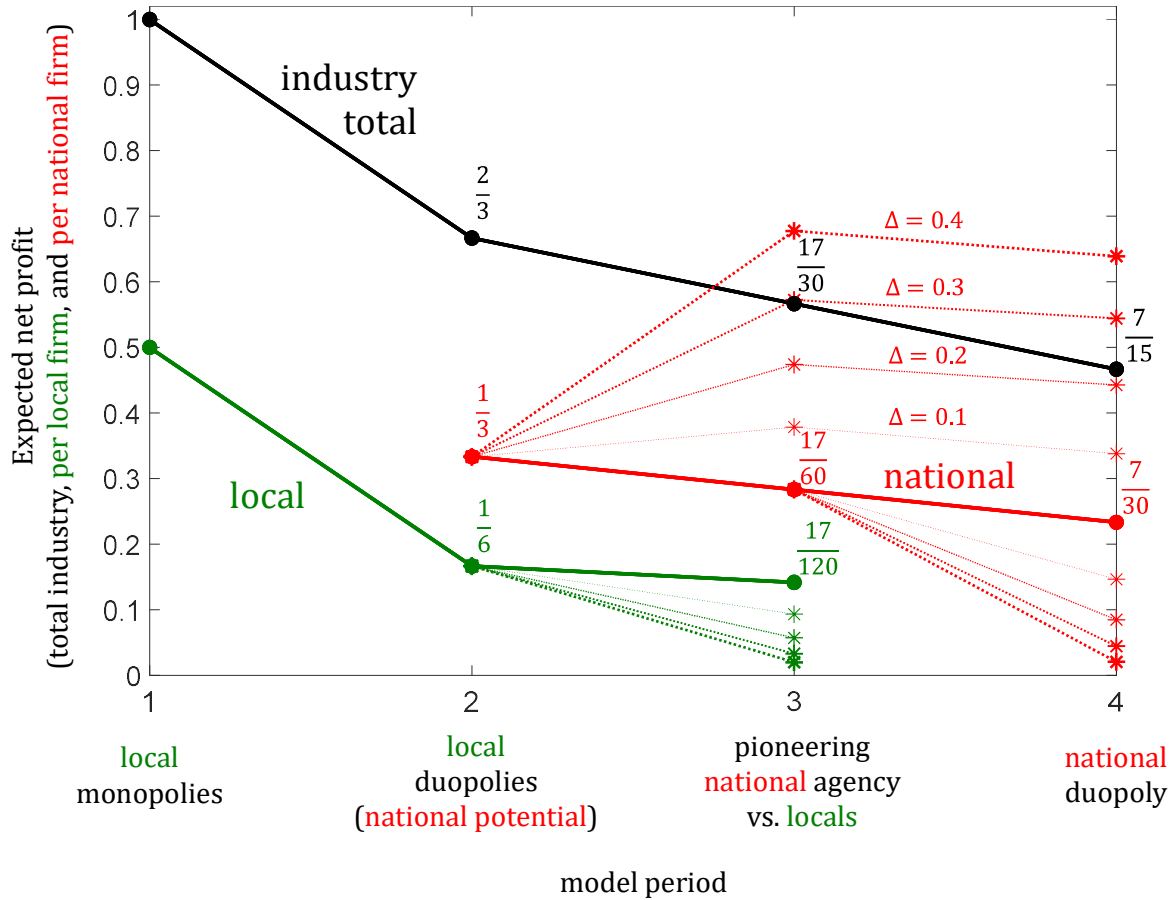
Note to Figure: Solid lines connect datapoints marked with circles corresponding to no quality differentiation (either all middlemen or both full-service in period 4). The black line indicates the total industry profits without quality differentiation. The green solid line connects the profits of local agencies when there is no differentiation. The red solid line connects the profit of national agencies when there is no differentiation (including the period-2 pair of local agencies that have national potential and become the pioneering national agency in period 3). The dashed lines connect profits of agencies when differentiation occurs (and include one prior period for reference). The thickness of the dashed lines corresponds to the Δ parameter as indicated. The K parameter is selected to be half way between $\frac{\Delta}{15}(15 - 20\Delta + 20\Delta^3 - 12\Delta^4)$ and $\frac{\Delta}{60}(55 + 70\Delta + 20\Delta^2 + 8\Delta^4)$, which ensures differentiation in equilibrium of both period 3 and period 4.

Collusion incentives

The price competition at the national level in the last period of the model is very intense, as captured in our next result:

Proposition 3: Whenever only one national agency switches sides to become full-service in the last two periods, it earns less in the fourth period than it did in the third period. When both national agencies switch sides in the last period, they earn less than the pioneering full-service agency earned in the third period.

A visual description of the essence of Proposition 3 is that in Figure 3, all lines are downward sloping between period 3 and period 4. Moreover, we have already shown that whenever the pioneer agency does not switch sides in the third period, it wins more often but earns less than it did in the second period (the solid red line is downward sloping between period 2 and 3). In other words, the pioneer national space middleman would be better off if it could somehow commit to skip the national bidding round, and compete only at the local level.

