The Medium of Advertising

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October 31, 2022

Abstract

We present a model to study the role of media formats in advertising communication. A media platform using content to attract consumers must decide whether and when to expose them to ads. A consumer must decide, given her limited attention, what to pay attention to at each point in time. Advertising can deliver a product match signal stochastically while the consumer is paying attention to the ad. Based on the platform's capability to control the consumer's attention, we classify advertising formats into three basic types: static, sequential, and interactive. We show how different formats tangle with two fundamental problems in media advertising: incentive misalignment between the consumer and media platform, and the platform's inability to observe the ad signal. The analysis identifies the conditions for the difference and equivalence between different advertising formats. For interactive advertising to be attractive to both the consumer and media platform, the ad needs to be sufficiently informative. In contrast, a moderately informative ad under an interactive format is equivalent to sequential advertising and thus entails a negative externality to the consumer. We discuss the implications for the evolution and management of media formats as well as for consumer welfare.

Keywords: *advertising, media, information design, two-sided market, limited attention*

[*]The author thanks Anthony Dukes, Jiwoong Shin, Aniko Öry, and Birger Wernerfelt for helpful discussions and suggestions. This paper has also benefited from the comments of seminar participants at Peking University, Johns Hopkins University, City University of Hong Kong, Xiamen University, Yale University, and University of Science and Technology of China.

1. Introduction

Advertisements are often disseminated through a medium. The evolution of mass media to a large extent has advanced the formats of advertising. Printed advertisements emerged when printing technology was invented.[1] This advertising format has flourished for hundreds of years until the birth of electronic broadcasting technologies in early 20th century. Since then radio and television commercials had grown to be the leading advertising tools for marketers and revenue sources for media.[2] More recently, the landscape was disrupted with the arrival of Internet in late 20th century, marking the third revolution of media advertising. Digital advertising has quickly become indispensable for marketers.[3]

With the rapid growth of digital advertising, some lament that it will soon fuel the demise of traditional advertising (i.e., print and broadcast advertising). According to the 2022 CMO Survey,[4] spending on traditional advertising has declined consistently from 2012 to 2022. However, an inflection of this historical trend was observed in 2022 that traditional advertising spending was predicted to increase by 3% across different industries (Moorman et al. 2022). At the same time, the boundary between traditional and digital advertising has become increasingly blurred. On one hand, with improved technology, traditional media have achieved greater targeting accuracy. Many newspapers and television channels have become digitalized, harnessing the power of customer data while keeping the traditional formats. On the other hand, many ads that appear on-

[1]The first known movable-type printing system was developed by Bi Sheng around 1040 C.E. during the Northern Song dynasty (960-1279 C.E.). The earliest printed ad found in China is believed to appear around the same time. The known example is a print poster produced by a needle shop in Ji'nan. It is widely known that Johannes Gutenberg's introduction of mechanical movable type printing around 1439 C.E. marked the emergence of modern print media.

[2]The first paid-for radio commercial was run on the AT&T-owned New York station WEAF in 1922, promoting the apartments in Jackson Heights owned by Queensboro Corporation. The first legal television commercial was aired on NBC's WNBT-TV in 1941, and featured Bulova's watch for ten seconds.

[3]The first banner ad appeared on Hotwired.com in 1994, sponsored by AT&T.

[4]See https://cmosurvey.org/results/ for detail.

line are in no way different from traditional ads: Display ads or banners ads bear similarity to print ads on newspapers; Commercials shown on streaming platforms are similar to those on traditional televisions.

The resilience of traditional advertising coincides with the scaling back of spending on targeted digital ads by leading marketers. A case in point is Procter & Gamble, which was reported to cut more than $100 million in digital marketing spend in a quarter in 2017 (Terlep and Seetharaman 2017). Digital ads are often found to be ineffective and yield lower returns on investment than expected (Blake et al. 2015, Lewis and Rao 2015), which have become growing concerns for marketers. These observations beg for the question: What are the fundamental differences between traditional and digital advertising? How will the two continue to evolve in light of the improving (and regulated) targeting technology?

The pushback that digital advertising experiences can be explained, at least partly, by the fact that digital ads have been found to antagonize consumers the most among all types of media ads. A survey study conducted by HubSpot in 2016 reveals that people generally dislike digital ads: 73% of the responses dislike online pop-ups, 57% dislike online video ads.[5] These are in contrast with traditional ads: only 36% of responses dislike TV ads and the rate is reduced to 18% to magazine or print ads. Questions are, what causes these consumer heterogeneity? Why are digital ads less welcome by consumers (and when this is true)?

We seek to shed light on these questions with a simple model that can provide a unifying framework to understand how different media formats affect advertising communication. Advertising is a complex phenomenon with a wide variety of factors at play: to name a few, brand name, product category, ad copy, scheduling, and targeting. We therefore begin with a small set of ad elements that can help us understand the fundamental differences between various me-

[5] See detail at https://blog.hubspot.com/marketing/why-people-block-ads-and-what-it-means-for-marketers-and-advertisers.

dia advertising. To that end, we use a simple stochastic process to capture the informative role of advertising: an ad can inform a consumer of the existence of a product and whether it is a match to her. Because the ad is transmitted through a medium, at any point in time, the consumer can decide whether to pay attention to the ad or the media content or take the outside option. Due to limited attention, she cannot attend to both the ad and media content at the same time. Communication takes time. Even when the consumer is paying attention to the ad, the match signal arrives at a random time. It is likely that after a period of time, the consumer has learned nothing about the match and thus no conversion takes place. None of the parties possesses superior information about the match value *ex ante*. If the match signal arrives, only the consumer obtains it, whereas the media platform and the advertiser remain uninformed.

In a nutshell, the model features both an information acquisition problem of a consumer and a dynamic information design problem of a medium. These two jointly determine the equilibrium behaviors.

Due to the differences in the underlying technologies, media vary substantially in terms of how much control they possess over a consumer's attention. This motivates us to categorize media formats into three general types. In the first type, a media platform releases its content and ad simultaneously, and thus cannot force a consumer to pay attention to the ad. The consumer, however, can flexibly determine what to pay attention to. This format is called *static advertising*, and fits well with print media such as newspapers, magazines, and online websites with static display ads. In the second type, a media platform can choose when to expose a consumer to an ad or to content, but cannot display both at the same time. This flexibility allows the medium to show the ad exclusively at specific times, as in media like television and radio. We refer to this format as *sequential advertising*. In the third type, a media platform can release either the ad, the content, or both, at any point in time. The important feature here is that the platform can, after showing the ad for some time, "grant" a consumer an option to skip it. Essentially, such advertising has an interactive feature and thus is termed *interactive advertising*. Many online media such as video streaming platforms that allow viewers to skip ads fall into this category.

Naturally, the taxonomy captures chronologically the evolution of advertising media. More fundamentally, it underlines the important role of the ability to control consumer attention in shaping different media outcomes. One major theoretical insight is that the control capability has a deep implication for two fundamental problems in media advertising.[6] First, when making the attention decision, the consumer does not internalize the advertising benefit to the platform. Thus, the incentives of the consumer and media platform may not necessarily be aligned. Second, the platform does not directly observe the consumer's private information – whether she has obtained the ad signal.

Under static advertising, a media platform simultaneously introduces its content and an ad to a consumer, who then decides how to allocate her attention. Paying attention to the ad activates the signal arrival process. The consumer then needs to decide whether to pay attention to it and for how long. This is an infinite-horizon optimal stopping problem and the optimal solution to it is a simple threshold strategy. The consumer pays attention to the ad whenever the content value is below the threshold λv_c, where λ captures how soon the match signal arrives (i.e., the informativeness of the ad) and v_c is the consumer's expected surplus conditional on a match. She pays attention to the ad until the signal arrives. Introducing advertising is optimal to the platform only if the consumer has the interest to pay attention to it. In this case, the incentives of the consumer and media platform are aligned. However, for a relatively less informative ad, their incentives are not aligned. In the lack of control capability, the platform may lose advertising revenue.

Under sequential advertising, a media platform can expose a consumer to an ad or its content sequentially. Although the platform is equipped with greater power to control the consumer's attention, the lack of private information (i.e., the arrival of ad signal) has led the platform to impose a deterministic time

[6]We thank the anonymous reviewer for pointing this out.

window for the ad exposure. The consumer then faces a *finite-horizon* control problem, which in general is intractable. Owing to the exponential arrival assumption, the solution becomes tractable, which is the same as the one for the infinite-horizon counterpart. Anticipating the consumer's behavior, the platform's optimal policy is shown to be a simple function of the expected match values of advertising, the content value to the consumer, and the informativeness of the ad. If the ad is moderately informative, unlike the case in static advertising, the platform is still able to profit from advertising despite the misalignment of incentives.

Unlike static advertising, where the platform has "too little" control, and sequential advertising, where the platform has "too much" control, interactive advertising finds a sweet spot in between. In this case, a media platform can simultaneously offer an ad and its media content, giving the consumer the flexibility to choose between them. As a result, the platform needs to decide whether and when to introduce this option. If the ad is not too informative, then the consumer has no interest in paying attention to the ad even if it is available. This forces the platform to introduce a *minimum run time* to keep the consumer attended to the ad. Then the platform is faced with the same problem under sequential advertising – when to terminate the ad's run time. In contrast, if the ad is sufficiently informative, then the consumer becomes willing to pay attention to it. It is then optimal for the platform to introduce the skip option as early as possible, a situation equivalent to static advertising.

Building on these characterizations, we discuss the managerial and welfare implications in Section 4. First, as expected, an increase in the informativeness of an ad can always (weakly) improve the profit of all types of media. This suggests that improving targeting accuracy not only benefits digital media, but also traditional media, from the perspective of media profit. Furthermore, our analysis highlights that the interactive feature has given digital advertising an additional edge over traditional advertising. This is true only when ads are sufficiently informative, since then interactive advertising can bring strictly more media profit than either the sequential or static advertising. Nevertheless, it is possible that both sequential and static advertising can do just as well as interactive advertising under certain conditions. Specifically, when the ad is highly informative, interactive advertising is equivalent to the static format. This is consistent with many practices: some skippable ads can allow consumers to skip the ad from the start. However, when the ad is moderately informative, interactive advertising becomes equivalent to sequential advertising. This explains why some skippable ads entail a positive minimum run time. In essence, our equivalence results shed light on when skippable ads should be *completely* skippable or *partially* skippable.

