Personalized Recommendation with Superior Knowledge under Data Compliance

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Abstract

The use of consumer profiling for personalized recommendations has become increasingly prevalent. However, companies face a trade-off between recommending more relevant products and those that generate higher profits. In this study, we establish a micro-foundation for personalized recommendations that takes into account varying degrees of data usage. Specifically, we examine three scenarios where firms use personal data only (common knowledge), inferred consumer scores only (asymmetric information), or both personal data and inferred consumer scores (superior knowledge) to personalize recommendations. Our results suggest that having greater knowledge does not necessarily translate to more accurate recommendations, as it can lead to strategic ambiguity or even mixed equilibria. Moreover, only personalization using either common or superior knowledge may lead to Pareto-efficiency, whereas using only inferred consumer scores is never firm-optimal, even when the firm can perfectly identify consumers' intrinsic preference types.

Keywords:

Personalized recommendation; data profiling, data compliance; superior knowledge; personal data; intrinsic preference.

1. Introduction

The emergence of precision analytics has transformed marketers' ability to analyze consumer data and identify consumers’ intrinsic preferences. For instance, Netflix utilizes algorithms to classify 125 million viewers into 2000 taste groups, which in turn facilitates personalized movie recommendations. Edelweiss Tokio, a health insurance company, leverages consumer health-tracking data to determine individual-specific health risks to customize medical-plan recommendations (Swedloff, 2019). Additionally, modern computing and artificial intelligence (AI) technologies have accelerated the implementation of personalized recommendations. For example, Emirates Airlines has developed conversational chatbots that collect users' personal information, trip intentions, and browsing histories to tailor travel packages. An industry report indicates that algorithmic personalized recommendations have resulted in an 87% increase in acceptance rates (Dilmegani, 2022).

Firms contend that personalization technologies provide more precise services to consumers and improve recommendation efficiency (Oxera, 2017; Perez, 2019). However, there is growing concern among consumer advocates that companies may misuse their data advantage to provide biased recommendations (Townley et al., 2017; Davidowitz, 2017; Schelter & Kunegis, 2018; Heilweil, 2020). The prevalence of biased incentives in marketing and sales reinforces this concern. For instance, salespersons have a greater propensity to recommend seasonal or less popular products to novice consumers. Similarly, digital recommenders have a higher incentive to promote less relevant but more profitable products. For example, Netflix may have a higher profit margin on its original content than on expensive licensed movie titles, while Amazon Kindle Publishing may prioritize recommending specialized products with higher royalty fees. Given these conflicts of interest, consumers should be rationally suspicious about firm incentives and be prepared to disregard recommendations that may appear biased.

The potential for firms to possess superior knowledge of consumer preferences beyond consumers' own knowledge (Green 2018; Xu and Dukes 2019, 2022; Li and Xu 2022) exacerbates concerns about personalized recommendations. For instance, Netflix's vast collection of user data allows the company to employ Collaborative Filtering Algorithms,[1] which identifies customers with similar browsing histories and ratings and uses them to predict the user's preference for movies they have not yet watched (Koksal 2018). Consequently, firms may have better predictions of a consumer's preference than the consumer themselves.

Does possessing superior knowledge incentivize firms to distort personalized recommendations? Consumer advocates (Oxera, 2017; Perez, 2019; Ichihashi, 2020) argue that uninformed consumers are susceptible to belief manipulation and surplus exploitation, rendering them more likely to accept a less relevant but more profitable product. Consequently, they suggest implementing data compliance regulations that restrict the firm's data collection and profiling. Conversely, firms are concerned that precision analytics can lead to backfiring from consumers' suspicions, as uninformed consumers may reject the product that best aligns with their intrinsic preference (McDonald & Cranor, 2010; Furman & Simcoe, 2015). Therefore, a primary challenge of utilizing information advantage is to convince high-value consumers of their type. As it is difficult to establish credibility, the role of superior knowledge in personalized recommendations remains uncertain, especially when the recommender has biased incentives.

In this research, we build a game-theoretical model to investigate the strategic interaction between a firm with information advantage and partially uninformed consumers in the context of personalized recommendation. The model comprises three consumer segments, each with heterogenous intrinsic preferences denoted as $I = \{0, 0.5, 1\}$, which are unknown to the consumers but are known to the firm. The consumers learn their personal data, P, which is binary and correlated with I. The product space is represented on a Hotelling line between 0 and 1, along which the firm prefers the larger values and the consumers prefer the closer locations. We then

[1] See how Collaborative Filtering algorithm applies to a movie database, visit www.movielens.org.

examine three scenarios when the firm has difference access of data usage: (1) only P (common knowledge), (2) only I (asymmetric knowledge), or (3) both P and I (superior knowledge). These scenarios are related to the different methods of data access, which can be either direct, through information provided by consumers themselves, or indirect, through tracking technologies and inference data that predicts future consumer behaviors (Blanke, 2020). Therefore, the comparison of equilibria under the three scenarios can provide insights into the potential trade-offs associated with regulations that limit the usage of data for discriminatory purposes, thereby informing policy decisions pertaining to data privacy and consumer protection.

We find that in contrast to the pure recommendation strategies in the cases of common knowledge or asymmetric knowledge, the firm with superior knowledge of both consumer intrinsic preference and consumer prior beliefs may use a semi-separating mixed strategy. Therefore, higher degree of consumer knowledge may not translate into more accurate recommendations. Instead, the random and noisy personalization in equilibrium establishes a theoretical link between biased incentives and strategic ambiguity. Our model shows that the boundaries of pure equilibrium are limited by both superior knowledge and consumer types. In particular, no pure equilibrium survives the divinity criterion D1 for the high personal consumer segment, since it is more difficult for a firm with higher information asymmetry to convince high-value consumers.

Our research findings also highlight the trade-offs between a firm's profits and consumer surplus. Only the recommendations that are based on common knowledge or superior knowledge may be Pareto efficient, while three-segment discrimination with asymmetric knowledge is never optimal for the firm. Moreover, superior knowledge allows firms to fully exploit the average consumer surplus, even with strategic ambiguity, whereas asymmetric knowledge results in the highest consumer surplus. Consequently, our research suggests that policymakers need to consider the strategies employed by firms when formulating data compliance rules. A nuanced approach may be required to balance the benefits of personalized recommendations with the potential harm caused by consumer loss of information rent.

The remaining sections are organized as follows. In section 2, we discuss the relevance to the existing data compliance rules and our intended contributions to the literature. In section 3, we present the model. We analyze the equilibrium under common knowledge in section 4, starting from the benchmark of full information to using only personal data. In section 5 and 6, we examine the equilibrium when the firm uses only inferred data and both data, respectively. We then examine the welfare implications of each data compliance rule in section 7. Section 8 concludes.

2. Relevance to Literature

2.1 Data Compliance Rules and Policy Debates

Our research provides a positive theory of compliance rules on the levels of data access. The controversies on consumer data collection has led to long-time debate in regulations. At present, 137 countries have passed legislation to protect personal information and privacy, among which the most influential legal texts are the general data protection regulation (GDPR) of the European Union, the privacy protection law of California (CCPA & CPRA)[2], the personal information protection law of China (hereinafter referred to as PIPL), and the recent development on American Data Privacy and Protection Act (ADPPA)[3]. Table 1 (see Appendix A) illustrates the differences among the data compliance rules.

Our research examines data access scenarios while abstracting away from restrictions on data consent or disclosure. However, it is important to note that the four data compliance rules exhibit variations in their requirements for consent and

[2] These regulations stipulate the scope of legal use. Personal information shall not be processed without personal consent. And they all put forward substantive requirements for the validity of consent, that is, voluntary, clear and fully informed. The CCPA & CPRA still takes opt-out as the main mode, that is, unless the user refuses or exits, the company can continue to process the user's personal information. In addition, GDPR and PIPL clearly put forward general requirements for the increasingly intensive automatic decision-making (algorithm regulation), and emphasized the right of objection of the information subject. Our model examines the data aggregation and processing of firms under data compliance. With the law's permission, the firm can use consumer personal information with consumers' consent.