The intuition for Proposition 3 is that the full-service national agencies bid for the entire business of national advertisers using the agency average cost across all markets, which in turn blunts any local cost advantages the national agency may have. So the full-service national agencies look back fondly on the second period, in which they were merely local middlemen dealing in space. They do not even have to look backward in time because both historically and within our model, national agencies continue to bid for local business without providing added-value services. The local markets thus provide a clear focal point for collusion: the national agencies merely have to agree to charge national advertisers the same prices they are used to receiving in the local market. And since the prices in local markets are commissions of space-dealing middlemen, both the idea of receiving a commission and the level of the commission get perpetuated into compensation of full-service agencies working at the national level. If we take period 2 profits as the benchmark colluding agencies are trying to accomplish, we get the following Corollary to Proposition 3:

Corollary: National agencies prefer to collude on the commissions available in local transactional markets under the following conditions:
   a) When K is low enough relative to Δ that both national agencies become full service in the last period, or high-enough that both remain as space middlemen.
   b) When K is intermediate relative to Δ such that only one national agency becomes full-service in the last period and $K > -\frac{1}{10} + \frac{\Delta}{15}(15 + 20\Delta - 20\Delta^3 + 12\Delta^4)$

The a) part follows immediately, and the b) part follows from the comparison of the profit of the lone full-service agency and the period-2 profit aggregated to the national level, i.e. 1/3. Figure 4 shows the resulting region of the parameter space along with the two dashed blue lines from Figure 2 that delineate the three possible market structures in period 4. One way to interpret the figure is that collusion occurs under most parameter settings, and always when costs of ad development are low relative to the benefits. However, a region remains in which the one agency that becomes full-service in the last period prefers its profit to colluding. As it happens, Figure 3 illustrates several such possibilities.

Figure 4: When national agencies prefer to collude on commission levels from local markets
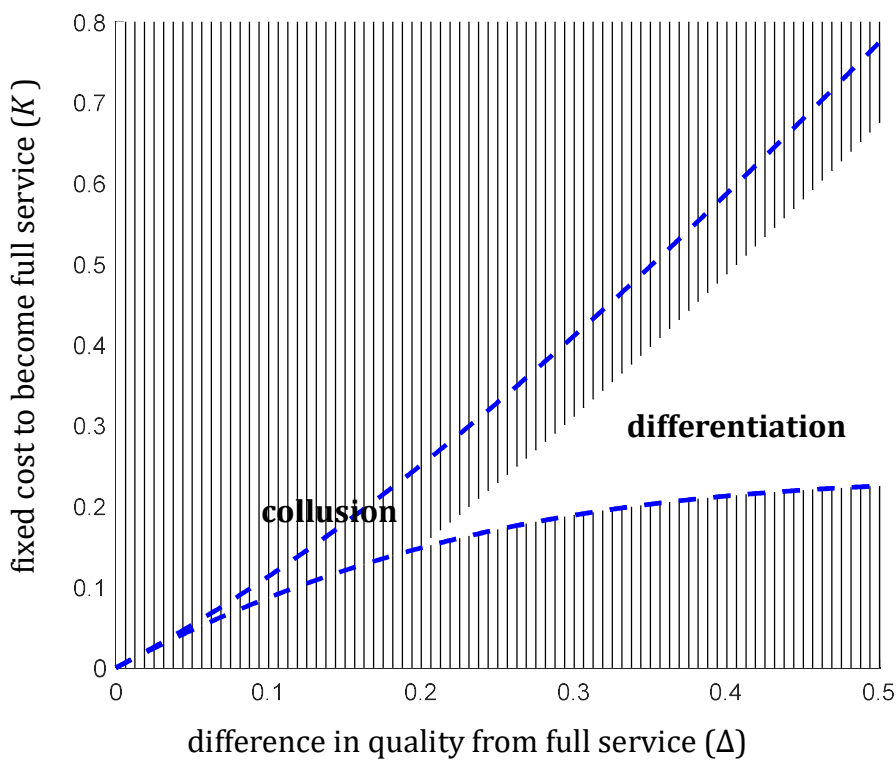
Possible extensions

Regarding the number of competing agencies, it is easy to extend the situation in period four to more than two national competitors, but analyzing a mixed two-stage competition between multiple national competitors and some local-only competitors (a natural extension of the situation in period three) is not feasible using the integration approach developed here. There are simply too many moving parts to integrate over in the second stage of the competition when no national player wins the first round.

The model also leaves out two historical features of the full-service agency contracts: their longer-term nature and the pledge not to serve multiple close competitors in the same industry. The longer-terms nature of actual full-service contracts allowed the agencies the "open contract" flexibility of exactly when and exactly where to place the ads, presumably reducing the expected cost of doing so. If this expected cost-reduction could be treated as a constant, it would be equivalent to an increase in the quality difference Δ withing our model. If this expected cost-reduction took the form of national agencies drawing their costs from some other distribution than the distribution of their local costs, the model would have to be re-solved and tractability may break down. However, we do not anticipate that small changes in the distributional assumptions would lead to large changes in the qualitative insights, such as the intense competition in the national market, the agencies switching sides in response to the emergence of national advertisers, or the perpetuation of commission compensation via co-existence of local and national markets.

The pledge of a full-service agency not to serve multiple close competitors can be accommodated within our model as the third period continuing even with two national agencies A and B: when the newly arriving national advertiser is a competitor of the one already served by agency A, only agency B will bid for the contract.