Second, when an ad becomes more informative, it does not always benefit consumers. Under sequential advertising, consumers can become worse off if the ad becomes more informative. This is because the more informative ad incentivizes the medium to extend the ad exposure time. The same force applies to interactive advertising as well when it is equivalent to sequential advertising (i.e. in the format of partially skippable ad). One implication of this finding is that, if targeting technology improves the informativeness of an ad, then the ad can result in greater consumer reactance. Indeed, Goldfarb and Tucker [2011] find that when intrusive online ads are more targeted, they become less effective. Our analysis further suggests that this result does not always hold: as an interactive ad becomes sufficiently more informative, it will allow consumers to skip the ad entirely and thus be less likely to trigger consumer reactance. These results together explain the consumer heterogeneity in ad annoyance across different media formats, and why digital ads, particularly those with the intrusive nature, are less welcome by consumers, even when they are targeted.

We next review the relevant literature. Section 2 sets up the model for our analysis. Section 3 characterizes the equilibrium behaviors under different media formats. Building on these characterizations, Section 4 discusses the implications for media profits and consumer welfare. In Section 5, we explore a few directions that the model can be extended. Section 6 concludes.

Related Literature

There is a vast literature on advertising, and much has been focused on the persuasive, informative, and complementary effects of advertising.[7] However, relatively few efforts have been devoted to understanding how advertising communication may be endogenously influenced by the type of media in which the ads are embedded. More recently, researchers have began to study media advertising in the context of of two-sided markets (e.g., Armstrong 2006, Rochet and Tirole 2006, Weyl 2010) – that is, media platforms seek to attract both advertisers and consumers and benefit from their interactions. Much of the focus has been on the pricing problem, particularly on how pricing policies are shaped by the cross-side externality, and whether market provision of advertising is efficient. In contrast, the focus of this paper is on the information design problem of media platforms, a problem that has received relatively little attention. At the same time, our model can micro-found the externality of advertising to consumers that has been assumed in the extant work on two-sided markets (e.g., Dukes and Gal-Or 2003, Dukes 2004, Anderson and Coate 2005, Anderson and Gans 2011).

The externality of advertising to consumers is, to a large extent, motivated by the phenomenon of ad avoidance. This has been particularly relevant for sequential advertising. Television viewers are known to use various ways to avoid commercials. For example, they can switch channels (Siddarth and Chattopadhyay 1998) or perform other tasks (Wilbur 2008). Zhou [2004] presents an early theoretical analysis on how viewers' avoidance behavior affects the choice of commercial breaks. In order to study both the frequency and length of commercial breaks, his model makes a convenient assumption that some viewers drop out of a TV program when a commercial break begins and once they leave the program, they never come back. This has led to the prediction that the TV program will never start with a commercial. We instead explicitly model a viewer's

[7] A comprehensive review here is unnecessary and beyond the scope of this paper. There are several excellent reviews with different focuses (e.g., Bagwell 2007, Renault 2015).

optimal decisions over time given the possible options at hand. Our analysis illustrates the possibility of a TV program to show an ad upfront before showing the program content. More recently, the incentive of consumers to switch away from ads has fueled the growth of ad-avoidance technologies, such as the TiVo digital video recorder and many ad-blocking softwares or mobile applications. These technologies can remove ads entirely from media platforms, posing a threat to the eco-system of the media market (Anderson and Gans 2011, Johnson 2013). One solution to the problem is to leverage the power of price discrimination by offering consumers a choice between ad-funded content and ad-free content at a higher price (Lin 2020). Our analysis of interactive advertising also suggests that leaving more control to consumers could be beneficial to a media platform.

Our theory highlights *interactivity* as one of the key features of digital advertising. Although this feature is naturally brought by the Internet technology, it has not gained much attention until recently when the so-called "skippable ads" become increasingly popular. YouTube's *TrueView* ad is a leading example. Dukes et al. [2022] present an early theoretical analysis of why and when skippable ads can be profitable to a media platform. They argue that the skippable format can encourage a platform to serve more ads, thereby exposing consumers to more beneficial transactions while not irritating them. This indirect demand-enhancing effect can eventually benefit the platform. We focus on different research questions and thus tailor a different model to answer them. In particular, we are interested in the fundamental differences among various types of media formats and how they affect consumer decision and welfare. We therefore allow for a richer media strategy space such that under interactive advertising, a media platform can determine at any point when to introduce a skip option. As a result, interactive ads (i.e., digital ads) can be either partially skippable, entailing compulsory view time just as sequential ads, or completely skippable, without forcing consumers to view an ad just as static ads.[8] Our continuous-time model provides a tractable framework for such an analysis. It further allows us to examine how the choice of media format can depend on the informativeness of an ad, which is not the focus of Dukes et al. [2022].

The optimal advertising design problem we study is also related to a traditional literature on the dynamic advertising allocation problem, which has a long history in marketing, economics, and operations research (e.g., Vidale and Wolfe 1957, Nerlove and Arrow 1962, Little 1979, Mahajan and Muller 1986). Sethi [1977] and Feichtinger et al. [1994] provide a comprehensive review of this line of research. Although our theory similarly builds on a continuous-time control problem, the approach taken is rather different. We explicitly model the interaction process between advertisers and consumers, instead of assuming an aggregate model that relates advertising spending to product sales. Thus, while the literature speaks to the problem of scheduling advertising over long periods (e.g., weeks or months), our model focuses on advertising allocation decisions for a single ad in a short period. More importantly, extant studies are largely motivated by the problem of advertising spending on television, and thus are unable to provide guideline for advertising in the digital age, which features both targetability and interactivity. The theory developed in this study fills this gap and provides insight into how various media formats influence advertising. Methodologically, the proposed theory solves both the media and consumers' control problems simultaneously, in contrast to studies that focuses on the firm's problem without a micro model of consumers.

The information design problem studied here also connects two streams of research in information economics. The first stream investigates how individuals (e.g., consumers, job seekers) actively acquire information over time, but assumes information is exogenously given. The seminal papers by McCall [1970] and Weitzman [1979] have inspired extensive work on optimal informa-

[8] In Dukes et al. [2022], skippable ads are always partially skippable, and thus they focus on comparing partially skippable ads to traditional ads in the sequential format.

tion search. The second stream focuses on how an individual (sender) persuades another (receiver) to change her actions within a symmetric information framework but assumes that receivers passively update beliefs given the information provided by senders. The static Bayesian persuasion framework was first introduced by Kamenica and Gentzkow [2011] and has subsequently been extended to dynamic settings in which senders can control the flow of information over time (e.g., Ely et al. 2015, Ely 2017). However, even in these dynamic settings, receivers are often assumed to be myopic with no dynamic incentive to acquire information. This study integrates both research streams in the analysis of a media market: media control the flow of information with the intention of influencing consumers, who in turn have dynamic incentive to acquire information.

2. The Model

We model a simple market with one media platform, one consumer, and one advertiser. The platform can produce media content M continuously over an infinite time horizon $T = \infty$, bringing informational or entertainment value to the consumer. In reality, media content can take a variety of forms, such as news articles, television programs, online videos, music streaming, news feeds on social media, etc. In general, the flow utility of the consumer's content consumption may vary over time. Without loss of generality, we assume that the consumer values the media content at a constant rate: $m(t) = m \geq 0$ for any time t.[9]

In addition to media content, the platform can expose the consumer to an ad A sponsored by the advertiser. In theory, at any time t, the full set of options for the platform's decision $d(t)$ is $\mathcal{D} = \{A, M, \{A, M\}\}$, where the subset $\{A, M\}$ implies that the platform allocates both advertising and media content

[9]Section 5 discusses an extension of the model with time-varying content.

simultaneously to the consumer.[10] In practice, however, there are technological constraints on the platform's actions. Thus, the platform's actions are often confined to a subset of $\mathcal{D}$. This observation suggests an approach that classifies advertising based on the media formats.

Definition 1 (A Taxonomy of Media Advertising Formats):

1. ***Static Advertising***: $\mathcal{D}_{Static} = \{\{A, M\}\}$. *The media platform can only run the ad and the media content simultaneously. The consumer can choose to process one of the two. Example: print media ads.*
2. ***Sequential Advertising***: $\mathcal{D}_{Sequential} = \{A, M\}$. *The media platform can run either the ad or the content in any sequence but cannot run them simultaneously. Example: television and radio commercials, online video ads.*
3. ***Interactive Advertising***: $\mathcal{D}_{Interactive} = \{A, M, \{A, M\}\}$. *The media platform can run the ad and the content either simultaneously or sequentially at any point in time. Example: online skippable ads like YouTube's "Trueview" ads.*

As will be shown in the following analysis, this taxonomy helps us to understand the fundamental differences and connections between different advertising formats. It is worth pointing out that ads on many online or digital media platforms are not necessarily interactive. For example, many websites display simple static banner ads that are combinations of text and images. YouTube not only allows advertisers to adopt the skippable Trueview ads, but it also provides the options of nonskippable in-stream ads or bumper ads. The variety of ad formats on digital media calls for a deeper understanding of the nature of them, which is precisely the focus of the main analysis.

Whenever the consumer decides to pay attention to the running ad, the ad information process is activated. This information process must be distinguished from the ad exposure process, as exposure does not guarantee that the

[10] Here, when both the ad and media content are bundled, it does not mean that the consumer will process both. Instead, she needs to choose which one to process due to limited attention, a point discussed in more detail below.

consumer will pay attention to the ad. Thus, it is useful to define the ad process time x, which is the time the consumer takes to process the ad. Note that this process time is endogenously related to the universal timeline t and hence the function $x(t)$ will be used in the analysis. The role of advertising is both to inform the consumer of the existence of the advertised product (or the advertised brand more generally) and to provide her with information about whether the product is a match to her.

This match information, or the signal, is stochastic in two dimensions. First, it arrives at a random time $\sigma \geq 0$, following an arrival distribution $F_\sigma(x) \equiv Pr(\sigma \leq x)$. To keep the analysis tractable, the signal arrival process is assumed to follow an exponential distribution with parameter λ. This parameter captures the *informativeness* of the ad: for a fixed time period, an ad with a higher value of λ is more likely to produce the signal. Second, upon arrival, there is a probability $\rho > 0$ the signal tells the consumer that there is a match, and a probability $1 - \rho$ that the signal indicates that there is no match. Conditional on a match, the one-off benefit to the consumer is $\tilde{v}_c \geq 0$ and to the advertiser is $\tilde{v}_a \geq 0$.[11] We assume that if the signal does not arrive and thus the consumer remains uninformed about the value of the advertised product, the consumer will not make a purchase.[12] To simplify notation, let $v_c \equiv \rho\tilde{v}_c$ and $v_a \equiv \rho\tilde{v}_a$ denote the expected ad values to the consumer and the advertiser conditional on the signal arrival. Unlike the signal arrival process that speaks to the efficiency of the ad, the realization of the match value captures the effectiveness of the ad. Further, the arrival of the signal and the realization of the match value are independent.[13] The setup of the information environment allows us to keep the

[11] These benefits can be interpreted as the surplus generated by the transaction between the consumer and advertiser. However, it is likely that consumers may continue to explore an advertised product before a purchase (Mayzlin and Shin 2011, Dukes et al. 2022). Then $\tilde{v}_c$ and $\tilde{v}_a$ represent the total expected value of continuing searching.