[3] Source: https://www.congress.gov/bill/117th-congress/house-bill/8152/text#toc-H0299B60817D742978DC3C447CD110A88, accessed in January, 2023.

disclosure. Specifically, GDPR and ADPPA mandate explicit and informed consent for collecting and processing personal data used for product recommendations or discrimination, while CCPA and PIPL allow for personal data use without explicit consent, but with the option for individuals to opt-out of data sale. The presence of consumer opt-in restrictions is unclear when the business practice does not involve price discrimination. Furthermore, ADPPA requires that any "derived data" created through the derivation of information, correlations, inferences, and predictions must be under consumer data ownership, granting consumers the right to access, correct, delete, and move the derived data indirectly drawn from their personal data (Section 203). Conversely, GDPR is less strict in their requirements for disclosure of inference data. For instance, GDPR (p. 88) suggests that derived data necessitates a balance of interests between the data subjects making requests, data controllers, and relevant parties. Therefore, discrimination based on derived data is possible without mandatory disclosure to consumers under GDPR (Yu 2018).

Similar to ADPPA, PIPL imposes stringent measures against the use of "data deduced by automatic algorithms" for discrimination. According to Article 24 of the regulation, the process of personalization based on inferred consumer data must ensure transparency and fairness. Consumers have the right to demand explanations for the use of inferred data and can reject the analytics algorithms employed by data controllers. Conversely, CCPA does not distinguish between inference data and personal data, nor does it mandate that firms disclose their analytics algorithms or trade secrets. Consequently, many firms argue that inference data constitutes trade secrets and are therefore not subject to discrimination restrictions (Wrabetz 2022).

To contextualize our research findings with data compliance, we align data access rules with the regulations as follows: (1) firms can use only two-segment personal data (as per ADPPA and PIPL); (2) firms can use only three-segment inferred data for discrimination (as per GDPR); and (3) no restriction on data usage for non-price discrimination (as per CCPA), allowing firms to discriminate consumers using all four segments. We identify the boundary conditions for each rule that may enhance consumer surplus. Our results are thus relevant to the legislative process regarding the welfare implications of data compliance regulations. Policymakers can gain valuable insights into the potential impacts of these regulations by comparing equilibrium across different scenarios, such as comparing a scenario with stringent data usage regulations to one with more relaxed regulations. These insights can enable policymakers to effectively weigh the benefits and costs of different regulatory measures, including their effects on companies' profitability, consumers' welfare, and the prevalence of discrimination. This information is instrumental in guiding policy decisions related to data privacy and consumer protection.

2.2 Literature in Information Economics and Marketing Models

Our study builds upon recent research on product recommendation and cheap talk models. Since Crawford and Sobel's (1982) seminal paper, many scholars have explored the effects of cheap talk on market outcomes (Farrell, 1993; Farrell and Rabin, 1996; Aumann and Hart, 2003; Chakraborty and Harbaugh, 2007, 2010). Aumann and Hart (2003) confirmed that long cheap talk may lead to mutually preferred outcomes compared to a single message. Chakraborty and Harbaugh (2010) demonstrated that cheap talk recommendations can benefit both the sender (seller) and the receiver (buyer) by increasing the likelihood of a sale and providing more information. However, previous studies have primarily focused on a single sender, neglecting the potential impact of competition on sender credibility (Bagwell and Ramey, 1993; Gardete, 2013; Gilligan and Krehbiel, 1989; Li et al., 2016; Wernerfelt, 1990). For example, Bagwell and Ramey (1993) found that companies' choice between cheap talk or differential advertising to communicate quality depends on the relationship between fixed cost and quality level. Kim and Kircher (2015) examined a large market where auctioneers compete for bidders by announcing cheap-talk messages and showed that effective first price auctions can lead to truthful revelation of types. Choudhary and Zhang (2019) demonstrated that consumer information can affect the bias in product recommendations in both ways, depending on search costs, while Zhou and Zou (2021) compared the accuracy and profitability of different recommendation systems and showed that a neutral recommendation system can be more profitable than a profit-based one. Moreover, Dzyabura and Hauser (2019) argued that traditional recommendation systems that prioritize products with the highest option value or most likely to be selected may not be optimal due to consumers' preference learning behavior.

Unlike traditional cheap-talk recommendation models that rely on transmitting noisy signals to estimate quality, our model focuses on a horizontally-differentiated product space where the firm offers a take-or-leave recommendation. By doing so, we can examine how biased recommendation incentives play a role in personalization. Our model also addresses strategic communication in personalized recommendation in two unique ways. Firstly, we incorporate the idea that the firm may possess superior knowledge of consumers' intrinsic preferences through data profiling, meaning recommendations contain information about both the product space and consumer types. Secondly, we take into account situations in which the firm may only have partial information about consumers, thus requiring recommendations that consider both the consumers' inference process and the uncertainty of consumer types.

Other information games have explored various aspects of using consumer data for discriminative treatments (Zhao 2000; Chen et al., 2001; Chen and Iyer 2002; Acquisti and Varian 2005; Fong 2005; Fudenberg and Villas-Boas 2006; Guo and Zhang 2012; Kremer and Debo 2016; Guo 2016; Guo and Wu 2016; Li and Jain 2016; Kummer 2019; Van de Waerdt 2020; Cao and Zhang 2020; Ke and Sudir 2022), such as optimal advertising and pricing strategies, effects of consumer learning, privacy protection policies, and impact of privacy rights on firms' price discrimination capabilities. For example, Kremer and Debo (2016) studied the impact of waiting time on consumer behavior, while Cao and Zhang (2021) explored the role of appropriate incentives in promoting consumer learning. Huang et al. (2018) developed an analytical framework to study the interaction between demand learning and preference learning, while Montes et al. (2019) investigated the welfare impact of privacy protection policies in online markets. Ke and Sudhir (2022) considered a two-period model where firms sell products to consumers and explored the impact of privacy rights on price discrimination capabilities.

By contrast, we departs from the traditional two-stage framework by examining the roles of different levels of consumer knowledge in personalized recommendation. Firstly, we consider a horizontally-differentiated product space where the firm makes personalized recommendation offers based on consumer preferences. Secondly, we examine the impact of biased recommendation incentives on consumer choices and welfare. Thirdly, we consider situations where the firm may only have partial information about consumer preferences, and the recommendations must take into account both consumer inference processes and uncertainty of consumer types.

Our research is closely related to the economics of superior knowledge (Xu and Dukes 2019, 2021; Li and XU 2022). Xu and Dukes found that even when a firm has better knowledge about consumers' match values than the consumers themselves, the firm cannot capture all the surplus due to consumer suspicion of overpaying. This is because the firm needs to use list price as a credible message to limit its strategy space of personalized pricing. The existing literature on superior knowledge is based on data aggregation, where each consumer type observes a noisy signal of their preference, and the firm can aggregate the noises to partition out the common environmental factors and deduce consumers' intrinsic preferences. By contrast, our model proposes a different micro-foundation for superior knowledge, which is based on data profiling from both personal data and inferred data. This distinction enables us to examine the different roles between superior knowledge and asymmetric knowledge, and the economic values between consumer beliefs and consumer preference types. Furthermore, our model assumes away price discrimination to focus on biased recommendations on a horizontally-differentiated product space. Therefore, we abstract away the price discrimination components from the model and focus on the non-pricing strategies that are permitted by the data compliance rules. To our knowledge, the role of superior knowledge from data profiling on biased recommendations has not been studied in prior literature.

3. Model

We set up the model with three notable features that capture the essence of ubiquitous practice of personalized product recommendation with data collection: *biased incentives*, *personalization*, and *information advantage from data profiling*. We explain each characteristic in detail as we introduce the model.

A firm of multiple products has a biased incentive to recommend its products to consumers with heterogenous preferences. To focus on the information asymmetry on consumers' horizontally differentiated preferences, we abstract away the quality dispersion and price differences, and represent the product space by a Hotelling line, $r \sim \mathrm{U}[0,1]$, where r denotes a product's location. Consumer ideal product preference I is classified as three positions on the Hotelling line, i.e., $I = \{0, 0.5, 1\}$. To interpret, the assumption that the product space is richer than the preference space, consider the example of Netflix that classifies 125 million viewers into 2,000 taste groups while there are over 5, 400 movies and TV shows on the platform.