Discussion

This paper provides one possible explanation why buyer's agents are often compensated with a commission despite the non-aligned incentives such a scheme entails in the relationship between the agents and their principals. We focus on the case of advertising agencies, which were compensated with 15% of media billings for over a century even though they were working on creative ideas for the advertisers. Our explanation centers on the agencies starting as ad-space salesmen employed by publishers, evolving into middlemen dealing in ad space, and then switching sides to work for advertisers instead of the publishers. The key aspect of the switch for the persistence of commissions is that it did not occur instantaneously for all types of agencies and advertisers: only large "national" agencies serving national advertising accounts switched, while smaller "local" agencies serving smaller accounts continued to operate as middlemen competing on price. We model the asymmetric competition among such heterogeneous agencies as a scoring auction, and show how the local competitors effectively set the price at which the national agencies win their contracts with the national advertisers. In other words, we propose that the commission compensation was perpetuated through competitive bidding (i.e. "pitching" in advertising agency language) from exclusive salesmen to space middlemen to full-service agencies. Our model

endogenizes the market asymmetry by showing that heterogeneity in business models can emerge in equilibrium, with some agencies switching sides and others symmetrically situated ones remaining in the middleman role. The intuition for this result is the basic principle of vertical differentiation ala Shaked and Sutton (1982), but our differentiation outcome does not require heterogeneity in the customer willingness to pay for quality. Instead, the two-stage nature of competition among agencies – starting at the national level, and following up at the local level whenever no national contract emerges – guarantees demand to the firm that does not invest in quality.

The fact that the agencies switched sides and started working for the advertisers at the end of the 19th century is only one piece of the puzzle that explains the long-run persistence of commissions for most of the 20th century. Once enough agencies switched sides, why didn't a more rational compensation scheme evolve through competition among them? The second piece of the puzzle we document is the recognition system—a collusive arrangement between the agencies and the publishers that fixed the commission amount at 15% and erected barriers to entry into the agency industry. Our model characterizes the conditions under which the agencies first voluntarily switch sides and then want their old commission compensation back. We show that switching the locus of competition from the local level to the national level intensifies the downward pressure on prices. So why would agencies first switch sides and then demand their old compensation back? Because the emerging national advertisers demanded national-level bids to benefit from the economies of scope involved in a single contract reaching multiple newspapers.

While our paper is laser-focused on the compensation of advertising agencies, other industries also involve buyer's agents receiving a commission. We know of at least three examples: real estate, life insurance, and commercial trash hauling. In several of these industries, at least some components of our explanation for the persistence of commissions seem to apply. For example, the recognition system we describe has a parallel in the U.S. real estate industry, where the National Association of Realtors (NAR) has managed to sustain a collusive system that also involves a fixed commission and barriers to entry (Levitt, Syverson, and Ferreira 2008). Analogously for advertising agencies receiving a seemingly misguided commission on media billings, buyer agents in the U.S. receive a fixed commission on the price their client pays for the house. Our model fits one aspect of the situation, namely the fact that real-estate agents started by representing sellers and only added buyer-side representation after the collusive Multiple Listing Service (MLS) was introduced. The NAR describes the MLS: "In the late 1800s, real estate brokers regularly gathered at the offices of their local associations to share information about properties they were trying to sell. They agreed to

compensate other brokers who helped sell those properties, and the first MLS was born, based on a fundamental principal that's unique to organized real estate: Help me sell my inventory and I'll help you sell yours."[9] Besides not being able to spell "principle," the NAR also signals the fundamental difficulty of compensating and motivating buyer's agents—agents in charge of buying (spending money) rather than selling (getting revenue)—a difficulty both the real estate and the advertiser agents share. Conceptually, the classic principal-agent results should apply on that side of the market as well, giving the buyer's agents an incentive to save money. However, such contracts are not common, at least to the extent of our knowledge. Instead, buyer's agents receive fixed commissions on the money spent or flat fees—both poorly aligned with the goals of the buyer. We hope our paper stimulates future research into the effective contracts for buyer's agents in both of these important markets and beyond.

Another important domain where buyer's agents receive a commission from the seller is the life insurance market in the U.S. Instead of buying from insurance "agents" who exclusively represent a single insurance company, buyers can also work with insurance "brokers", who give information about offerings from multiple companies and receive a percentage commission on policies they "recommend". This situation is analogous to the emergence of full-service advertising agencies: as independent middlemen, insurance brokers are clearly trying to switch sides and work for the buyer, but they remain compensated by a percentage commission because buyers can always get a quote from the insurance agents. Our model can likely contribute to the understanding of commission persistence in the life insurance market as also arising from price competition among heterogeneous agents.

Finally, Salz (2022) describes the New York City private trash-hauling market, in which broker intermediaries, who "*allow customers, who often operate on a national scale, to have a one-stop shop for dealing with the fragmented landscape of waste removal services.*" and are compensated by a large percentage commission on the business they arrange. Why would a business go through a broker instead of contacting the trash hauler directly? Salz shows that customers with high search costs use the brokers, and pay them a premium to save on search costs. As in our model, heterogeneous customers compete for business, and the result is a percentage commission to the brokers. One aspect of New York City trash-hauling that differs from our context is that the brokers are barred from accepting a commission from the trash hauling companies, presumably to prevent corruption. The three examples of industries in which buyer's agents receive a percentage

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[9] https://www.nar.realtor/nar-doj-settlement/multiple-listing-service-mls-what-is-it, accessed 7/17/2021. The NAR is currently being investigated by the DOJ for anti-competitive practices.

commission illustrate both the importance of our research question and the potential impact of our model beyond the domain of advertising agencies.