[12] This can occur if the prior belief about the product value is sufficiently low, or if the alternative way of acquiring information is too costly.

[13] This assumption avoids the complication that consumers may infer product match based on the mere fact that a targeted ad is delivered to them. See a recent analysis by Shin and Yu [2021] building on that logic.

decision problems of both the consumer and the media platform tractable.[14]

It is assumed that at any point in time the consumer can access to an outside option B, which captures any option available other than the media content or the ad. This option may generate reward (possibly stochastically) to the consumer over time. Examples of an outside option include a bathroom break while watching a television program, chatting with a friend while reading a newspaper, or checking emails while watching an online video. This outside option can be viewed as the opportunity cost of paying attention to the ad. As such, it can be interpreted as a search cost.[15] To keep the analysis simple, it is assumed that this option yields a constant reward $b(t) = b$. To avoid triviality, we assume that $b < m$. In later part of the analysis, b will be further normalized to zero.

The consumer has a unit budget of attention to allocate. At any point in time, she chooses an attention action $c(t)$ from a choice set $\mathcal{C}(t)$ that is partially determined by the media platform. Formally, $\mathcal{C}(t) = \{B, d(t)\}$. For example, if the platform runs the ad at time t, $d(t) = A$, then the consumer can choose to either pay attention to the ad (i.e., $c(t) = A$), or take the outside option (i.e., $c(t) = B$). In both cases, she does not consume the media content. If, however, $d(t) = \{A, M\}$, then she has the additional option of skipping the ad and consuming the content (i.e., $c(t) = M$).

The consumer discounts future rewards. The standard approach is to assume that the present value of a future reward at time t is discounted exponentially by a factor of e^{-rt}. Here we adopt the interpretation that the discounting is due to a random termination of the problem which, if occurs, results in no ad processing and content consumption. This stochastic approach simplifies the analysis because all of the players share the same discount factor, which leads

[14] In a more general and realistic setup, one may allow an ad to generate signals multiple times over time. However, such a model can easily become intractable. As shown later, the consumer is faced with a finite-horizon dynamic problem. In general, the optimal solution to such problems is hard to solve in closed-form.

[15] As argued in Nelson [1974], the marginal cost to a consumer looking at an ad is primarily a time cost. "This time cost will vary by the alternative use of the time used in watching the advertisements." (p.745, Nelson 1974)

to a simple formula for the optimal ad allocation.

Note that the media platform does not observe either the arrival of an ad signal or the realization of the match value. In general, it does not observe the consumer's action.[16] This information structure fits well with the reality — newspapers, magazines, television and radio producers do not know whether a particular consumer has paid attention to an ad. Given this information structure, the platform's action at time t is independent of the history of the consumer's actions prior to time t. Hence, under Model $\mathcal{D}_k$, $k \in \{Static, Sequential, Interactive\}$, the platform chooses the action $d(t) \in \mathcal{D}_k$ at time t; the consumer then selects an action $c(t) \in \mathcal{C}(t)$ based on the information she has accumulated as of time t.

An equilibrium solution requires simultaneously solving both the platform's allocation problem and the consumer's attention problem. The equilibrium behaviors, of course, depend on the media format. The next section is devoted to characterizing these equilibrium behaviors under the three different advertising formats.

3. Consumer Attention and Media Policy

3.1 Static Advertising

Under this advertising model, the consumer can decide freely whether to pay attention to the ad or to the media content. At the same time, the media platform has limited capability to divert the consumer' attention to the ad. The platform's action, once determined at the start of the game, is fixed over time. It remains to analyze the optimal strategy of the consumer given this information environment. The analysis also lays the foundation for analysis of more complex advertising models. Lemma 1 below presents the result.

Lemma 1 (Consumer Attention under Static Advertising):

[16] In the case of interactive advertising, a media platform can observe a consumer's choice if it offers her the choice of an ad or content. However, in this scenario, the observation does not change the platform's incentive.

1. *If $\lambda v_c \leq m$, the consumer pays attention to the media content only.*
2. *If $\lambda v_c > m$, the consumer pays attention to the ad first and consumes the content after the ad signal has arrived.*

PROOF: As the consumer has access to both the outside option and the media content at any decision time and $b < m$, she will never choose the outside option. The consumer problem is reduced to an optimal control problem with two options A and M. This is in fact a two-armed bandit problem with a safe arm M and an uncertain arm A.[17] The well-known solution to this problem is the Gittins Index solution (Gittins 1979). Before the arrival of the match signal, we can define the index of the ad process as a function of the state $x(t)$, which is the process time of the ad:

$$G_A(x) \equiv \sup_{t>x} \frac{v_c \int_x^t f(s)e^{-rs}ds}{\int_x^t [1-F(s)]e^{-rs}ds} = \lambda v_c. \tag{1}$$

The equality of the above equation follows from the distributional assumption that F is exponential. Thus, the index value does not depend on the process time as long as the signal has not arrived, simplifying the solution: the consumer processes the ad if only if the ad index is greater than the reward of the safe arm, $\lambda v_c > m$. Once the signal has arrived, the uncertainty is resolved and thus the consumer consumes the content thereafter. ■

If $\lambda v_c \leq m$, following the optimal strategy, the consumer expects a total value of

$$V^*_{Static} = \int_0^\infty m e^{-rt} dt = \frac{m}{r}. \tag{2}$$

As the platform extracts all the consumer surplus, its profit is $\Pi^*_{Static} = V^*_{Static} = m/r$. In contrast, if $\lambda v_c > m$, the consumer's total expected value following the

[17] The uncertain arm A defined in the model is equivalent to a *job process* in the job scheduling literature in operations research.

optimal strategy is

$$V^*_{Static} = \int_0^\infty (v_c + \frac{m}{r})e^{-rt}dF(t) = \frac{\lambda(rv_c + m)}{r(\lambda + r)}. \tag{3}$$

The media platform collects both consumer and advertiser surplus, earning

$$\Pi^*_{Static} = \int_0^\infty (v_c + v_a + \frac{m}{r})e^{-rt}dF(t) = \frac{\lambda(r(v_c + v_a) + m)}{r(\lambda + r)}. \tag{4}$$

To profit from advertising, the platform needs to ensure that the profit expressed in Equation (4) is greater than the ad-free profit m/r. This requires that $\lambda(v_c + v_a) > m$, a condition that can be satisfied as long as $\lambda v_c > m$. The optimal policy for the platform can then be summarized as follows.

Proposition 1 (Media Policy under Static Advertising): *Under Model $\mathcal{D}_{Static}$, the optimal media policy is to introduce the ad only if $\lambda v_c > m$, and offer no-ad content otherwise.*

Remark 1. For the consumer to process the ad and the platform to profit from it, the ad needs to be sufficiently informative and valuable, $\lambda v_c > m$. In this case, the incentives of both the consumer and the platform are perfectly aligned. Therefore, even though the platform does not possess the signal arrival information, it can allow the consumer to control her attention in a way that is desirable to the platform as well. However, for a slightly less informative or less valuable ad that satisfies $\lambda v_c < m < \lambda(v_c + v_a)$, the incentives of both the consumer and the platform are not aligned. This is more likely to occur as v_a becomes larger. The platform wishes the consumer to pay attention to the ad, but the latter prefers to skip it. Because the consumer has full control of her attention, the platform is unable to force her to attend to the ad and has to abandon the ad altogether. It turns out that sequential advertising introduced next addresses this problem.

3.2 Sequential Advertising

Under this class of model, media have more control over consumer attention. Specifically, at any point in time, a media platform can choose to either run an ad or offer the media content, but not both simultaneously. Consumers' attention decisions therefore depend on how the platform allocates the two. Next, we first analyze the optimal strategy of a consumer under any arbitrary policy, and then solve for the optimal media policy.

The sequential nature of the media format raises the issue of commitment problem. However, because the media platform does not observe the consumer's action, the commitment problem becomes irrelevant here. Hence, the media strategy has the simple form of history-independent allocation of advertising. Consider an arbitrary allocation policy π of a media platform that runs an ad for a total duration of $S = \int \mathbb{1}\{d(t) = A\}dt$. Note that the class of policies with the same ad duration S is quite large. A policy might specify that the ad is interrupted at some point in time and then resumes at a later point. Lemma 2 summarizes the optimal strategy of the consumer.

Lemma 2 (Consumer Attention under Sequential Advertising):

1. *If $\lambda v_c \leq b$, the consumer always chooses the outside option when the ad is running, and consumes the content whenever it is available;*
2. *If $\lambda v_c > b$, the consumer pays attention to the ad when it is running until the match signal arrives or the ad terminates, whichever comes first; she consumes the content whenever it is available.*

PROOF: Given any arbitrary allocation policy π with ad exposure time S (possibly infinite), let us consider the reduced problem π_0: the ad is run *continuously* from the start for a duration of S without interruption, followed by the provision of media content thereafter. That is,

$$\pi_0: \quad d(t) = \begin{cases} A & \text{if } t < S; \\ M & \text{if } t \geq S. \end{cases}$$

The consumer again faces a bandit problem similar to that in static advertising but with two important distinctions. First, there is a fixed time window S for the ad process. Second, the media content is not available when $t < S$ but becomes the only available option when $t \geq S$. These two features together suggest that the problem can be decomposed into two parts: the first part is an optimal control problem with $c(t) \in \mathcal{C}(t) = \{A, B\}$ for $t < S$, and the second part is simply choosing the media content, that is, $c(t) = M$ for all $t \geq S$.

The first part is a *finite-horizon* bandit problem, under which the Gittins Index strategy is not guaranteed to be optimal. However, under the assumption that the ad process follows an exponential process, the flow payoff of paying attention to advertising is non-increasing with probability one, which satisfies the *deteriorating arm* condition (Weber 1992). Then the index strategy remains optimal under a finite horizon.[18] Following the index strategy, the consumer chooses to process the ad if and only if $\lambda v_c > b$.