We specify consumer I's utility of accepting the recommended product r as,

$$U_I = 0.5 - |r - I|,$$

where the base value 0.5 denotes consumers' utility of purchasing the ideal product, which ensures that the recommendations have a critical weight in consumers' decision making [4]. The distance $|r - I|$ captures the disutility from product-preference mismatch.

Contrary to consumers' incentive of purchasing a product that matches their ideal preference I, the firm generally has a biased incentive to recommend certain products that lead to higher payoffs. Consider the following examples: the licensing fees of movie titles largely differ on Netflix – therefore, it may have stronger incentives

[4] Suppose that the base value is greater than 1, then the consumers obtain a positive surplus by accepting even the most mismatched recommendation. We restrict that the base value to be sufficiently small so that the extreme types, such as $I = 0$, obtain negative value from accepting the product from the other turf, such as $r \in (0.5,1]$. In this way, the model captures the necessity of consumers' strategic response upon observing a recommendation.

to recommend the self-made shows rather than the more-expensive licensed ones; Amazon has higher profit margins from some products than others in a same category; Sales people may have higher compensations by pushing less popular products. To examine the biased incentives, we assume that the firm obtains $\Pi = r$, as long as the consumer accepts the recommended product. This stylized setup captures the first feature: *biased incentives*, since the firm has a higher incentive to recommend a product toward the right side, whereas the consumers prefer those closer to their preference location. The unique feature of our model lies in that in a horizontally differentiated market, the firm not only has an incentive to increase matching accuracy to ensure the consumers' acceptance, but also has a strategic incentive to recommend its favored product.

The second feature: *personalization*, is reflected by the heterogeneous consumer values. Suppose that consumers reject r, then they may find their ideal product with probability $\rho \in (0,1)$. For example, consumers can search and experience different movies and find the one they like by trial and error. In this case, the consumer obtains $U = \frac{1}{2}$, and the firm obtains I. With probability of $(1-\rho)$, however, the consumers cannot find the ideal product and leave the market with zero payoff. The probability ρ can be affected by various factors such as product assortment, easiness of search on the firm's site, etc. From the firm's perspective, the expected profit is (ρI) if consumer type I rejects the recommendation, which we call is I's reservation value. Therefore, the firm has incentives to personalize recommendations to consumers with different preferences and, if they are partially informed, different beliefs.

We model the third feature, *information advantage in data profiling,* by building a micro-foundation of information structure that differentiates the roles among common knowledge, asymmetric knowledge, and superior knowledge. We assume that the consumers' intrinsic preferences, I, is *ex-ante* unknown to the consumers and can only be deduced by the firm using data analytics. The consumers' prior knowledge is denoted by the personal data, P, has a noisy correlation with the intrinsic preference

I, so that induction from P to I or vice versa may not be always possible. To highlight the role of data profiling, we differentiate the concepts of *asymmetric knowledge* and *superior knowledge*. Specifically, we refer the inference data on consumers' intrinsic preference as the firm's asymmetric knowledge, and the combination of both inference data and personal data as the firm's superior knowledge, since in this case the firm's knowledge set contains the consumers' knowledge set.

> **Definition 1**: The firm obtains *asymmetric knowledge* if it's inference of consumers' preference is more accurate than the consumers' prior, i.e., $Var[I_i|P_i] > 0$. Data profiling of $\{P_i, I_i\}$ creates *superior knowledge*, since $P_i \in \{P_i, I_i\}$.

To simplify the analysis, we assume $P_i \in \{0,0.5\}$ with equal prior probability, and $I_i \in \{P_i, P_i + 0.5\}$ with the conditional probability $\Pr[I_i = P_i] = \beta$. This assumption prevents the consumers to perfectly deduce the preference type using any P_i and the firm to deduce the consumers' personal data using $I_{0.5}$. For example, a consumer of P_0 can only deduce the preference type as either $I_i = 0$ or $I_i = 0.5$, so that the firm's inferred data I_i has information value to the consumer. But the consumers' prior knowledge P_0 also has information value to the firm that observes $I_i = 0.5$, as the firm can differentiate between $(P_0, I_{0.5})$ that underestimates their type and $(P_{0.5}, I_{0.5})$ that overestimates their type. The prior distribution of I_i is $\Pr[I_i = 0] = \beta/2$, $\Pr[I_i = 0.5] = 1/2$, and $\Pr[I_i = 1] = (1-\beta)/2$. To examine the role of biased incentives, we focus on the parameter range $\beta \geq 1/2$ so that there are fewer consumers who prefer the high-value products. In this case, rational consumer suspicion may lead to rejecting the recommendation.

Figure 1 illustrates the information structure of the game and the correlations among personal data and intrinsic preference types.

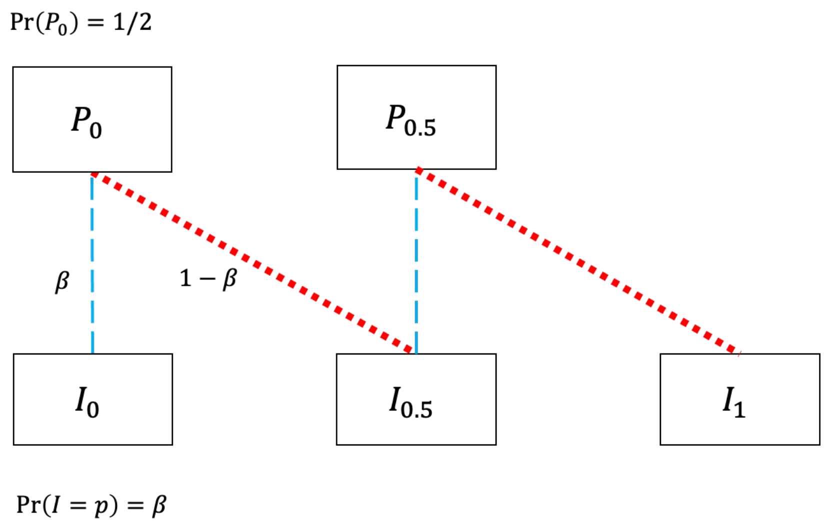

Figure 1.: Information Structure of the Game

Table 1 summarizes the notations. For clarity, we use uppercase letters for the categories, Roman letters for parameters, and lowercase letters for decision variables. Note that throughout the analysis we restrict that $0 < \rho \leq \frac{1}{2} \leq \beta < 1$.

Table 1: Table of Notations

Categorical Variables	P	Personal Data	$P \in \{0,1\}$
	I	Intrinsic Preference	$I \in \{0, 0.5, 1\}$
Exogenous Parameters	β	Correction between P and I	$\Pr[I = P] \equiv \beta$
	ρ	Retention probability	Probability of retention after rejection
Decision Variables	r	recommendation	$r \sim U(0,1)$
	m	mismatch	Probability of false recommendation
	a	acceptance	Probability of accepting r

The timeline is as follows. In period 0, the regulator announces the compliance rule on which data is allowed for personalized recommendation. In period 1, nature draws the consumer intrinsic type I, and consumers observe only their own personal data. In period 2, the firm announces the data collection policy based on the restrictions of regulation. Then the firm uses the collected data for recommendation r. In period 3, consumer I observes r and chooses whether to accept it (or accept at a probability a). If she accepts, the game ends and she receives $0.5-|I-r|$. The firm profits r. If she rejects, she expects the utility of $\rho/2$, and the firm expects (ρI).

4. Recommendation with Common Knowledge

In this section, we analyze the equilibrium under different information settings. In section 4.1, we begin with the benchmark case in which both the firm and consumers have full information of the preference type to examine the economic value of recommendation and consumers' acceptance rules. We then consider the case when the firm can use only the personal data for personalized recommendation in section 4.2. In both cases, there is no signaling in the recommendation, since both the firm and consumers have common knowledge.