Our findings also have implications for today's advertising market, which is undergoing a rapid shift to programmatic digital display advertising, with spending on display ads surpassing the spending on search ads since 2016. Intermediaries involved in programmatic advertising (e.g., media agencies, platforms, ad exchanges, and other aggregators) are compensated with a commission on the money passing through them. Often called "tech fees," these commissions are ultimately paid by the advertiser (Beals and Elliott 2019). The Association of Nation Advertisers (ANA) recently conducted several member surveys about "media agency compensation" and discovered both percentage commissions and a lack of transparency in the way media agencies and other intermediaries spend the advertising budgets (Association of National Advertisers, Inc. 2013,2016, Beals and Elliott 2019). So, history is repeating itself, and advertisers are compensating their agencies on a commission basis. Much like the newspaper advertising market of the latter half of the 19th century, the programmatic display advertising market is currently in flux, evidenced by a lack of standardization and rapid changes in both the major players and market rules (Beals and Elliott 2019, Choi et al. 2020). As the market matures in the coming years, our findings suggest some major intermediaries will seek to collude with the publishers and retain the current lucrative billings-based compensation. Going forward, advertisers and regulators need to be careful about blindly embracing any sort of association of "certified" or "recognized" media agencies in the programmatic space, and they should be skeptical about any "standard contract" such an association promotes. Will the government have to step in to break up such an arrangement? Will some of the intermediaries eventually switch sides to work more explicitly for the advertisers, and start exclusively providing value-added creative services to large national or even global brands?

Appendix: proofs of propositions

Proof of Proposition 1: The proof is mostly in the text. The last statement is that the absolute value of the difference between the bottom two lines in Figure 1 is $\left|\frac{\Delta}{120}(-5 + 10\Delta + 30\Delta^2 - 50\Delta^3 + 18\Delta^4)\right|$, and a simple plot reveals that this absolute value is below $1/100$ for all $\Delta < \frac{1}{2}$.

Proof of Proposition 2: There are three claims, and they can be shown as follows:

a) For every $\frac{1}{2} > \Delta > 0$, if K is low-enough that the local agencies facing a single full-service national competitor invest in quality, then so does the second national agency. But for every $\Delta > 0$, there is also a set of K, namely $\frac{\Delta}{120}(65 - 70\Delta - 10\Delta^2 + 50\Delta^3 - 18\Delta^4) < K < \frac{\Delta}{15}(15 - 20\Delta + 20\Delta^3 - 12\Delta^4)$, in which both national agencies invest but local agencies facing a single full-service national competitor do not.

b) $\frac{\Delta}{15}(15 - 20\Delta + 20\Delta^3 - 12\Delta^4) > \frac{\Delta}{120}(65 - 70\Delta - 10\Delta^2 + 50\Delta^3 - 18\Delta^4)$ because the difference is $\frac{\Delta}{120}(55 - 90\Delta + 10\Delta^2 + 110\Delta^3 - 78\Delta^4)$, which is increasing for all $\frac{1}{2} > \Delta > 0$.

c) The difference between the two uppermost lines in Figure 2 is $-\frac{1}{12}\Delta(1 + 2\Delta - 4\Delta^2 - 16\Delta^3 + 8\Delta^4)$, which remains within about 0.02 in terms of absolute value.

Proof of Proposition 3: Suppose K is low-enough that both national agencies operating in the fourth period switch sides to become full-service, and each thus earns $\frac{7}{30} - K \approx 0.233 - K$. This profit is certainly less than $\frac{17}{60} + \frac{\Delta}{60}(55 + 70\Delta + 20\Delta^2 + 8\Delta^4) - K$ the pioneer full-service agency received in the third period because

$$\frac{17}{60} + \frac{\Delta}{60}(55 + 70\Delta + 20\Delta^2 + 8\Delta^4) - K > \frac{7}{30} - K$$

$$\iff \frac{17}{60} + \frac{\Delta}{60}(55 + 70\Delta + 20\Delta^2 + 8\Delta^4) - \frac{7}{30} = \frac{1}{20} + \frac{\Delta}{60}(55 + 70\Delta + 20\Delta^2 + 8\Delta^4) > 0$$

Interestingly, the profit is also (much) lower than that received back in the day of local competition when the agency with national potential did not spend K and earned $2 \times \frac{1}{6} = \frac{1}{3} > \frac{17}{60} > \frac{7}{30} > \frac{7}{30} - K$.