Note that the original problem under policy π can be viewed as introducing interruptions to the ad process given in the reduced problem π_0, while keeping the total ad exposure time S fixed. During these interruptions, e.g., when the ad is not running, the consumer's only choice is to consume the media content. These interruptions neither affect the state (process time) nor the payoff of the ad process. Thus, following the index strategy for problem π_0 is optimal for the original problem π. ■

Given the optimal strategy of the consumer, the platform can choose an optimal allocation policy to maximize profit. In this simple model, the platform can extract all the value generated by a match between the consumer and the advertiser by charging the consumer a subscription fee and charging the advertiser an advertising fee. However, the platform cannot base its fee on whether the consumer actually pays attention to the ad, because the attention is unob-

[18]Intuitively, in solving the index value as defined in Equation 1, the consumer finds it optimal to stop immediately after the next infinitesimal time interval. This implies that the time horizon appears to be irrelevant in her decision. She acts as if she is myopic despite her forward-looking incentive.

servable and thus it is not a feasible basis for a contract (Rochet and Tirole 2006). In essence, the platform is maximizing the total welfare of all parties. The following proposition summarizes the result.

Proposition 2 (Media Policy under Sequential Advertising): *Under Model $\mathcal{D}_{Sequential}$,*

1. *if $\lambda v_c \leq b$, the media platform does not run the ad; and*
2. *if $\lambda v_c > b$, the optimal media policy is to run the ad continuously from the start for a period of $S^* = \max\{\frac{1}{\lambda}\ln\frac{\lambda(v_c+v_a)}{m}, 0\}$ and to present the media content thereafter.*

PROOF: The first part follows straightforwardly from Lemma 2. To prove the second part, note that if the platform chooses A at time t, then Lemma 2 implies that its flow payoff is $(v_c + v_a)f(x(t))$. If the platform chooses M at any time, the flow payoff is always m. Note further that if the platform suspends the ad at some time t and then resumes the ad at a later time $t' > t$, the state does not change. Then the platform's problem is an optimal stopping problem: if it is optimal to choose M at time t, it will continue to do so thereafter. Hence, the optimal policy takes the form of continuously running the ad for a period of S and then running the content thereafter. The optimal S must solve:

$$\max_{S\geq 0} \Pi_{Sequential}(S) = \int_0^S (v_c + v_a)e^{-rs}dF(s) + \int_S^\infty me^{-rs}ds. \tag{5}$$

The first-order condition is given by

$$\lambda(v_c + v_a)e^{-(\lambda+r)S} - me^{-rS} = 0. \tag{6}$$

It immediately follows that the interior solution is

$$S^* = \frac{1}{\lambda}\ln\frac{\lambda(v_c + v_a)}{m}, \tag{7}$$

which is positive if $\lambda(v_c+v_a) > m$. Otherwise, the corner solution $S^* = 0$ applies. ■

Intuitively, the optimal ad exposure time S^* strikes a balance between the values generated by the consumer-advertiser match and the opportunity cost of delaying content provision to the consumer. This optimal ad exposure time is a simple function of the market parameters, allowing us to make sharp predictions about how these parameters influence the time windows for advertising. The following proposition characterizes these relationships.

Proposition 3 (Comparative Statics under Sequential Advertising): *The optimal exposure time of sequential advertising*

1. *weakly decreases with the content quality* m*,*
2. *weakly increases with the expected match values for the consumer and advertiser* (v_c, v_a)*, and*
3. *weakly increases as the ad informativeness* λ *increases if* $\lambda < em/(v_c + v_a)$*, but weakly decreases in* λ *if* $\lambda > em/(v_c + v_a)$*.*

PROOF: The first two parts are trivial given the solution in Equation 7. The third result is obtained by examining the derivative of the interior solution S^* with respect to λ,

$$\frac{\partial S^*}{\partial \lambda} = \frac{1}{\lambda^2}\big[1 - \ln \frac{\lambda(v_c + v_a)}{m}\big], \tag{8}$$

which is positive if $\lambda < em/(v_c + v_a)$, and negative if $\lambda > em/(v_c + v_a)$. ■

The first two parts of Proposition 3 are fairly intuitive. If the media content becomes more attractive to the consumer, then the platform should reduce the exposure time of the ad. However, if the total value brought about by successfully matching the consumer and advertiser is higher, then the consumer should be exposed to the ad for a longer period to increase her chance of obtaining the match signal.

The third part of the proposition illustrates the nonmonotonic relationship between the optimal exposure time for advertising and the informativeness of the ad captured by λ. Figure 1 provides an illustration. Because λ is the hazard rate of the exponential distribution, the probability of observing the signal, given no signal before time x, is $\lambda = f(x)/(1 - F(x))$, $\forall x$. If λ is very small, an increase in λ can effectively increase the chance of observing the signal. This motivates the platform to run the ad for a longer period. Conversely, if λ is sufficiently large, the ad signal arrives much sooner. In that case, it is more important for the platform to reduce the ad exposure time so that it can introduce the content sooner.

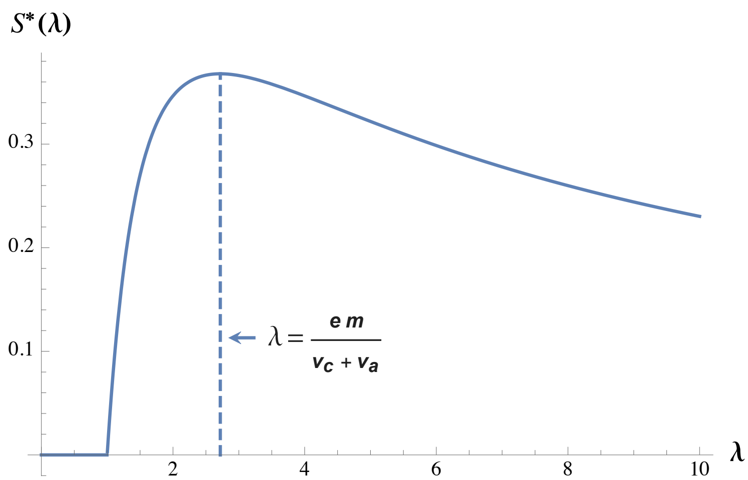

Figure 1: Optimal Ad Exposure as a Function of Ad Efficiency ($v_c + v_a = 1$, $m = 1$)

Remark 2. Recall that static advertising suffers from the incentive misalignment problem when $\lambda v_c < m < \lambda(v_c + v_a)$ (see Remark 1). This problem is alleviated under sequential advertising. In particular, the media platform now can draw the consumer's attention to the ad, even though she prefers to enjoy the media content. Yet, the ability to control the consumer's attention has an undesirable consequence: because of the lack of private information (i.e., the arrival of the match signal), the platform needs to determine the ad exposure window in an inefficient way. This problem becomes prominent for a very informative ad $\lambda v_c > m$, under which it would be optimal for the consumer to terminate the attention to the ad only when the signal has arrived. Next, we show how interactive advertising can resolve this issue.

3.3 Interactive Advertising

Under the interactive advertising model, the media platform has one additional control option $d(t) = \{A, M\}$, which allows the consumer to choose between processing the ad and consuming the media content. Essentially, interactive advertising integrates both static and sequential advertising by enabling one of them at any time t. Whenever the platform is focusing on sequential advertising, either the ad or the content is running exclusively. Again, when running the ad exclusively, the platform does not directly observe the consumer's action and thus the commitment issue is irrelevant. However, when the platform switches to static adverting, allowing the consumer to make a choice between the ad and the content, it can observe the decision outcome. For example, an online video streaming platform can allow users to skip an ad to watch a video. By clicking on the skip option, a user reveals to the platform that she prefers to watch the video content instead of attending to the ad. Observing the consumer's action opens up the possibility for the platform to condition its policy on her action whenever $d(t) = \{A, M\}$ is introduced.

This commitment problem introduces complexity to the analysis. We first draw the connection to the earlier results on the consumer's problem by assuming that the platform can commit to a policy that is independent of the consumer's decision. That is, whenever $d(t) = \{A, M\}$ is introduced, it specifies for how long this option will last before expiration and commits to it.

Lemma 3 (Consumer Attention under Interactive Advertising with Commitment):

1. *When the platform exclusively runs A or M, the consumer follows the strategy under sequential advertising in Lemma 2.*
2. *When the platform introduces the option $\{A, M\}$, the consumer follows the strategy under static advertising in Lemma 1.*

PROOF: The first part immediately follows by noticing the equivalence of the consumer problem to that under sequential advertising analyzed in subsection

3.2. The second part can be obtained by extending the result in subsection 3.1 to the finite-horizon case and following the same logic adopted in the proof of Lemma 2. ■

Note that the above result does not fully solve the consumer problem because it is restricted to the committed case. However, as we show next, this indeed is the problem that the consumer faces in an equilibrium. The following lemma suggests that the optimal media policy has a simple form that entails commitment.

Lemma 4 (Format of Media Policy under Interactive Advertising): *Under Model $\mathcal{D}_{Interactive}$, the optimal media policy takes the following form:*

1. *the media platform runs the ad A exclusively from the start for a period of S_m, followed by granting the consumer the optional control $\{A, M\}$ thereafter, and*
2. *the media platform never overturns the consumer's decision.*

PROOF: See the appendix. ■

Lemma 4 suggests that the optimal strategy is featured by a *minimum run time* S_m that forces the consumer to attend to the ad and a *skip* control that allows the consumer to skip through the ad to consume the media content immediately. Indeed, this format is widely adopted in practice. For example, YouTube's skippable video ads entail a period of five seconds that an ad must be watched and an option to skip after the five seconds. At first glance, such a practice appears puzzling: after all, the platform has already decided to let consumers decide whether to watch an ad depending on their interests. Yet, our analysis reveals why this could arise.

To understand the intuition behind Lemma 4, we need to distinguish two cases. First, if $\lambda v_c \leq m$, then the consumer prefers to consume the media content over the ad when both are available. This holds true regardless of whether or not the consumer has already obtained the ad signal prior to the introduction of the skip option. Hence, even after observing that the consumer has chosen the media content, the platform has no information about the signal arrival. The problem to the media platform reduces to an optimal stopping one: when to introduce the skip control, or equivalently, the choice of S_m.

Second, if $\lambda v_c > m$, then the consumer prefers to process the ad over the media content when both are available, as long as the ad signal has not arrived. In this case, the consumer's action can reveal the signal arrival. At the point of S_m when the platform observes that the consumer does not choose A, it infers that she has already obtained the ad signal and thus it is optimal for the platform to let her continue to consume the content. It has no incentive to force the consumer to pay attention to the ad. In contrast, when the platform observes that the consumer continues to choose A after the skip option is introduced, it learns that the signal has not arrived. It is then in the interest of both the consumer and the platform to continue processing the ad until the signal arrival. Any strategy interrupting this process is suboptimal.