4.1 Benchmark: Full Information

Before analyzing the firm's strategic communication incentives, we first seek to understand how consumer heterogeneity and recommendations may drive the consumers' acceptance. We abstract away information asymmetry and assume that both the firm and consumers know perfectly the preference type. Since the consumers have an outside option of rejection the recommendation, $\frac{\rho}{2}$, they should accept the recommendation if and only if $U_I \geq \frac{\rho}{2}$. For convenience of the analysis, we restrict the acceptance region to be semi-open. This technical assumption ensures that the firm will never recommend the product on the left boarder. We derive the acceptance rule for three preference types in Lemma 1:

Lemma 1: *Under full information, consumer* I_1 *accepts if and only if* $r \in \left(\frac{1+\rho}{2}, 1\right]$*; Consumer* $I_{0.5}$ *accepts if and only if* $r \in \left(\frac{\rho}{2}, \frac{2-\rho}{2}\right]$*; and consumer* I_0 *accepts if and only if* $r \in \left(0, \frac{1-\rho}{2}\right]$ *. The firm always has incentives to induce acceptance. The optimal recommendation strategy is to offer* $r_1 = 1$*,* $r_{0.5} = \frac{2-\rho}{2}$*, and* $r_0 = \frac{1-\rho}{2}$*.*

Lemma 1 implies an economic value of product recommendation. This is because even when consumers are fully informed of their preference types, recommendation can still enhance matching efficiency, thus the consumers can mitigate the potential loss of leaving without purchasing by accepting the recommendation. The economic value of recommendations is independent from the consumers' type. But since the consumers with moderate preference ($I_{0.5}$) can accept products from both sides relative to those with extreme preference (I_0, I_1), the acceptance region is greater for $I_{0.5}$.

From Lemma 1, the retention probability ρ negatively correlates with the economic value of recommendations. As the outside option improves, i.e., ρ increases, the product space that can induce consumers I's acceptance of a recommendation becomes more targeted. That is, a consumer is more likely to reject a less-relevant recommendation. Therefore, the firm needs to provide more accurate recommendation to convince the consumer to accept, which restricts the firm's strategy space.

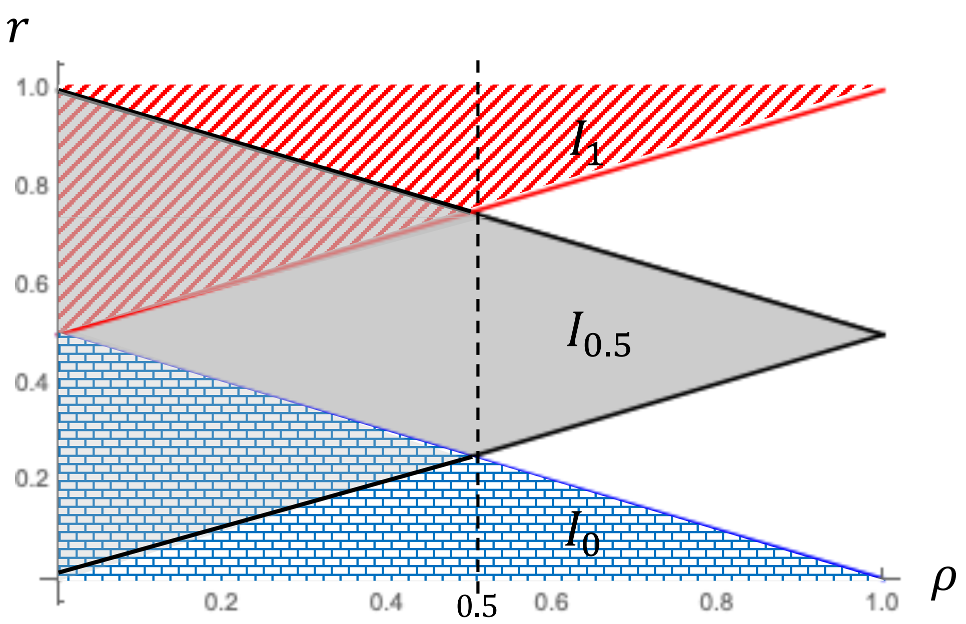

Figure 2: Acceptance Region under Different Retention Probability

Figure 2 illustrates Lemma 1. The upper shaded triangle region represents the acceptance region for I_1; the middle gray region represents that for $I_{0.5}$; and the lower brick-gridded region represents for I_0. Clearly, when ρ increases, the acceptance regions decrease for each consumer type. Note that when $\rho > 1/2$, there exists recommendations that will not be accepted by any consumers. By contrast, if $\rho \leq 1/2$, then any recommendation in the product space is at least accepted by one consumer.

To ensure that the recommendation space is economic efficient everywhere, we restrict the parameter space: $\rho \leq \frac{1}{2}$. Note that the restriction does not affect the equilibrium analysis but simplifies the recommendation strategy space. Therefore, we report only the equilibrium under the region $\rho \leq \frac{1}{2}$ throughout the remaining sections. The complete proofs for the extended parameter space of ρ are provided in the appendix.

4.2 Personal-based Discrimination

By contrast, if the consumers only learn their personal data P_i, and thus are partially informed of their preference types. Their prior belief is $\mu(I_{\mathrm{i}}) = \beta$ and $\mu(I_{i+0.5}) = 1 - \beta$. For instance, P_0 consumers know their preference type is either I_0 with probability β or $I_{0.5}$ with probability $(1-\beta)$. Thus, they accept recommendation r if and only if

$$\frac{1}{2} - \left[\beta r + (1-\beta)\left|r - \frac{1}{2}\right|\right] \geq \frac{\rho}{2} \Leftrightarrow r \leq \min\left\{\frac{2-\beta-\rho}{2}, \frac{\beta-\rho}{2(2\beta-1)}\right\}.$$

Similarly, $P_{0.5}$ consumers know their preference type is either $I_{0.5}$ with probability β and I_1 with probability $(1-\beta)$. Thus, $P_{0.5}$ consumers accept the recommendation so long as

$$\frac{1}{2} - \left[\beta\left|r - \frac{1}{2}\right| + (1-\beta)(1-r)\right] \geq \frac{\rho}{2} \Leftrightarrow r \in \left(\frac{1-\beta+\rho}{2}, \frac{3\beta-\rho-1}{2(2\beta-1)}\right].$$

Similar to the benchmark case, the firm compares the highest payoff from recommendations (i.e., the upper bound of consumers' acceptance range) and the reservation value once consumers reject the recommendation to decide the optimal recommendation strategy. Proposition 1 derives the optimal recommendation strategies when personalization is restricted to personal data. All proofs are relegated to the appendix.

Proposition 1: *If the firm recommends by only personal data, then* $r^*(P_0) =$

$min\left\{\frac{2-\beta-\rho}{2}, \frac{\beta-\rho}{2(2\beta-1)}\right\}$, $r^*(P_{0.5}) = \min\left\{\frac{3\beta-\rho-1}{2(2\beta-1)}, 1\right\}$. *Consumers accept the recommendations.*

Note that the firm does not have incentives to recommend a less-relevant product outside of the acceptance region in this situation. Suppose that the firm induces a P_0 consumer to reject, then its expected profit is $\rho(1-\beta)/2$, which must be lower than the payoff under a successful acceptance $r^*(P_0)$. Similarly, the expected profit from a rejecting $P_{0.5}$ is $\rho(1-\beta)$, which is strictly lower than $r^*(P_{0.5})$.

5. Recommendation with Asymmetric Knowledge

When the firm uses asymmetric knowledge that identifies the consumer's intrinsic preference, its personalized recommendation tradeoffs between efficiency in preventing consumers from rejection and credibility in convincing consumers of their type. Specifically, we analyze the case when the firm uses only the *I* data. This case is consistent with the regulations that are more lenient on inference data (as per GDPR), which allows using inferred consumer segments but prohibits further discrimination by using personal data directly. To differentiate the scenarios between asymmetric knowledge and superior knowledge, we call the former case three-segment discrimination of $\{(I_0), (I_{0.5}), (I_1)\}$, and the latter four-segment discrimination of $\{(P_0, I_0), (P_0, I_{0.5}), (P_{0.5}, I_{0.5}), (P_{0.5}, I_1)\}$.