Now suppose K is intermediate such that only one national agency switches sides to become full-service in the last two periods. Then, the full-service national agency earn less in the fourth period than in the third period.

$$\frac{17}{60} + \frac{\Delta}{60}(55 + 70\Delta + 20\Delta^2 + 8\Delta^4) - K > \frac{7}{30} + \frac{\Delta}{15}(15 + 20\Delta - 20\Delta^3 + 12\Delta^4) - K$$

$$\iff \frac{17}{60} + \frac{\Delta}{60}(55 + 70\Delta + 20\Delta^2 + 8\Delta^4) - \frac{7}{30} + \frac{\Delta}{15}(15 + 20\Delta - 20\Delta^3 + 12\Delta^4)$$

$$= \frac{1}{20} - \frac{\Delta}{12}(1 + 2\Delta - 4\Delta^2 - 16\Delta^3 + 8\Delta^4) > 0$$

where the last inequality is easy to confirm numerically.

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Web appendix: Detailed derivations of results:

Local advertiser: Competition between agents/middlemen and full service agency

The full-service agent wins more than half of the contests, and receives an expected payoff of:

$$\int_0^{\min(\beta+\Delta,1)}(\beta+\Delta-\gamma)\,d\gamma - K = \begin{matrix} \beta+\Delta < 1: \frac{(\beta+\Delta)^2}{2} - K \\ \beta+\Delta > 1: \beta+\Delta-\frac{1}{2} - K \end{matrix}.$$

So ex ante: $\int_0^{1-\Delta}\frac{(\beta+\Delta)^2}{2}\,d\beta + \int_{1-\Delta}^1 \beta+\Delta-\frac{1}{2}\,d\beta = \frac{1}{6}+\frac{\Delta}{6}[3+\Delta(3-\Delta)]$

The lower-quality agent is derived analogously: the agent wins when $q_1-(1-\beta) < q_0-(1-\gamma)$, i.e. when $q_1-q_0+\beta < \gamma$. To ensure the winner's payoff required by the scoring auction rules, namely $[q_0-(1-\gamma)]-[q_1-(1-\beta)]$, advertiser pays $(1-\beta)-\Delta$ when the lower-quality agent wins. The expected payoff of such an agent is

$$\int_0^{\max(\gamma-\Delta,0)}(\gamma-\Delta-\beta)\,d\beta = \begin{matrix} \gamma < \Delta: 0 \\ \gamma > \Delta: \frac{(\gamma-\Delta)^2}{2} \end{matrix}.$$

So ex ante: $\int_\Delta^1 \frac{(\gamma-\Delta)^2}{2}\,dy = \frac{(1-\Delta)^3}{6}$

Summary of the game: With equal quality, both make $\frac{1}{6} \approx 0.166$. So both invest in quality when K is low enough that the difference between $\frac{1}{6}$ and the profit of the non-investing agency,

, namely $\frac{1-(1-\Delta)^3}{6}$ is more than K. Otherwise, when $\frac{1-(1-\Delta)^3}{6} < K < \frac{\Delta}{6}[3+\Delta(3-\Delta)]$

, only one of the agencies invests. Finally, if $K > \frac{\Delta}{6}[3+\Delta(3-\Delta)]$, neither agency invests

National advertiser: Competition between local middlemen and national "one-stop-shop" agency

The cheaper of $(1-\beta_{L,1})+(1-\beta_{L,2})$ and $(1-\beta_{N,1})+(1-\beta_{N,2})$ wins the contract and receives

$$B < 1: \int_0^B (B-\gamma)\gamma\,d\gamma = \frac{B^3}{6}$$

$$B > 1: \int_0^1 (B-\gamma)\gamma\,d\gamma + \int_1^B (B-\gamma)(2-\gamma)\,d\gamma = \frac{1}{3}+B(B-1)-\frac{B^3}{6}$$

When the national agency loses the chance to represent the advertiser nationally, it still participates in the local contests. It obviously cannot win both of them, but can win some. Take the position of the national agency in one of the markets, let β be its cost there, and let γ be the cost of the local competitor and x be the cost of the local competitor in the other market.

Given a fixed $B<1$, β is uniform on $[0,B]$. The expected earnings in one local market should the national pitch fail are:

$$\int_0^B \frac{1}{B}\int_0^\beta \int_{B-\gamma}^1 (\beta-\gamma)\,dx\,d\gamma d\beta = \int_0^B \frac{1}{B}\int_0^\beta (1-B+\gamma)(\beta-\gamma)\,d\gamma d\beta = \frac{(4-3B)B^2}{24}$$

where the key simplification afforded by the uniform assumptions is that the pdf of (x,γ) is 1, and $B<G$ only influences the limits of integration.

Given a fixed $B>1$, β is uniform on $[B-1,1]$, so the analogous calculation yields:

$$\int_{B-1}^1 \frac{1}{2-B}\int_{B-1}^\beta \int_{B-\gamma}^1 (\beta-\gamma)\,dx\,d\gamma d\beta = \frac{(2-B)^3}{24}$$

Since there are two symmetric local markets, the total expected earnings of a national agency with $B<1$ are $\frac{B^3}{6}+\frac{(4-3B)B^2}{12}$

And, the total expected earnings of a national agency with $B>1$ are $\frac{1}{3}+B(B-1)-\frac{B^3}{6}+\frac{(2-B)^3}{12}$.