Based on the simplifying results in Lemma 4, we can readily obtain the optimal media policy under interactive advertising, as summarized in the following proposition.

Proposition 4 (Media Policy under Interactive Advertising): *Under Model $\mathcal{D}_{Interactive}$,*

1. *if $\lambda v_c \leq b$, the media platform does not run the ad, and*
2. *if $\lambda v_c > b$, the optimal media policy is to continuously run the ad from the start for a minimum period of $S_m^* = \frac{1}{\lambda} \ln \frac{\lambda(v_c+v_a)}{m}$ if $\lambda v_c < m < \lambda(v_c + v_a)$, and $S_m^* = 0$ otherwise. After S_m^*, the skip option is introduced.*

PROOF: By Lemma 3, the consumer will pay attention to the ad until the signal arrival or the termination of the ad, whichever comes first. For time $t > S_m$, the consumer chooses to skip the ad and consume the media content if $\lambda v_c \leq m$. Hence, the optimal allocation policy is the same as that under sequential advertising: the media platform runs the ad exclusively for a fixed time window S_m^* as long as $m < \lambda(v_c + v_a)$, and $S_m^* = S^*$ in Equation 7.

If $\lambda v_c > m$, the consumer will continue to pay attention to the ad for any time $t > S_m$ as long as the signal has not arrived. In this case, any platform's choice of $S_m > 0$ will be strictly dominated by $S_m = 0$. To see that, suppose $S_m > 0$. If the ad signal arrives at some time $t < S_m$, then the consumer has no access to the media content and thus can only choose the outside option within the time period $[\sigma, S_m]$. This implies that the platform could have improved the profit by letting the consumer choose M within $[\sigma, S_m]$, a contradiction. Hence, it must be that $S_m^* = 0$ in this case. ■

Remark 3. Recall that static advertising features "too little control" of the platform, which gives the consumer full autonomy to decide what to pay attention to. This can better align the incentives of the consumer and the platform only if the ad is very informative, but otherwise it suffers from incentive misalignment (see Remark 1). In stark contrast, sequential advertising implies that the platform has "too much control" as it leaves only one option at a time to the consumer. While sequential advertising can alleviate the loss due to incentive misalignment if the ad is moderately informative, it inevitably leads to efficiency lost for a more informative ad because of the lack of information about the signal arrival (see Remark 2). Interactive advertising resolves the issues by striking a balance between the two extremes of control situation: if the ad is highly informative, then the platform can lift the control, handing it over to the consumer; if the ad is only moderately informative, then the platform can tighten up the control just like in sequential advertising to profit from advertising.

4. Implications

The equilibrium characterizations above reveal that different media formats can lead to different communication outcomes between advertisers and consumers. In this section, we further discuss the implications for media profitability and consumer welfare. In particular, we focus on how the informativeness of an ad can influence the outcomes.

4.1 Media Profitability and Equivalence

Because interactive advertising admits the richest set of actions for a media platform, this format clearly dominates, at least weakly, the other media formats. The more interesting question is, however, under what circumstances can static advertising and sequential advertising achieve the same media profits as the interactive format. Given the results in Section 3, the answer can be readily obtained as follows (with a graphical representation in Figure 2).

Proposition 5 (Profitability of Media Formats): *Ad informativeness always weakly increases media profit for all formats; Furthermore,*

1. *For highly informative ads,* $\lambda > \frac{m}{v_c}$*:* $\Pi_{Interactive} = \Pi_{Static} > \Pi_{Sequential}$*,*
2. *For moderately informative ads,* $\frac{m}{v_c+v_a} < \lambda < \frac{m}{v_c}$*:* $\Pi_{Interactive} = \Pi_{Sequential} > \Pi_{Static}$*;*
3. *For less informative ads,* $\lambda < \frac{m}{v_c+v_a}$*, then* $\Pi_{Interactive} = \Pi_{Sequential} = \Pi_{Static}$*.*

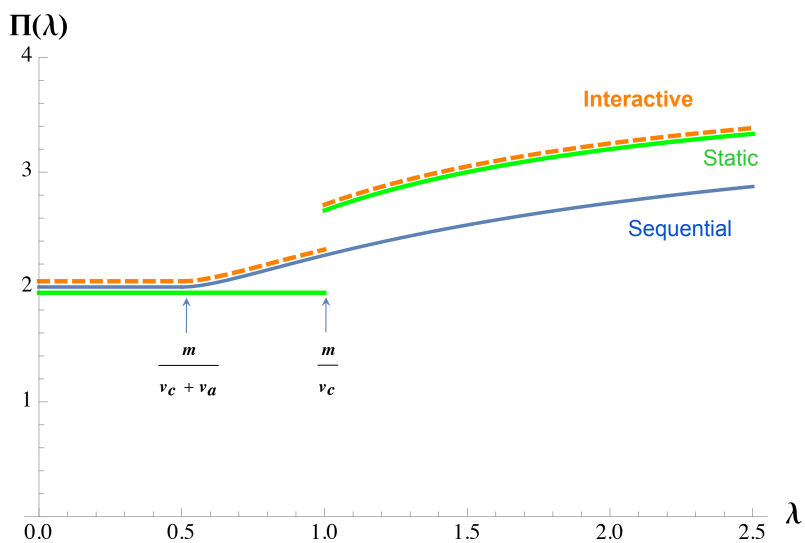

Figure 2: Media Profit as a Function of Ad Informativeness ($v_c = v_a = 1$, $m = 1$, $r = 0.5$)

If an ad is sufficiently informative, $\lambda > m/v_c$, then interactive advertising becomes equivalent to static advertising. In this case, the media platform benefits more from matching the advertiser and consumer than from profiting solely from the media content. It is optimal for the platform to let the consumer pay attention to the ad until a signal arrives. Both static advertising and interactive advertising achieve exactly that. In contrast, sequential advertising can make the platform worse off because it restricts the ad exposure time.

If an ad is moderately informative, $m/(v_c + v_a) < \lambda < m/v_c$, then interactive advertising is reduced to sequential advertising. This result immediately follows from Proposition 4: it is sufficient for the platform to let the consumer choose the media content after showing her the ad for a period of time, because she is motivated to do so anyway. However, under static advertising, granting the consumer the complete control of attention encourages her to skip the ad entirely. This approach leaves money on the table because the platform could have extracted the value from matching the advertiser and the consumer.

Last, if an ad is relatively uninformative, $\lambda < m/(v_c + v_a)$, all three advertising formats lead to the same outcome, that is, the platform finds it optimal to stop offering advertising to focus on the media content. Note that the threshold for this to occur increases with higher value of the media content, but decreases with higher match values between the consumer and advertiser. This observation provides a complementary perspective to the role of uninformative advertising as an invitation to search suggested by Mayzlin and Shin [2011]. While an uninformative ad can be employed by a high-quality firm to signal its quality, this comes at the (opportunity) cost of the media platform, which could potentially profits more from media content. Hence, to compensate the media platform, a sufficiently large v_a is necessary. That is, the condition can be satisfied even for an uninformative ad if v_a is sufficiently large.

Proposition 5 provides an alternative, but complementary, view on the superiority of digital advertising over traditional advertising such as newspapers and television. Conventional wisdom suggests that it is the ability to target specific consumer segment or individuals that has fueled the growth of digital advertising. Proposition 5 highlights that the *interactive* nature of digital media can also be an important driver. Traditional print and broadcast media lack the technological capability to engage in instantaneous two-way communication, which can help balance between media consumption and ad exposure. The interactive feature can become even more important as traditional media become increasingly equipped with targeting capability.

At the same time, Proposition 5 sheds light on when digital advertising is simply reduced to the more traditional sequential and static formats. This insight is particularly relevant in light of the observation that, in practice, skippable ads can be either *completely* skippable (i.e., ads can be skipped from the start) or *partially* skippable (i.e., ads can only be skipped after a minimum run time). Completely skippable ads are in theory equivalent to static advertising with the key feature of allowing consumers to have the full control of their attentions. This works well when an ad is sufficiently informative (relative to media content) as both the medium and consumers benefit from the efficiency. In contrast, partially skippable ads are essentially the same as sequential advertising with the feature of limiting the consumers' attention control. This occurs when an ad is neither too informative nor too uninformative, a situation where the incentives of consumers and advertisers are not entirely aligned.

4.2 Advertising Externality to Consumers

There are mixed views on how consumers perceive advertising. It is widely held that ads typically contain information about products or brands, thus facilitating consumers' decision making (Nelson 1974). This implies that consumers generally expect a positive net benefit from viewing an ad.[19] However, this argument is often made without considering the medium through which an ad is delivered. When the media environment is factored in, advertising may cause consumer resistance. Common complaints from consumers about advertising include, for example, "the ad is too distracting", "it is just wasting my time", and

[19] At a minimum, consumers do not lose much (e.g., not purchase a product) if an ad turns out to not deliver much information.

"it is not relevant/helpful at all". Hence, many analyses on two-sided media market have conveniently assumed that advertising is a nuisance to consumers (e.g., Dukes and Gal-Or 2003, Dukes 2004, Anderson and Coate 2005, Anderson and Gans 2011). A few cautiously point out that this assumption does not hold in all media formats (e.g., Kaiser and Song 2009). The annoyance effect is particularly pronounced in studies of radio and television advertisements. In print media such as newspaper and magazines, however, consumers often view advertising more positively.

This subsection provides a unified analysis to resolve these disparate views of advertising. To that end, we must be precise about what it is meant by the externality of advertising to a consumer. We note that a frequent complaint in the ad nuisance narratives is that ads distract consumers' attention from media content. Hence, the time meant for content consumption that is lost to advertising can form the basis for defining advertising externality. Let $V^0 = m/r$ denote the consumer's expected value under a counterfactual scenario, in which only the media content is available whereas no ad is shown. Let V^* denote the total value the consumer expects to gain on the platform by following the optimal attention strategy in equilibrium.[20] The externality of advertising, ΔV, is thus defined as follows:

Definition 2 (Advertising Externality): *The externality of advertising to the consumer is the difference between the expected value of the optimal attention strategy under advertising and the expected value when no advertising is present:* $\Delta V \equiv V^* - V^0$.

This definition of advertising externality is in line with the literature on media markets and advertising (e.g., Becker and Murphy 1993, Anderson and Gabszewicz 2006, Dukes et al. 2022). Under this definition, negative externality may arise when a consumer finds that the benefit of advertising falls short of the benefit of alternative use of her time. Without loss of generality, in this subsection the outside option is normalized to zero, i.e., $b = 0$. It is straightforward

[20]Note that this value will be fully extracted by the media platform.

that negative externality can hardly arise under static advertising, as stated in the following proposition.