When the firm perfectly identifies consumers' intrinsic type I and uses a separating strategy, it must credibly communicate with the uninformed consumers about their type. However, since the firm cannot commit "unbiased recommendation," as in any standard signaling game, credible communication incurs indirect signaling cost. Again, we resort to the divinity criterion D1 to refine the belief updates. Specifically, to convince the consumers that they are not steered into misbeliefs, the consumers must be able to deduce the firm's incentives of personalizing recommendation in each segment.

Alternatively, the firm may use a pooling strategy $r^*(I_0) = r^*(I_{0.5}) = r^*(I_1)$ to avoid signaling to the consumers about their type. However, this strategy is subjective to two constraints: First, since the firm can identify the intrinsic type, it cannot deviate by convincing the high-value segment, say, I_1, to purchase a more-profitable product. Second, the pooling strategy must induce all consumers to accept in equilibrium. Otherwise, the strategy cannot be optimal since either the I_0 or I_1 segment will reject. Lemma 2 examines the conditions under which a pure separating or pooling equilibrium may exist.

Lemma 2. *When the firm uses the three-segment discrimination, a pooling equilibrium exists if and only if* $\rho \in \left[\frac{1}{4\beta}, \frac{1}{2\beta} - (1-\beta)\right]$ *with* $r^* = \frac{\beta-\rho}{2(2\beta-1)}$. *Otherwise, there does not exist any separating equilibrium or pooling equilibrium.*

We then examine the possible pure strategy of semi-separating, i.e., $r^*(I_0) = r^*(I_{0.5}) \neq r^*(I_1)$, or $r^*(I_0) \neq r^*(I_{0.5}) = r^*(I_1)$. On the equilibrium, consumers must accept the recommendations $r^*(I_1)$ and $r^*(I_0)$, since the semi-separating strategy must be optimal for each intrinsic segment to prevent deviation. However, not all consumers must accept $r^*(I_{0.5})$ on the equilibrium. This is because when the strategy is semi-separating, consumers may update their beliefs differently upon observing $r^*(I_{0.5})$. For example, if $r^*(I_0) \neq r^*(I_{0.5}) = r^*(I_1)$, then $P_{0.5}$ consumers will maintain their beliefs $\mu(I_{0.5}|P_{0.5}, r^*(I_{0.5})) = \beta$, but P_0 consumers know that there is zero probability that they are the I_1-type, thus they can deduce $\mu(I_{0.5}|P_0, r^*(I_{0.5})) = 1$. The diverging beliefs updates then characterize the semi-separating equilibrium, which we specify in Proposition 2.

Proposition 2. *When the firm uses only the inferred data, the optimal recommendation strategy is as follows:*

(P2.1) $r^*(I_0) = r^*(I_{0.5}) = \frac{2-\beta-\rho}{2}$, $r^*(I_1) = \min\{2-\beta-\frac{3\rho}{2}, 1\}$, *if and only if* $\rho < \min\left\{\frac{2-2\beta-\beta^2}{2-\beta}, \frac{4-6\beta+\beta^2}{5-4\beta}\right\} < 1-\beta$; *All accept;*

(P2.2) $r^*(I_0) = r^*(I_{0.5}) = \frac{\beta-\rho}{2(2\beta-1)}$, $r^*(I_1) = \frac{\beta-\rho}{2\beta-1} - \frac{\rho}{2}$, *if* $\rho \in \left[1-\beta, \frac{1}{4\beta}\right)$; *All accept;*

(P2.3) Otherwise, $r^*(I_0) = \frac{1-\rho}{2}$; $r^*(I_{0.5}) = r^*(I_1) = \min\left\{\frac{3\beta-\rho-1}{2(2\beta-1)}, 1\right\}$; P_0 *rejects* $r^*(I_{0.5})$;

(P2.4) The semi-separating strategy dominates the pooling strategy in Lemma 2.

6. Recommendations with Superior Knowledge

In this section, we examine the core characteristic of data profiling: using both types of data creates superior knowledge. Specifically, we analyze the case when the firm has no restriction in recommendation personalization, which is consistent with the regulations that are lenient on both personal data and inference data (as per CCPA).

6.1 Pure Equilibrium

Consider whether a separating equilibrium may exist for P_0 consumers. The consumers need to deduce whether they are I_0 or $I_{0.5}$ upon observing r^*. On the separating equilibrium, we need to have $\mu\big(r^*(P_0, I_{0.5})\big) = 1$ and $\mu\big(r^*(P_0, I_0)\big) = 0$. Therefore, if separating is possible, then the firm must steer the consumers with the same prior into different actions upon observing the recommendations. Specifically, the firm must induce I_0-type to accept $r^*(P_0, I_0)$ and the $I_{0.5}$-type to reject $r^*(P_0, I_{0.5})$. To interpret this, suppose that the consumers unanimously accept on the equilibrium, then it is always profitable to deviate with $\max\{r^*(P_0, I_0), r^*(P_0, I_{0.5})\}$ in both consumer segments. This opportunistic incentive induces consumer suspicion and adversely-updated beliefs. Therefore, the only credible signal is that no consumer accepts $r^*(P_0, I_{0.5})$, even if it can convince the consumers of their type. The analysis for $P_{0.5}$ consumers is similar. Lemma 3 summarizes this discussion.

Lemma 3. *There does not exist any separating equilibrium that survives D1 with the superior knowledge.*

Next, we examine whether there exists a pooling equilibrium for the consumers with the same prior. Since the pooling recommendation does not convey any information about the firm's asymmetric knowledge on intrinsic preference, consumers maintain their prior belief $\mu(r^*) = 1 - \beta$. By Lemma 1, P_0 accepts if and only if $r \leq \min\left\{\frac{2-\beta-\rho}{2}, \frac{\beta-\rho}{2(2\beta-1)}\right\}$, and $P_{0.5}$ accepts if and only if $r \in \left[\frac{1-\beta+\rho}{2}, \frac{3\beta-\rho-1}{2(2\beta-1)}\right]$. Therefore, the firm may set the pooling recommendation as $r(P_0, I_0) = r(P_0, I_{0.5}) = \min\left\{\frac{2-\beta-\rho}{2}, \frac{\beta-\rho}{2(2\beta-1)}\right\}$, and $r(P_{0.5}, I_{0.5}) = r(P_{0.5}, I_1) = \min\left\{\frac{3\beta-\rho-1}{2(2\beta-1)}, 1\right\}$. Lemma 4 affirms that the pooling equilibrium always survives the divinity criterion D1.

Lemma 4. *With superior knowledge, there always exists a pooling equilibrium that survives D1:* $r(P_0, I_0) = r(P_0, I_{0.5}) = \min\left\{\frac{2-\beta-\rho}{2}, \frac{\beta-\rho}{2(2\beta-1)}\right\}$, $r(P_{0.5}, I_{0.5}) = r(P_{0.5}, I_1) = \min\left\{\frac{3\beta-\rho-1}{2(2\beta-1)}, 1\right\}$*, which consumers accept.*

Note that relative to the case of using only personal data, the superior knowledge does not affect the feasible region of the pooling strategy. In principle, however, the firm with superior knowledge may have stronger incentives to deviate from the pooling strategy by allowing the high-valuation consumers, such as the identified $P_{0.5}$ segment to reject rather than the P_0 segment, thus violating the belief refinement criterion D1. Lemma 4 thus suggests that the deviation incentives are not strong enough to induce consumer suspicion. Therefore, the equilibrium with superior knowledge is identical to the personal-based discrimination without superior knowledge. This implies that accurate consumer knowledge may not necessarily lead to accurate recommendations in equilibrium.

6.2 Semi-Separating Mixed Strategy

Since there exists a region under which there is no pure strategy for P_1, in this section we examine the case when the firm uses a mixed strategy. In this way, the firm may partially but credibly discriminate between consumers. We examine the existence of a semi-separating equilibrium in which the firm sometimes sends (P_i, I_i) customers an informative recommendation that convinces them of their intrinsic type, and, sometimes distorts the recommendation to mix them with the $(P_i, I_{i+0.5})$ customers.