It is interesting to examine the profits a national agency could make if it somehow skipped the national round and instead only competed locally. As a function of one of the local discounts, this is just $\frac{\beta^2}{2}$. It is useful to also compute this as a function of B in one of the markets.

$$\int_0^B \frac{1}{B}\int_0^\beta \int_0^1 (\beta-\gamma)\,dx\,d\gamma d\beta = \int_0^B \frac{1}{B}\int_0^\beta (\beta-\gamma)\,d\gamma d\beta = \frac{B^2}{6}$$

Since there are two such markets, the expected profit is $\frac{B^2}{3}$.

Comparing with full competition: $\frac{B^2}{3}-\frac{B^3}{6}-\frac{(4-3B)B^2}{12}=\frac{B^3}{12}>0$

Analogously for $B>1$:

$$\int_{B-1}^1 \frac{1}{2-B}\int_0^\beta \int_0^1 (\beta-\gamma)\,dx\,d\gamma d\beta = \int_{B-1}^1 \frac{1}{2-B}\int_0^\beta (\beta-\gamma)\,d\gamma d\beta = \frac{1-B+B^2}{3}$$

Comparing with full competition: $\frac{1-B+B^2}{3}-\left[\frac{1}{3}+B(B-1)-\frac{B^3}{6}+\frac{(2-B)^3}{12}\right]=\frac{(3B-2)(2-B)^2}{12}>0$

Why is profit of the national agency lower with the first round? $B<G$, there is obviously no difference from above. When $B>G$, the national agency wins always – too often: the difference is precisely $\int_0^B 1(\beta<\gamma)(\beta-\gamma)\Gamma\,d\Gamma<0$

$$\int_0^B (B-\Gamma)\Gamma\,d\Gamma=\int_0^{\beta_1+\beta_2}(\beta_1+\beta_2-\gamma_1-\gamma_2)(\gamma_1+\gamma_2)\,d(\gamma_1+\gamma_2)$$

What is the payoff of one of the local agencies given their γ? Given a fixed γ, Γ is uniform on $[\gamma, \gamma+1]$, so:

$$\int_{\gamma}^{\gamma+1} \int_0^{\gamma} \int_0^{min(\Gamma-\beta,1)} (\gamma-\beta)\, dx\, d\beta\, d\Gamma =$$

$$\int_{\gamma}^{1} \int_0^{\gamma} (\Gamma-\beta)(\gamma-\beta)\, d\beta\, d\Gamma + \int_1^{\gamma+1} \left[\int_0^{\Gamma-1}(\gamma-\beta)\, d\beta + \int_{\Gamma-1}^{\gamma}(\Gamma-\beta)(\gamma-\beta)\, d\beta\right] d\Gamma$$

$$= \frac{(6+8\gamma-3\gamma^2)\gamma^2}{24}$$

National advertiser: Competition between local middlemen and national full-service agency

The higher of $2q_0 - (1-\beta_{L,1}) - (1-\beta_{L,2})$ and $2q_1 - (1-\beta_{N,1}) - (1-\beta_{N,2})$ wins the contract and receives the difference between the total "quality-adjusted" discounts. The national agency's expected payoff from winning with a given total discount B is just a shift of the equal-quality competition. Let $\tilde{B} = B + 2\Delta$, then:

$$\tilde{B} < 1: \int_0^{\tilde{B}}(\tilde{B}-\gamma)\gamma\, d\gamma = \frac{\tilde{B}^3}{6}$$

$$2 > \tilde{B} > 1: \int_0^1(\tilde{B}-\gamma)\gamma\, d\gamma + \int_1^{\tilde{B}}(\tilde{B}-\gamma)(2-\gamma)\, d\gamma = \frac{1}{3} + \tilde{B}(\tilde{B}-1) - \frac{\tilde{B}^3}{6}$$

$$\tilde{B} > 2: \int_0^1 (\tilde{B}-\gamma)\gamma\, d\gamma + \int_1^2 (\tilde{B}-\gamma)(2-\gamma)\, d\gamma = \tilde{B} - 1$$

When the national agency loses the chance to represent the advertiser nationally, it still participates in the local contests. It obviously cannot win both of them, but can win some. Take the position of the national agency in one of the markets, let β be its cost there, and let γ be the cost of the local competitor and x be the cost of the local competitor in the other market.

Given a fixed $B<1$, β is uniform on $[0,B]$. The expected local-market earnings are thus:

$$\int_0^B \frac{1}{B} \int_{max(B+2\Delta-1,0)}^{\beta+\Delta} \int_{B+2\Delta-\gamma}^{1} (\beta+\Delta-\gamma)\, dx\, d\gamma\, d\beta$$

When $\tilde{B} < 1$, the expected earnings in one local market should the national pitch fail are:

$$\int_0^B \frac{1}{B} \int_0^{\beta+\Delta} \int_{B+2\Delta-\gamma}^{1}(\beta+\Delta-\gamma)\, dx\, d\gamma\, d\beta = \int_0^B \frac{1}{B} \int_0^{\beta+\Delta}(1-B-2\Delta+\gamma)(\beta+\Delta-\gamma)\, d\gamma\, d\beta=$$

$$=\frac{(4-3B)B^2}{24} + \frac{\Delta}{12}[B(6-15\Delta)-8B^2+2\Delta(3-5\Delta)]$$