Proposition 6 (Externality under Static Advertising): *The externality of static advertising to the consumer is always nonnegative:* $\Delta V_{Static} \geq 0$.

PROOF: If $m > \lambda v_c$, then $V^*_{Static} = V^0 = \frac{m}{r}$. If $m < \lambda v_c$, then $V^*_{Static} = \frac{\lambda(rv_c+m)}{r(\lambda+r)} > \frac{m}{r}$. ■

Proposition 6 states that there is no nuisance cost of advertising so long as the consumer follows the optimal attention strategy under static advertising. This is fairly intuitive. A consumer with full control of the decision to either consume the media content or to process an ad should not be annoyed by the availability of advertising as she can always choose to ignore it.

Turning to sequential advertising, we first note that the total value the consumer expects in equilibrium is

$$V^*_{Sequential} = \int_0^{S^*} v_c e^{-rt} dF(t) + \int_{S^*}^{\infty} m e^{-rt} dt. \tag{9}$$

Following Definition 2, the externality of advertising is measured by comparing $V^*_{Sequential}$ with the no-advertising benchmark $V^0 = m/r$. The following proposition presents the result of this comparison.

Proposition 7 (Externality under Sequential Advertising): *There exists a threshold* $\hat{\lambda}$ *such that the externality of sequential advertising to the consumer,* $\Delta V_{Sequential}$, *is positive if* $\lambda > \hat{\lambda}$, *negative if* $m/(v_c + v_a) < \lambda < \hat{\lambda}$, *and zero if* $\lambda < m/(v_c + v_a)$.

PROOF: See the appendix. ■

The key difference between Proposition 7 and Proposition 6 is that, under sequential advertising, as the media platform has more control over what the consumer can attend to, the consumer may be hurt by having advertising. While this makes intuitive sense, it is not at all obvious *a priori*, because the consumer can always choose not to pay attention to an ad. Indeed, under static advertising, the consumer is not hurt precisely because she can withhold her attention at any time.

The situation of sequential advertising is different. The ad is run in a fixed time window and because the ad signal arrives stochastically, there is always a positive probability that some time is wasted— that is, the consumer obtains the ad signal but has to wait for the media content. When an ad is very informative (i.e., $\lambda > \hat{\lambda}$), the consumer does not loss much if she pays more attention to the ad. When an ad is very uninformative (i.e., $\lambda < m/(v_c + v_a)$), it is optimal for the platform to abandon advertising. It is only when the ad informativeness is within an intermediate range that the consumer finds the ad annoying because she has to wait until S^* to enjoy the media content.

It is worth highlighting that under sequential advertising, the content value m *per se* has no direct impact on the consumer's attention strategy (recall Lemma 2 above). It only affects consumer utility indirectly through the endogenous change of the exposure time S^*, which is determined by the media platform. A similar logic holds for the match value v_a that is expected by the advertiser. Note that v_a captures the degree of incentive misalignment between the consumer and the media platform. Although v_a does not directly affect consumer's attention, it endogenously changes the ad's exposure time. That is, the consumer becomes more annoyed by the ad if the advertiser reaps more profit from the match, because in this situation the platform is motivated to extend the ad exposure time.

Corollary 1 *The externality of sequential advertising to the consumer decreases with advertiser's expected value* v_a.

PROOF: See the appendix. ■

Turning to the case of interactive advertising, we note that, given the equilibrium policy derived in Proposition 4, the consumer's expected value of following the optimal strategy is

$$V^*_{Interactive} = \begin{cases} V^*_{Static} & \text{if } \lambda > m/v_c; \\ V^*_{Sequential} & \text{if } \lambda < m/v_c. \end{cases} \tag{10}$$

It is then straightforward to characterize the advertising externality under this format of advertising. The result is presented in the following proposition:

Proposition 8 (Externality under Interactive Advertising): *The externality of interactive advertising to the consumer, $\Delta V_{Interactive}$, is positive if $\lambda > m/v_c$, negative if $m/(v_c + v_a) < \lambda < m/v_c$, and zero if $\lambda < m/(v_c + v_a)$. In addition, $\Delta V_{Interactive} \geq \Delta V_{Sequential}$.*

PROOF: See the appendix. ■

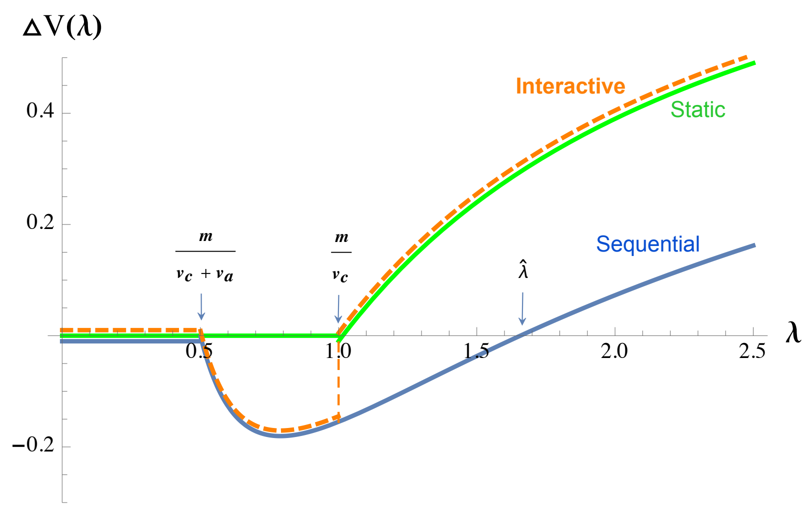

Figure 3: Externality of Advertising as a Function of Ad Informativeness ($v_c = v_a = 1$, $m = 1$, $r = 0.5$)

Figure 3 compares the pattern of advertising externality under all three advertising formats. Similar to sequential advertising, interactive advertising can also lead to a negative externality. As discussed in Section 4.1, although an interactive ad provides the skip option to the consumer, it may be only *partially* skippable with a minimum exposure time for the ad. As such, consumers may be distracted away from media consumption. Nevertheless, consumers generally expect an overall higher value under interactive advertising than under sequential advertising, *ceteris paribus*. First, the range of parameters for which negative externality occurs is reduced under interactive advertising because $\hat{\lambda} > m/v_c$ (recall that $\hat{\lambda}$ is the threshold that defines the negative externality under sequential advertising). Second, when an ad becomes sufficiently informative, i.e., $\lambda > m/v_c$, interactive advertising is reduced to the static format which allows consumers to *completely* skip the ad from the start. Thus, consumers become better off than under sequential advertising.

An interesting pattern emerging from the analysis is that the impact of ad informativeness on consumer welfare is non-monotonic under interactive and sequential advertising. In practice, one way to improve the informativeness of an ad (i.e., λ) is through increasing the targeting precision. Our result can imply that, even for a more targeted ad, consumers can become more annoyed by it if they cannot easily skip it. This is consistent with the empirical finding in Goldfarb and Tucker [2011]: if obtrusive digital ads that are hard to skip are also targeted, they can become less effective. Furthermore, our result also suggests that, if an ad becomes sufficiently informative, a digital media platform may find it optimal to let consumers decide whether to completely skip it, alleviating the problem of ad annoyance. Conversely, targeting precision may be handicapped by privacy protection policies (e.g., removing cookies that can track user browsing behaviors). This can potentially reduce the informativeness of an ad. The impact on the externality to consumers will also follow a non-monotonic pattern: it will first decrease but then increase as the informativeness drops to a certain level.

5. Beyond the Simple Model

The preceding analysis has illustrated how the simple framework developed allows us to succinctly analyze the influence of media formats on advertising communication. To make progress, as well as for expositional simplicity, we have made a number of simplifying assumptions. In this section, we briefly discuss some directions for stretching the model to accommodate other aspects of media advertising.

5.1 Time-Varying Content Value

One direction for enriching the model is to allow time-varying media content. To illustrate the general idea, let us focus on the case of sequential advertising.[21] A few additional assumptions and notations are necessary. Assume that the content process follows a Markov process captured by state $y(t)$, which is independent of the advertising process. Whenever the consumer processes the content at time t, she obtains a random reward $m(y(t))$. If the consumer freezes the content process but activates the ad process instead, then the state $y(t)$ remains unchanged.

Proposition 9 *Under time-varying media content, a media platform shows an ad whenever $\lambda(v_a + v_c) > M^*(y)$, where $M^*(y)$ is defined as the solution to the following equation:*

$$M^*(y) \equiv \sup_{\tau} \frac{\mathbb{E}[\int_0^{\tau} m(y(s))e^{-rs}ds|y(0) = y]}{\mathbb{E}[\int_0^{\tau} e^{-rs}ds|y(0) = y]}. \tag{11}$$

This result follows straightforwardly from the index policy. The ad allocation policy in the main analysis can be understood as a special case of the above result. In the more general media environment in which content value fluctuates, the optimal policy in Proposition 9 implies that a media platform may insert an ad a number of times throughout a content exposure session. This is consistent with the observation that television programs place commercials throughout the programming period.

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[21] The model here remains silent on many institutional details that may further influence media behavior. Zhou [2004] provides an early analysis on how a monopoly television network can structure commercial breaks based on the appeal of a program.

5.2 Multiple Ads

Media platforms often show multiple ads from different advertisers. The model can be readily extended to capture this practice. We again focus on the case of sequential advertising with constant media content. Suppose that there are K advertisers available in the advertising market. The k-th advertiser can gain v_{ak} and deliver an expected value of v_{ck} if its ad signal reaches the consumer. The ad process of the k-th advertiser follows an exponential process with hazard rate λ_k and is independent across ads. Again, same as the base model, v_{ak}, v_{ck}, and λ_k are common knowledge.

Proposition 10 *Under sequential advertising with multiple ads, a media platform shows the ad from advertiser k as long as $\lambda_k(v_{ak}+v_{ck}) > m$, and sequentially in the order of decreasing magnitude of $\lambda_k(v_{ak}+v_{ck})$. Furthermore, the exposure time of each ad is $S_k^* = \frac{1}{\lambda_k} \ln \frac{\lambda_k(v_{ck}+v_{ak})}{m}$.*

Proposition 10 illustrates that the decision to show an ad depends on the threshold rule that is independent across ads. This is quite natural given the independence assumption. More importantly, the proposition sheds light on how media platforms should prioritize different ads. Ads with higher index value $\lambda_k(v_{ak}+v_{ck})$ should be prioritized and shown first.