Figure 3 illustrates the mixed strategy.

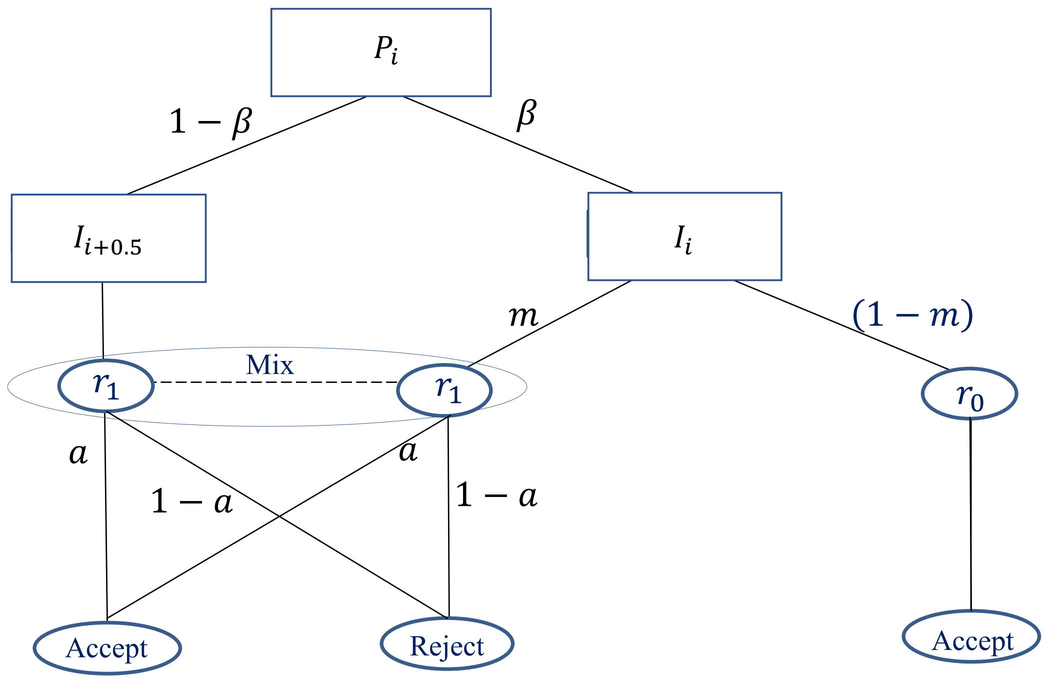

Figure 3: Game Tree of the Mixed Strategy

The P_i consumers who observe r_0 will perfectly deduce their type of (P_i, I_i). By contrast, since r_1 is mixed between the two consumer segments and the equilibrium belief must be consistent, the consumers deduce their type using Bayesian updates:

$$\mu = \Pr(I_{i+0.5}|r_1) = \frac{1-\beta}{1-\beta+\beta m} \in (1-\beta, 1)$$

Since the firm must be indifferent between recommending r_1 and r_0 to the (P_i, B_0) consumers, the profit margin must be the same:

$$r_1 a + \rho I_i(1-a) = r_0$$

And the consumers are indifferent between acceptance and rejection. Thus,

$$\mu\left(\frac{1}{2} - |I_{i+0.5} - r_1|\right) + (1-\mu)\left(\frac{1}{2} - |I_i - r_1|\right) = \frac{\rho}{2}.$$

Since $I_{i+0.5} - I_i = \frac{1}{2}$, we can simplify the firm's expected profit from this mixed strategy as follows:

$$\Pi^{M*} = \beta[(1-m)r_0 + m r_1 a + m(1-a)I_i\rho] + (1-\beta)[r_1 a + I_{i+0.5}\rho(1-a)]$$
$$= r_0 + \frac{(1-\beta)\rho}{2}(1-a).$$

The above analysis yields the results in Proposition 3.

Proposition 3. *When the firm uses both data for personalized recommendation, there exists a mixed strategy for $P_{0.5}$ consumers but not P_0. Specifically, when $\rho > 1-\beta$, $r^*(P_{0.5}, I_{0.5}) = \frac{2-\rho}{2}$, $r^*(P_{0.5}, I_1) = 1$, and $m^* = \frac{(1-\beta)(1-\rho)}{\beta\rho}$. The consumers always accept $r(P_{0.5}, I_{0.5})$, but only accept $r(P_{0.5}, I_1)$ at the probability $a^* = 1 - \frac{\rho}{2-\rho}$. The mixed strategy dominates the pure strategy.*

7. Welfare Implications

We now examine the economic implications of data compliance rules for both the firm and consumers. To do so, we first compare the firm profits under each compliance rule. Then we examine the optimal regions of each data compliance rule that maximize either the consumer surplus or the total social welfare.

7.1. Profit Comparison

We first examine the profit implications of data compliance. To differentiate, we denote the *ex-ante* profit using only personal data as $\text{E}\Pi^{\text{P}}$, that using only inferred intrinsic data as $\text{E}\Pi^{\text{I}}$, and that using both data as $\text{E}\Pi^{\text{PI}}$. From the previous analysis, the firm expects the following profit under each rule:

(7.1.1). If $\rho < 1-\beta$, then $\text{E}\Pi^{\text{P}} = \frac{4-\beta-\rho}{4}$; otherwise $\text{E}\Pi^{\text{P}} = \frac{4\beta-2\rho-1}{4(2\beta-1)}$

(7.1.2) If $\rho < \min\left\{\frac{2-2\beta-\beta^2}{2-\beta}, \frac{4-6\beta+\beta^2}{5-4\beta}\right\}$, then $\text{E}\Pi^{\text{I}} = \frac{1}{4}\left[(1+\beta)(2-\beta-\rho) + 2(1-\beta)\min\left\{2-\beta-\frac{3\rho}{2}, 1\right\}\right]$; If $\rho \in \left[1-\beta, \frac{1}{4\beta}\right]$, then $\text{E}\Pi^{\text{I}} = \frac{(1-2\rho)(\beta-\beta^2)+2(\beta-\rho)}{4(2\beta-1)}$; Otherwise, $\text{E}\Pi^{\text{I}} = \frac{1}{4}\left[\beta+\rho-2\beta\rho+\min\left\{\frac{3\beta-\rho-1}{(2\beta-1)}, 2\right\}\right]$.

(7.1.3) If $\rho < 1-\beta$, then $\text{E}\Pi^{\text{PI}} = \frac{4-\beta-\rho}{4}$; otherwise, $\text{E}\Pi^{\text{PI}} = \frac{\rho^2(2\beta-1)(1-\beta)+(2-\rho)(5\beta-2\rho\beta-2)}{4(2-\rho)(2\beta-1)}$.

Suppose that the firm can commit the use of data to the consumers, then the optimal strategy of data usage can be derived from static comparison among the four cases of (7.1.1) to (7.1.3). We summarize the results in Proposition 4.

Proposition 4. *The firm strictly prefers superior knowledge, if and only if $\rho > \frac{5-\sqrt{1+16\beta}}{6-4\beta}$; It* *strictly prefers personal data only, if and only if* $\rho \in \left(1-\rho, \frac{5-\sqrt{1+16\beta}}{6-4\beta}\right]$*. Otherwise, the firm is indifferent between the two data-usage rules.*

Proposition 4 suggests that the three-segment discrimination based on inferred data only are never optimal. The intuition is that even with the asymmetric knowledge on consumers' intrinsic preferences, the benefit of personalization is limited without perfect information on personal segment. Since the additional knowledge may signal the consumers of their intrinsic types, which enable consumers to learn about their preferences better than the firm and thus avoid accepting less-relevant recommendations, the firm always prefers either learning superior knowledge to the full extent (the case of the four-segment discrimination) or commit to learn no additional knowledge at all (the case of the personal-based discrimination).