When $\tilde{B} > 1 > B$, a small adjustment is needed because γ does not range all the way down to 0:

$$\int_0^B \frac{1}{B} \int_{B+2\Delta-1}^{\beta+\Delta} \int_{B+2\Delta-\gamma}^1 (\beta + \Delta - \gamma)\, dx\, d\gamma d\beta = \int_0^B \frac{1}{B} \int_0^{\beta+\Delta}(1 - B - 2\Delta + \gamma)(\beta + \Delta - \gamma)\, d\gamma d\beta =$$

$$= \frac{(1-\Delta)^4 - (1-B-\Delta)^4}{24}$$

Given a fixed $B>1$, β is uniform on [B-1,1], so the analogous calculation for $2 > \tilde{B} > B > 1$ yields:

$$\int_{B-1}^1 \frac{1}{2-B} \int_{B+2\Delta-1}^{\beta+\Delta} \int_{B+2\Delta-\gamma}^1 (\beta + \Delta - \gamma)\, dx\, d\gamma d\beta = \frac{(2 - B - \Delta)^4 - (\Delta)^4}{24(2 - B)}$$

Since there are two symmetric local markets, the total expected earnings of a national agency with $B < 1$ are $\frac{\tilde{B}^3}{6} + \frac{(4-3B)B^2}{12} + \frac{\Delta}{6}[B(6 - 15\Delta) - 8B^2 + 2(3 - 5\Delta)\Delta]$.

Ex ante, all this amounts to: $\frac{17}{60} + \frac{\Delta}{60}[55 + 70\Delta + 20\Delta^2 + 8\Delta^4]$

It is interesting to examine the profits a national agency could make if it somehow skipped the national round and instead only competed locally. As a function of one of the local discounts, this is just $\frac{(\beta+\Delta)^2}{2}$. It is useful to also compute this as a function of B in one of the markets.

Assuming

$$\int_0^B \frac{1}{B} \int_0^{\min(\beta+\Delta,1)} \int_0^1 (\beta + \Delta - \gamma)\, dx\, d\gamma d\beta =$$

$B + \Delta< 1$: $\int_0^B \frac{1}{B} \int_0^{\beta+\Delta}(\beta + \Delta - \gamma)\, d\gamma d\beta = \frac{B^2}{6} + \frac{\Delta(\beta+\Delta)}{2}$

$B + \Delta> 1 > B$: $\int_0^{1-\Delta} \frac{1}{B} \int_0^{\beta+\Delta}(\beta + \Delta - \gamma)\, d\gamma d\beta + \int_{1-\Delta}^B \frac{1}{B}\left(\beta + \Delta - \frac{1}{2}\right) d\beta = \Delta - \frac{1-B}{2} + \frac{(1-\Delta)^3}{6B}$

Since there are two such markets, the expected profit is double...

Analogously for $B>1$:

$$\int_{B-1}^1 \frac{1}{2-B} \int_0^{\min(\beta+\Delta,1)} \int_0^1 (\beta + \Delta - \gamma)\, dx\, d\gamma d\beta=$$

$$1 < B < 2 - \Delta: \int_{B-1}^{1-\Delta} \frac{1}{2-B} \frac{(\beta + \Delta)^2}{2}\, d\beta + \int_{1-\Delta}^1 \frac{1}{2-B}\left(\beta + \Delta - \frac{1}{2}\right) d\beta$$

$$= \frac{1 - B + B^2}{3} + \frac{\Delta}{6}\left(3B + \Delta\left(3 - \frac{\Delta}{2-B}\right)\right)$$

$$B > 2 - \Delta: \int_{B-1}^1 \frac{1}{2-B}\left(\beta + \Delta - \frac{1}{2}\right) d\beta = \frac{B - 1}{2} + \Delta$$

What is the payoff of one of the local agencies given their $\gamma > \Delta$? Given a fixed γ, Γ is uniform on [γ, $\gamma+1$], so:

$$\int_{\gamma}^{\gamma+1} \int_{0}^{\gamma-\Delta} \int_{0}^{min(\Gamma-2\Delta-\beta,1)} (\gamma - \Delta - \beta)\, dx\, d\beta d\Gamma =$$

$$\int_{\gamma}^{1+2\Delta} \int_{0}^{\gamma-\Delta} (\Gamma - \beta - 2\Delta)(\gamma - \beta - \Delta)\, d\beta d\Gamma$$

$$+ \int_{1+2\Delta}^{\gamma+1} \left[\int_{0}^{\Gamma-2\Delta-1} (\gamma - \beta - \Delta)\, d\beta + \int_{\Gamma-2\Delta-1}^{\gamma-\Delta} (\Gamma - \beta - 2\Delta)(\gamma - \beta - \Delta) d\beta \right] d\Gamma =$$

$$\frac{(6+8\gamma-3\gamma^2)\gamma^2}{24} - \frac{\Delta}{12}[8(\Delta)^3 + (10 - 24\gamma)(\Delta)^2 + 3\Delta(-1 - 8\gamma + 8\gamma^2) + 2\gamma(3 + 9\gamma - 5\gamma^2)]$$

Ex ante, all this amounts to an integral of the above from delta to 1

$$: \frac{17}{120} - \frac{\Delta}{120}[65 - 70\Delta - 10\Delta^2 + 50\Delta^3 - 18\Delta^4]$$

Summary of game: With equal quality, national makes $\frac{17}{60} \approx 0.283$ and each local makes $\frac{17}{120} \approx 0.142$. Note that locals earn less than before, and the national makes less than two locals did before. So everyone invests in quality when K is low enough that the locals do, i.e. when difference between $\frac{17}{120}$ and their profit without extra quality, namely $\frac{\Delta}{120}[65 - 70\Delta - 10\Delta^2 + 50\Delta^3 - 18\Delta^4]$ is more than K.