5.3 Ads with Entertainment Value

Ads often carry more than just product information. To draw attention, advertisers deliberately make ads interesting, humorous, and entertaining, and they may involve storytelling (Dukes and Liu 2020). These ads typically deliver benefits to consumers, in addition to product information. The simplest way to capture this idea is to assume a flow utility w being attached to an ad, independent of the ad's signal arrival process. Consumers are more likely to attend to an ad that offers a flow benefit, motivating media platforms to show such ads and to increase their exposure time.

Proposition 11 *Under sequential advertising, a media platform shows an ad with constant entertainment value as long as* $\lambda(v_a + v_c) > m - w$ *for a duration of* $S^* = \frac{1}{\lambda} \ln \frac{\lambda(v_c+v_a)}{m-w}$. *Within the ad exposure session, the consumer pays attention to the ad as long as* $\lambda v_c + w > b$ *and the signal has not arrived.*

Proposition 11 illustrates that as an ad contains more entertainment value, consumers become more willing to process the ad. Furthermore, the media platform becomes more willing to extend the ad exposure time, which in turn increases the chance that consumers will receive the ad signal. The complementarity between entertainment value and informational value to some extent explains why a considerable fraction of many television commercials is devoted to entertaining consumers.

6. Concluding Remarks

Despite the reliance of advertising on media to reach consumers, the influence of media formats on advertising communication has received relatively little attention in the advertising literature. We present a unifying framework that allows us to understand the fundamental differences and equivalences between various media formats. The model captures the information design problem of media platforms, which need to determine the allocation of ads and media content given that consumers determine what to pay attention to over time. Based on a platform's capability to control a consumer's attention, we can classify advertising formats into three basic types: static, sequential, and interactive. We show how different formats tangle with two fundamental problems in media advertising: incentive misalignment between the consumer and media platform, and the platform's inability to observe the ad signal. The analysis allows us to shed light on a number of issues such as how digital advertising is fundamentally different from traditional advertising, how they evolve in light of the improving (and regulated) targeting technology, and what causes the consumer heterogeneity in ad attitudes on different media.

Clearly, this preliminary study raises more questions than it answers. Nevertheless, as shown in Section 5, the model can be flexibly extended to study a host of problems. Still, more directions deserve investigation. The simple model presented here does not take into account consumer heterogeneity. An obvious area of inquiry is to look at the implications of consumer heterogeneity for advertising allocation policy, particularly when media platforms do not have the capability of targeting individuals or the targeting technologies are imperfect. As consumer heterogeneity is assumed away, pricing decisions are also nullified in this study, allowing us to focus on the information design problem. Future research could examine the interaction between pricing and information design. Another interesting direction for future studies is how multiple media may jointly affect advertising communication. This is particularly relevant when media platforms offer multiple channels or when multiple media platforms are competing for consumer attention. Last, although the consumer in our model is fully rational with forward-looking capability, it turns out that the optimal attention strategy is a myopic (or one-step look-ahead) policy. Future studies that involve complex media environments may consider assuming myopic consumers to simplify analysis without much loss of generality.

Declaration

Funding and Competing Interests. All authors certify that they have no financial or non-financial interests in the subject matter or materials discussed in this manuscript. The authors have no funding to report.

Appendix

A. Proof of Lemma 4

To prove this lemma, we first prove a set of useful intermediate results.

Lemma A.1 (Advantage of Optional Control (Part I)): *Suppose the ad signal has not arrived at time t_0, and consider an exogenous deadline T (possibly infinite). Then within the period $[t_0, T]$, running the media content exclusively is weakly dominated by offering and committing to the optional control $\{A, M\}$ throughout.*

PROOF: First, consider the case $\lambda v_c \leq m$. By Lemma 3, the consumer will always choose to consume the media content M when the optional control $\{A, M\}$ is available. Hence, the total profit the media can extract under the optional control is the same as that under running M exclusively.

Second, consider the case $\lambda v_c > m$. By Lemma 3, when the optional control $\{A, M\}$ is available within $[t_0, T]$, the consumer will first pay attention to the ad A until the signal arrival, or the deadline T, whichever comes first. There are two possible outcomes, depending on the arrival of the ad signal.

Case (a): The ad signal does not arrive before T (i.e., $\sigma > T$). In this case, the consumer pays attention to the ad throughout $[t_0, T]$. The profit accrued is then higher than the profit under running M exclusively because $\lambda v_c > m$.

Case (b): The ad signal eventually arrives before T (i.e., $\sigma < T$). In this case, the profit yielded after σ is the same across the two policies because both lead the consumer to consume M only. However, within $[t_0, \sigma]$, the optional-control policy allows the consumer to process the ad. Thus, by the same argument in Case (a), the profit is higher under the optional-control policy.

Summarizing, by committing to the optional control within $[t_0, T]$, the media platform can achieve a higher profit than running the media content exclusively throughout. ■

Lemma A.2 (Advantage of Optional Control (Part II)): *Suppose the ad signal has not arrived at time t_0, and consider an exogenous deadline T (possibly infinite). Suppose further that $\lambda v_c > m$. Then within the period $[t_0, T]$, exposing the ad exclusively for a period of S followed by running the media content exclusively is weakly dominated by offering and committing to the optional control $\{A, M\}$ throughout.*

PROOF: When $\lambda v_c > m$, by Lemma 3, under the optional control $\{A, M\}$ the consumer will first pay attention to the ad A until the signal arrival, or the deadline T, whichever comes first. Let us examine the two possible cases of signal arrival.

Case (a): The ad signal does not arrive before S (i.e., $\sigma > S$). In this case, prior to $t = S$, the consumer pays attention to the ad under both policies. After $t = S$, however, the consumer continues to process the ad until the signal arrival under the optional-control policy, whereas she will be diverted to media content under the sequential policy. Hence, starting from $t = S$, the situation is the same as that in Lemma A.1. Then, the optional-control policy dominates the sequential policy.

Case (b): The ad signal arrives before S (i.e., $\sigma < S$). In this case, prior to $t = \sigma$, the consumer pays attention to the ad under both policies. After $t = S$, however, the consumer switches to the media content under the optional-control policy, whereas she has to turn to the outside option for the period of $[\sigma, S]$ under the sequential policy. Thus, there is profit gain under the optional-control policy.

Summing up, by committing to the optional control within $[t_0, T]$, the media platform can achieve a higher profit than running the ad and media content sequentially throughout. ■

We are now prepared to prove Lemma 4.

PROOF: By Lemma A.1 and Lemma A.2, the optimal media policy entails sequencing of two components: running the ad A exclusively and running the optional control $\{A, M\}$. This is because any M-exclusive run will be weakly dominated by the optional control $\{A, M\}$. Under the optional control $\{A, M\}$, the media platform may or may not commit the course of action throughout. Next, we show that the media platform has no incentive to overturn the consumer's decision whenever the optional control $\{A, M\}$ is introduced. Take the first sequence such that the platform starts with running A exclusively for a period of S_1, followed by a period S_2 of optional control $\{A, M\}$.

Case (1): $\lambda v_c \leq m$. By Lemma 3, the consumer will keep paying attention to the ad during the period of $[0, S_1]$ unless the ad signal has arrived. If the ad signal does arrive before S_1, that is, $\sigma < S_1$, then it will be always optimal for the consumer to choose M after $t = S_1$, provided it is available. If at any point later than S_1, the platform disables the access to M, then it will incur a loss of profit. If the ad signal does not arrive before S_1, that is, $\sigma > S_1$, then the consumer does not have any incentive to process A after $t = S_1$. Given the availability of M after $t = S_1$, the consumer will consume it. Therefore, regardless of whether the signal has arrived or not, the consumer will choose M as long as it is available. Hence, by observing the decision of the consumer at the point of introducing $\{A, M\}$, the platform does not update its information thereafter. Its optimal policy is to let the consumer continue to consume the media content M. This can be achieved by either continuing to offer the optional control $\{A, M\}$ or restricting to the media content M.

Case (2): $\lambda v_c > m$. Again, by Lemma 3, the consumer will keep paying attention to the ad during the period of $[0, S_1]$ unless the ad signal has arrived. If the ad signal does arrive before S_1, that is, $\sigma < S_1$, then it will be always optimal for the consumer to choose M after $t = S_1$, provided it is available. However, if the ad signal does not arrive before S_1, that is, $\sigma > S_1$, then the consumer will continue to process the ad after $t = S_1$. If the platform wants to overturn the consumer's decision, then it can instead introduce the media content exclusively, implying the failure to commit to the optional control $\{A, M\}$ throughout $[S_1, S_2]$. However, according to Lemma A.2, this is weakly dominated by the committed optional control. Therefore, it is never optimal for the media platform to violet the commitment for any point after $t = S_1$.

The two cases taken together imply that once the platform has introduced the optional control $\{A, M\}$, it is committed to providing this option thereafter.

■

B. Proof of Proposition 7

The proof evokes a variant of Bernoulli's inequality, which is stated below without proof:

Lemma B.1 (A Variant of Bernoulli's Inequality): $(1+y)^{\theta} < \frac{1+y}{1+y-\theta y}$, *for any real numbers* $\theta > 1$ *and* $y \in (-1, 0)$.

We next present the proof of Proposition 7.

PROOF: The total discounted value a consumer expects following the optimal strategy is:

$$\begin{aligned} V^*_{Sequential} &= \int_0^{S^*} v_c e^{-rt} dF(t) + \int_{S^*}^{\infty} m e^{-rt} dt && \text{(A-1)} \\ &= \frac{\lambda v_c}{r+\lambda}(1 - e^{-(r+\lambda)S^*}) + \frac{m}{r} e^{-rS^*} && \text{(A-2)} \\ &= \frac{\lambda v_c}{r+\lambda}(1 - e^{-(r+\lambda)S^*}) + \frac{\lambda(v_c + v_a)}{r} e^{-(r+\lambda)S^*} && \text{(A-3)} \\ &= \frac{\lambda v_c}{r+\lambda} + \left(\frac{\lambda(v_c + v_a)}{r} - \frac{\lambda v_c}{r+\lambda}\right) e^{-(r+\lambda)S^*} && \text{(A-4)} \\ &= \frac{\lambda v_c}{r+\lambda} + \left(\frac{\lambda(v_c + v_a)}{r} - \frac{\lambda v_c}{r+\lambda}\right)\left(\frac{m}{\lambda(v_c + v_a)}\right)^{1+\frac{r}{\lambda}}. && \text{(A-5)} \end{aligned}$$

where the third equality (A-5) follows from the first-order condition in Equation (6). Let $\Delta V_{Sequential} = V^*_{Sequential} - V^0$, where $V^0 = m/r$. Note that, if $\lambda < m/(v_c + v_a)$, sequential advertising entails zero ad exposure, and thus the externality is zero, $\Delta V_{Sequential} = 0$. We only need to consider what happens if $\lambda > m/(v_c + v_a)$. Let us analyze two cases.