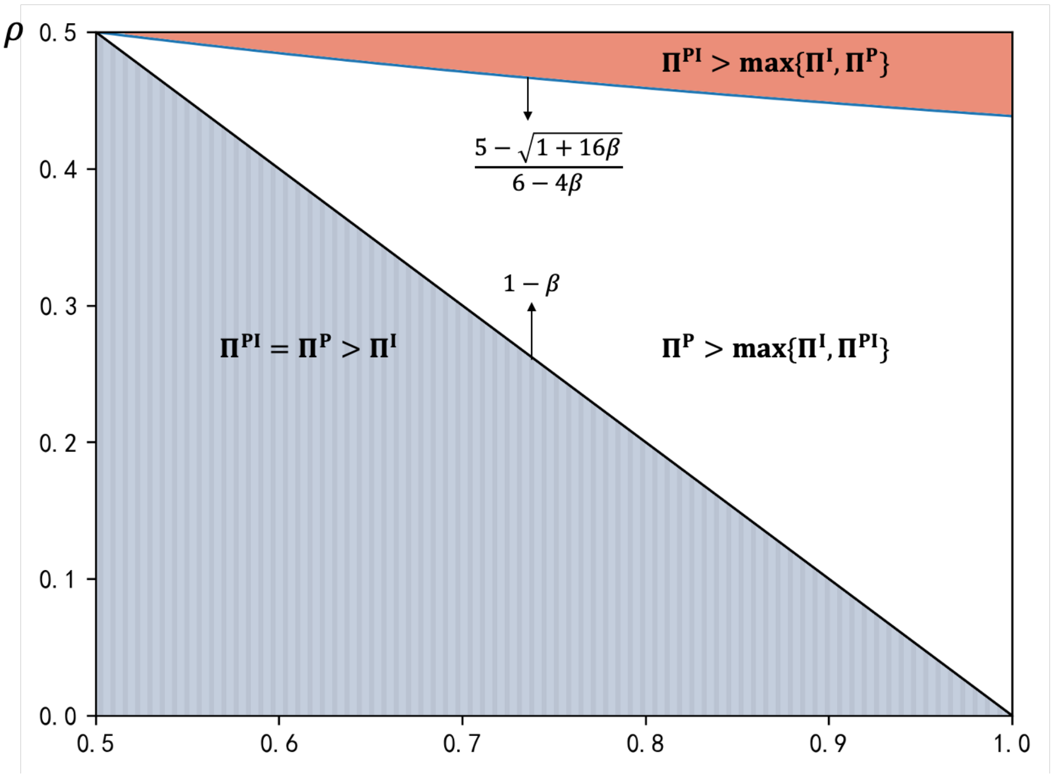

Figure 4: Optimal Regions for Each Data Usage Rule (Colored Online)

Figure 4 illustrates the optimal regions for data usage. Specifically, the white region characterizes the optimal conditions for the personal-based discrimination without superior knowledge. The dark region in the upper right corner supports the four-segment discrimination with both data. The shaded regions represent the conditions under which the firm is indifferent between personal-based discrimination and the four-segment discrimination with superior knowledge.

Note that in the optimal region for the firm to strictly prefer personal-based discrimination, all consumers accept their personalized recommendations. With an alternative data usage, P_0 rejects the pooling strategy; $P_{0.5}$ sometimes rejects the four-segment mixed strategy; and I_0 deduces that they have the lowest-value. The intuition for the no-superior knowledge to be profitable is that when consumers have sufficiently high outside options (ρ is high), they will be less likely to accept the recommendations due to rational suspicions. Therefore, the firm may have incentives to avoid signaling to consumers of their types, which results in loss of accepting consumers. Thus, the personal-based discrimination is the optimal data usage rule with sufficient accuracy in discrimination without raising consumer suspicion of data abuse.

The results also shed light on the compliance implications. The current policy debates on data access focuses on whether discrimination based on personal data should be illegal *per-se*. Our results show that although the personal-based discrimination may not always be optimal, the firm still prefers to use personal data directly in the four-segment discrimination with superior knowledge. Therefore, the restrictions against personal-based discrimination strictly reduces the firm's profit under any parameter space.

7.2. Consumer Surplus

In this section we examine the implications of data collection on consumers' *ex-ante* utility. Denote $\mathrm{CS^P}$, $\mathrm{CS^I}, \mathrm{CS^{PI}}$ as the consumer surplus under the scenarios of personal data only, inference data only, and both data, respectively.

(7.2.1). If $\rho < 1-\beta$, then $\mathrm{CS^P} = \frac{1-\beta+\rho}{4}$; Otherwise $\mathrm{CS^P} = \frac{\rho}{2}$

(7.2.2) If $\rho < \min\left\{\frac{2-2\beta-\beta^2}{2-\beta}, \frac{4-6\beta+\beta^2}{5-4\beta}\right\}$, then $\mathrm{CS^I} = \frac{1}{4}[\rho+\beta^2+\beta\rho+(1-\beta)\min\{1,3-2\beta-3\rho\}]$; If $\rho \in \left[1-\beta, \frac{1}{4\beta}\right]$, then $\mathrm{CS^I} = \frac{1}{4(2\beta-1)}[(2-\beta)-(1-\beta^2)(1+2\rho)]$; Otherwise, $\mathrm{CS^I} = \max\left\{\frac{\rho}{2}, \frac{1-\beta+\rho}{4}\right\}$.

(7.2.3) $\mathrm{CS}^{\mathrm{PI}} = \frac{\rho}{2}$

Proposition 5 *Average consumers are indifferent if and only if* $\rho \geq \frac{1}{4\beta}$. *Otherwise, asymmetric knowledge maximizes consumer surplus. In addition, we can identify two Pareto-efficient data usage rules: (1) superior knowledge, if and only if* $\rho \geq \frac{5-\sqrt{1+16\beta}}{6-4\beta}$*; and (2) the personal-data only, if and only if* $\rho \in \left[\frac{1}{4\beta}, \frac{5-\sqrt{1+16\beta}}{6-4\beta}\right)$*; There is no Pareto-efficiency, if* $\rho < \frac{1}{4\beta}$.

Figure 5 illustrates the optimal region for the compliance rule that maximizes the consumer welfare. Specifically, the dark region in the upper right corner indicates no restriction is necessary, since the average consumer surplus is the same. The white region in the middle represents the conditions under which the discrimination based on partial data (either personal or inference) maximizes the average consumer surplus. This indicates that a stringent regulation against using superior knowledge (as per European GPDR and American ADPPA) can benefit the consumers.

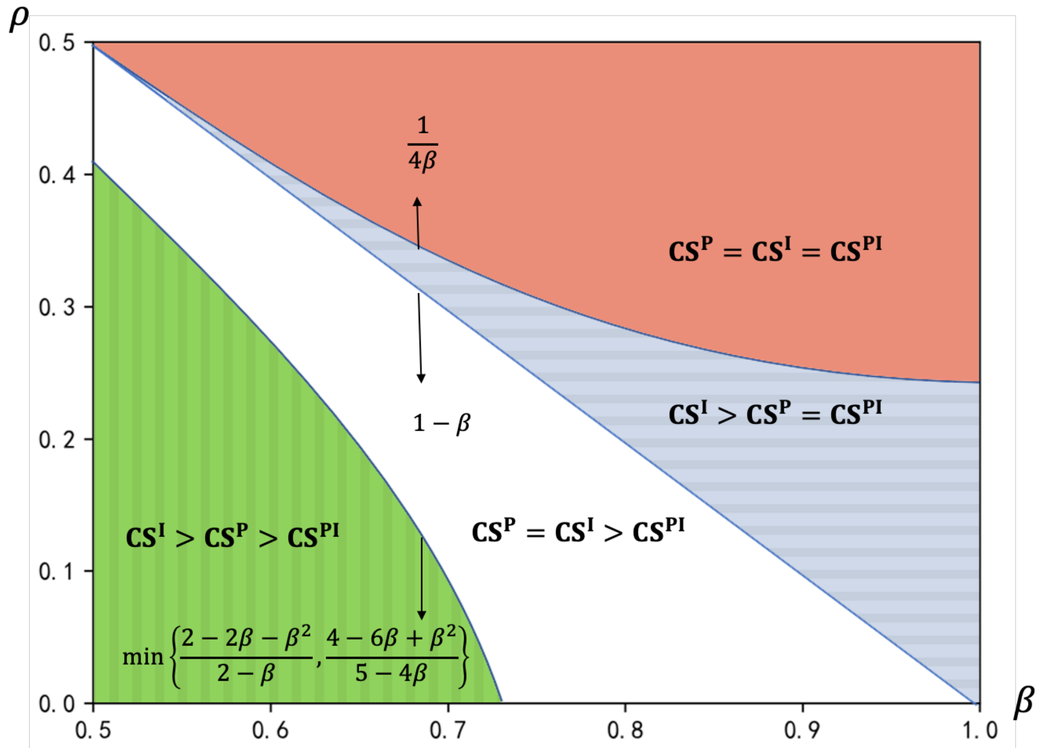

Figure 5: Data-Usage Rules for Consumer Surplus (Colored Online)

Contrary to the classic consumer advocates' arguments, our results identify a region under which discrimination by personal data is Pareto-efficient that can benefit the firm without harming consumers relative to the other cases of data usage. In addition, when the firm strictly prefers using both data for personalized recommendation, the consumers are not harmed. Therefore, superior knowledge is Pareto-efficient whenever it is strictly profitable. As a result, we suggest that the regulator should restrict the use of personal data for the rules of reason rather than illegal per se to benefit the consumers.