Otherwise, when $\frac{\Delta}{120}[65 - 70\Delta - 10\Delta^2 + 50\Delta^3 - 18\Delta^4] < K < \frac{\Delta}{60}[55 + 70\Delta + 20\Delta^2 + 8\Delta^4]$

, only the national agency invests. Finally, when $K > \frac{\Delta}{60}[55 + 70\Delta + 20\Delta^2 + 8\Delta^4]$, nobody invests.

Otherwise, the national middleman remains a middleman and there is only one full-service national agency.

National advertiser: Competition between two national agencies

Suppose both offer the same quality. Then, this is just like the first stage of the national competition without quality differentiation:

The greater of B and G wins the contract and receives

$$B < 1: \int_{0}^{B} (B - G)G\, dG = \frac{B^3}{6}$$

$$B > 1: \int_{0}^{1} (B - G)G\, dG + \int_{1}^{B} (B - G)(2 - G)\, dG = \frac{1}{3} + B(B - 1) - \frac{B^3}{6}$$

So ex ante, each player receives

$$\int_{0}^{1} \frac{B^3}{6} B dB + \int_{1}^{2} \left(\frac{1}{3} + B(B - 1) - \frac{B^3}{6} \right)(2 - B) dB = \frac{7}{30} \approx 0.233$$

Let $\tilde{B} = B + 2\Delta$. When only one of the agencies offers full service, it receives:

$$\tilde{B} < 1: \int_0^{\tilde{B}} (\tilde{B} - \gamma)\gamma \, d\gamma = \frac{\tilde{B}^3}{6}$$

$$2 > \tilde{B} > 1: \int_0^1 (\tilde{B} - \gamma)\gamma \, d\gamma + \int_1^{\tilde{B}} (\tilde{B} - \gamma)(2 - \gamma) \, d\gamma = \frac{1}{3} + \tilde{B}(\tilde{B} - 1) - \frac{\tilde{B}^3}{6}$$

$$\tilde{B} > 2: \int_0^1 (\tilde{B} - \gamma)\gamma \, d\gamma + \int_1^2 (\tilde{B} - \gamma)(2 - \gamma) \, d\gamma = \tilde{B} - 1$$

So ex ante:

$$\int_0^{1-2\Delta} \frac{\tilde{B}^3}{6} B \, dB + \int_{1-2\Delta}^1 \left(\frac{1}{3} + \tilde{B}(\tilde{B} - 1) - \frac{\tilde{B}^3}{6} \right) B \, dB$$
$$+ \int_1^{2-2\Delta} \left(\frac{1}{3} + \tilde{B}(\tilde{B} - 1) - \frac{\tilde{B}^3}{6} \right)(2 - B) \, dB + \int_{2-2\Delta}^2 (\tilde{B} - 1)(2 - B) \, dB =$$

$$\frac{7}{30} + \frac{\Delta}{15}[15 + 20\Delta - 20\Delta^3 + 12\Delta^4]$$

The agency that does not offer full service receives:

$$0 < G - 2\Delta < 1: \int_0^{G-2\Delta} (G - 2\Delta - B)B \, dB = \frac{\tilde{G}^3}{6}$$

$$G - 2\Delta > 1: \int_0^1 (G - 2\Delta - B)B \, dB + \int_1^{G-2\Delta} (G - 2\Delta - B)(2 - B) \, dB\gamma = \frac{1}{3} + \tilde{G}(\tilde{G} - 1) - \frac{\tilde{G}^3}{6}$$

So ex ante:
$$\int_{2\Delta}^1 \frac{\tilde{G}^3}{6} G \, dG + \int_1^{1+2\Delta} \frac{\tilde{G}^3}{6} (2 - G) \, dG + \int_{1+2\Delta}^2 \left[\frac{1}{3} + \tilde{G}(\tilde{G} - 1) - \frac{\tilde{G}^3}{6} \right](2 - G) \, dG =$$

$$\frac{7}{30} - \frac{\Delta}{15}[15 - 20\Delta + 20\Delta^3 - 12\Delta^4]$$

Summary of game: With equal quality, both make $\frac{7}{30} \approx 0.233$. So both invest in quality when K is low enough that the difference between $\frac{7}{30}$ and the profit of the national middleman, namely $\frac{\Delta}{15}[15 - 20\Delta + 20\Delta^3 - 12\Delta^4]$, is more than K. Otherwise, when

$$\frac{\Delta}{15}[15 - 20\Delta + 20\Delta^3 - 12\Delta^4] < K < \frac{\Delta}{15}[15 + 20\Delta - 20\Delta^3 + 12\Delta^4]$$

the national middleman remains a middleman and there is only one full-service national agency. Finally, when $K > \frac{\Delta}{15}[15 + 20\Delta - 20\Delta^3 + 12\Delta^4]$, nobody invests.