Case 1: $m/(v_c + v_a) < \lambda < m/v_c$. We shall show that $\Delta V_{Sequential} < 0$ under this case. Note that this case is equivalent to $\lambda v_c < m < \lambda(v_a + v_c)$. It would be more convenient to characterize the change of $\Delta V_{Sequential}$ with respect to m. Note that

$$\frac{\partial \Delta V_{Sequential}}{\partial m} = \underbrace{\Big(\frac{\lambda(v_c+v_a)}{r} - \frac{\lambda v_c}{r+\lambda}\Big)}_{>0}\big(\lambda(v_c+v_a)\big)^{-1-\frac{r}{\lambda}}\big(1+\frac{r}{\lambda}\big)m^{\frac{r}{\lambda}} - \frac{1}{r} < 0$$

for m sufficiently small, and

$$\frac{\partial^2 \Delta V_{Sequential}}{\partial m^2} = \Big(\frac{\lambda(v_c+v_a)}{r} - \frac{\lambda v_c}{r+\lambda}\Big)\big(\lambda(v_c+v_a)\big)^{-1-\frac{r}{\lambda}}\big(1+\frac{r}{\lambda}\big)\frac{r}{\lambda}m^{\frac{r}{\lambda}-1} > 0.$$

Together with the facts that $\Delta V_{Sequential}(m=0) = \lambda v_c/(r+\lambda)$ and that $\Delta V_{Sequential}(m = \lambda(v_c+v_a)) = 0$, it follows that, there exists a threshold $\hat{m} \in (0, \lambda(v_c+v_a))$ such that $\Delta V_{Sequential} > 0$ if $m < \hat{m}$, and $\Delta V_{Sequential} < 0$ if $m \in (\hat{m}, \lambda(v_a+v_c))$. Furthermore, we need to show that $\hat{m} < \lambda v_c$ to complete the proof of this case. It suffices to prove that if $m = \lambda v_c$, $\Delta V_{Sequential} < 0$. Indeed, following Equation A-5 we have

$$\begin{aligned}
\Delta V_{Sequential} \equiv V^*_{Sequential} - V^0 &= \frac{\lambda v_c}{r+\lambda} + \Big(\frac{\lambda(v_c+v_a)}{r} - \frac{\lambda v_c}{r+\lambda}\Big)\Big(\frac{v_c}{v_c+v_a}\Big)^{1+\frac{r}{\lambda}} - \frac{\lambda v_c}{r} \\
&= -\frac{\lambda^2 v_c}{r(r+\lambda)} + \frac{\lambda\big(\lambda(v_c+v_a)+rv_a\big)}{r(r+\lambda)}\Big(\frac{v_c}{v_c+v_a}\Big)^{1+\frac{r}{\lambda}}, \\
&< -\frac{\lambda^2 v_c}{r(r+\lambda)} + \frac{\lambda\big(\lambda(v_c+v_a)+rv_a\big)}{r(r+\lambda)} \cdot \frac{\lambda v_c}{\lambda(v_c+v_a)+rv_a} \\
&= 0,
\end{aligned}$$

where the second equality is obtained by rearranging terms and the inequality follows from Lemma B.1 by noting that $y = -v_a/(v_a+v_c)$ and $\theta = 1 + r/\lambda$.

Case 2: $\lambda > m/v_c$. We shall show that under this case, there exists $\hat{\lambda}$ such that, $\Delta V_{Sequential} < 0$ if $\lambda \in (m/v_c, \hat{\lambda})$, but $\Delta V_{Sequential} > 0$ if $\lambda > \hat{\lambda}$. This can be proved by demonstrating that (a) $\Delta V_{Sequential} < 0$ at $\lambda = m/v_c$, (b) $\Delta V_{Sequential} > 0$ at $\lambda \to \infty$, and (c) $\partial \Delta V_{Sequential}/\partial\lambda > 0$. Note that (a) has been proved above already. (b) is easily checked by taking $\lambda \to \infty$ under Equation A-5. The last step is to verify (c). Note that

$$\frac{\partial \Delta V_{Sequential}}{\partial \lambda} = \frac{(\frac{\lambda(v_c+v_a)}{m})^{-1-\frac{r}{\lambda}}}{\lambda(1+\lambda)^2} \cdot H$$

where

$$\begin{aligned} H =& v_c r \lambda (\frac{\lambda(v_c+v_a)}{m})^{1+\frac{r}{\lambda}} - \big(v_a r^2 + (v_c + 2v_a) r\lambda + v_a \lambda^2\big) \\ &+ (r+\lambda)(v_a r + (v_a+v_c)\lambda) \ln \frac{\lambda(v_c+v_a)}{m}. \end{aligned}$$

It suffices to show that $H > 0$ for $\lambda > m/v_c$, or equivalently, $m < \lambda v_c$. Note that $\partial H/\partial m < 0$. The only thing left to show is $H(m = \lambda v_c) > 0$. Indeed,

$$\begin{aligned} H|_{m=\lambda v_c} =& v_c r \lambda (\frac{v_c+v_a}{v_c})^{1+\frac{r}{\lambda}} - \big(v_a r^2 + (v_c + 2v_a) r\lambda + v_a \lambda^2\big) \\ &+ (r+\lambda)(v_a r + (v_a+v_c)\lambda) \ln \frac{(v_c+v_a)}{v_c} \\ >& v_c r \lambda (\frac{v_c + v_a + \frac{r}{\lambda} v_a}{v_c}) - \big(v_a r^2 + (v_c + 2v_a) r\lambda + v_a \lambda^2\big) \\ &+ (r+\lambda)(v_a r + (v_a+v_c)\lambda) \ln \frac{(v_c+v_a)}{v_c} \\ >& v_c r \lambda (\frac{v_c + v_a + \frac{r}{\lambda} v_a}{v_c}) - \big(v_a r^2 + (v_c + 2v_a) r\lambda + v_a \lambda^2\big) \\ &+ (r+\lambda)(v_a r + (v_a+v_c)\lambda) \frac{v_a}{v_c+v_a} \\ =& \frac{(r+\lambda) r v_a^2}{v_a+v_c} \\ >& 0. \end{aligned}$$

The first inequality makes use of the Bernoulli's Inequality in Lemma B.1 by taking again $y = -v_a/(v_a+v_c)$ and $\theta = 1 + r/\lambda$. The second inequality follows from the standard logarithm inequality, i.e., $\ln(1+z) \geq z/(1+z)$ for $z > -1$.

$\blacksquare$

C. Proof of Corollary 1

PROOF: Note that V^0 is independent of v_a. Thus, the impact of v_a on $\Delta V_{Sequential}$ is the same as its impact on $V^*_{Sequential}$. Using the expression of $V^*_{Sequential}$ in Equation A-4 and taking its derivative over v_a, we have

$$\begin{aligned}
\frac{\partial \Delta V_{Sequential}}{\partial v_a} &= \frac{\lambda}{r} e^{-(r+\lambda)S^*} + \big(\frac{\lambda(v_c+v_a)}{r} - \frac{\lambda v_c}{r+\lambda}\big)\frac{\partial e^{-(r+\lambda)S^*}}{\partial v_a} \\
&= \frac{\lambda}{r} e^{-(r+\lambda)S^*} - \big(\frac{\lambda(v_c+v_a)}{r} - \frac{\lambda v_c}{r+\lambda}\big)\frac{r+\lambda}{\lambda(v_c+v_a)} e^{-(r+\lambda)S^*} \\
&= -\frac{v_a}{v_c+v_a} e^{-(r+\lambda)S^*} \\
&< 0.
\end{aligned}$$

■

D. Proof of Proposition 8

PROOF: Part(i): Based on Equation 10, we have

$$\Delta V_{Interactive} \equiv V^*_{Interactive} - V^0 = \begin{cases} V^*_{Static} - V^0 & \text{if } m < \lambda v_c; \\ V^*_{Sequential} - V^0 & \text{if } m > \lambda v_c. \end{cases}$$

Then both Proposition 6 and Proposition 7 immediately suggest that (a) $\Delta V_{Interactive} = \Delta V_{Sequential} = \Delta V_{Static} = 0$ if $\lambda < m/(v_c + v_a)$, (b) $\Delta V_{Interactive} = \Delta V_{Sequential} < 0$ if $m/(v_c + v_a) < \lambda < m/v_c$, and (c) $\Delta V_{Interactive} = \Delta V_{Static} > 0$ if $\lambda > m/v_c$.

Part(ii): To prove the second result that $\Delta V_{Interactive} \geq \Delta V_{Sequential}$, it suffices to show $V^*_{Static} > V^*_{Sequential}$ if $m < \lambda v_c$. Indeed,

$$\begin{aligned}
V^*_{Static} - V^*_{Sequential} &= \frac{\lambda(rv_c + m)}{r(\lambda + r)} - \Big[\frac{\lambda v_c}{r+\lambda} + \big(\frac{\lambda(v_c+v_a)}{r} - \frac{\lambda v_c}{r+\lambda}\big)\big(\frac{m}{\lambda(v_c+v_a)}\big)^{1+\frac{r}{\lambda}}\Big] \\
&= \frac{\lambda}{r(r+\lambda)}\Big[m - \big(\lambda(v_c+v_a) + rv_a\big)\cdot\big(\frac{m}{\lambda(v_c+v_a)}\big)^{1+\frac{r}{\lambda}}\Big]
\end{aligned}$$

$$
\begin{aligned}
&> \frac{\lambda}{r(r+\lambda)}\Big[m - \big(\lambda(v_c+v_a)+rv_a\big)\cdot\frac{m}{(r+\lambda)(v_c+v_a)-\frac{rm}{\lambda}}\Big] \\
&= \frac{m\lambda}{r(r+\lambda)}\Big[1 - \frac{\lambda(v_c+v_a)+rv_a}{(r+\lambda)(v_c+v_a)-\frac{rm}{\lambda}}\Big] \\
&> \frac{m\lambda}{r(r+\lambda)}\Big[1 - \frac{\lambda(v_c+v_a)+rv_a}{(r+\lambda)(v_c+v_a)-rv_c}\Big] \\
&= 0,
\end{aligned}
$$

where the first inequality makes use of Lemma B.1 by treating $y = m/\lambda(v_a+v_c)-1$ and $\theta = 1 + r/\lambda$, and the second inequality follows from $m < \lambda v_c$. ■

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