Our results also indicate that restrictions on personal-data usage are consumer beneficial whenever the retention rate is sufficiently low, i.e., $\rho < 1 - \beta$. Since the firm never prefers using only the inferred-data, the relevant compliance rules can improve the consumer surplus at the cost of the firm's profit. In addition, the degree of regulation stringency is non-monotone with the retention rate. Specifically, when ρ is moderate, i.e., $\rho \in \left(\min\left\{\frac{2-2\beta-\beta^2}{2-\beta}, \frac{4-6\beta+\beta^2}{5-4\beta}\right\}, 1-\beta\right)$, the regulator may implement the most strictly rule that forbids any use of personal data. But when ρ is either extremely low or high, the consumer-surplus-driven regulators should consider lenient rules that allow for discrimination based on inferred consumer data.

8. Concluding Remarks

Our research objective is to explore the consequences of information asymmetry on biased personalized recommendations. We establish a micro-foundation for biased personalized recommendations, incorporating different levels of information advantage, and investigate the welfare implications of various data compliance rules in the U.S., Europe, and China. Our model contributes to both theory and practice in the following three ways:

First, we extend the classic signaling models to partial information exchange in which receiver has private information that sender does not know. This is a challenge for any theoretical attempt to formally examine the implications of data compliance, because rules (as per GDPR) may require the firm not to use personal information for discrimination treatments. In this way, the signal message (product recommendation) contains information that receivers (consumers) do not know, but the receiver also has private information that the sender (firm) cannot access, which complicates the consumers' inferences of the sender incentives upon observing personalized recommendation.

Second, we examine "strategic ambiguity" in personalized recommendation, where firms collect more consumer data but fail to provide more accurate recommendation. While conventional wisdom attributes this to limitations in data technologies or noisy learning algorithms, we approach the issue from the perspective of the firm's biased incentives. Through our game theoretical model, we identify the boundaries of pure equilibrium and the existence of mixed strategy when the sender has superior knowledge, shedding light on why more data collection may not lead to more accurate personalization.

Finally, our research has broad implications for data collecting firms, digital product platforms, algorithmic recommendation engineers, and the public. By contributing to international discussions on consumer data collection and compliance, our model may help firms better understand consumers' strategic responses to personalized recommendations, including reluctance due to rational suspicion. Additionally, our findings can inform a revised definition of data advantage that takes into account the values of data collection and personalization. Rather than attributing negative consumer experiences solely to algorithm inaccuracy, our approach examines the biased incentives in the strategic interactions between firms and consumers.

References

Aumann, R.J., Hart, S., 2003. Long cheap talk. *Econometrica* 71, 1619–1660.

Acquisti, A. and Varian, H.R., 2005. Conditioning prices on purchase history. *Marketing Science*, 24(3), 367–381.

Banks, J.S. and Sobel, J., 1987. Equilibrium selection in signaling games. *Econometrica,* 647-661.

Bagwell, K., Ramey, G., 1993. Advertising as information: Matching products to buyers. J*ournal of Economics & Management Strategy* 2, 199–243.

Bar-Gill, O. 2019. Algorithmic price discrimination: When demand is a function of both preferences and (mis)perceptions. *University of Chicago Law Review,* 86(2), article 12.

Blanke, Jordan M., 2020. Protection for 'Inferences Drawn:' A Comparison between the General Data Protection Regulation and the California Consumer Privacy Act. *Global Privacy Law Review* 81

Cao, X., Zhang, J., 2021. Preference learning and demand forecast. *Marketing Science,* 40, 62–79.

Chakraborty, A., Harbaugh, R., 2007. Comparative cheap talk. Journal of Economic Theory 132, 70–94. Chakraborty, A., Harbaugh, R., 2010. Persuasion by cheap talk. *American Economic Review* 100, 2361–82.

Choudhary, V., Zhang, Z., 2019. Product recommendation and consumer search. Available at SSRN3398994.

Crawford, V.P., Sobel, J., 1982. Strategic information transmission. *Econometrica*, 1431–1451

Cao, X., and Zhang, J. 2020. Preference learning and demand forecast. *Marketing Science*.

Chen, Y. and Iyer, G., 2002. Consumer addressability and customized pricing. *Marketing Science*, 21(2), 197-208.

Chen, Y., Narasimhan, C. and Zhang, Z.J., 2001. Individual marketing with imperfect targetability. *Marketing Science*, 20(1), 23–41.

Cho, I.K. and Kreps, D.M., 1987. Signaling games and stable equilibria. *Quarterly Journal of Economics*, 102(2), 179-221.

Davidowitz, S. Everybody lies: big data, new data, and what the Internet can tell us about who we really are. *Harper Collins*, 2017.

Dilmegani, C. Top 25 chatbot case studies & success stories (with tips), *AI Multiple*, https://research.aimultiple.com/top-chatbot-success/, accessed in November 2022.

Dzyabura, D., Hauser, J.R., 2019. Recommending products when consumers learn their preference weights. *Marketing Science* 38, 417–441.

Farrell, J., 1993. Meaning and credibility in cheap-talk games. *Games and Economic Behavior* 5, 514–531.

Farrell, J., Rabin, M., 1996. Cheap talk. *Journal of Economic Perspectives* 10, 103–118.

Fong, Y.F., 2005. When do experts cheat and whom do they target? *RAND Journal of Economics*, 36(1), 113-130.

Fudenberg, D. and Villas-Boas, J.M., 2006. Behavior-based price discrimination and consumer recognition. *Handbook on economics and information systems*, 1, 377–436.

Furman, J., and Simcoe, T. 2015. *The economics of big data and differential pricing*. The White House President Barack Obama.

Gardete, P.M., 2013. Cheap-talk advertising and misrepresentation in vertically differentiated markets. *Marketing Science* 32, 609–621.

Gilligan, T.W., Krehbiel, K., 1989. Asymmetric information and legislative rules with a heterogeneous committee. *American Journal of Political Science* 459–490.

Green, A. 2018. Google knows you better than you know yourself. *Komando* (April 1).

Guo, L. 2016. Contextual deliberation and preference construction. *Management Science*, 62(10), 2977-2993.

Guo, L. and Wu, Y., 2016. Consumer deliberation and quality signaling. *Quantitative Marketing Economics*, 14, 233–269.

Guo, L., and Zhang, J. 2012. Consumer deliberation and product line design. *Marketing Science*, 31(6), 995-1007.

Huang, T., Liang, C., Wang, J., 2018. The value of “bespoke”: Demand learning, preference learning, and customer behavior. *Management Science* 64, 3129–3145.

Heilweil, R. 2020. Tinder may not get you a date. It will get your data. *Vox* (Feb 14).

Ke, T. Tony and K. Sudhir (2022), “Privacy Rights and Data Security: GDPR and Personal Data Markets,” *Management Science*.

Kim, K., Kircher, P., 2015. Efficient competition through cheap talk: the case of competing auctions. *Econometrica* 83, 1849–1875.

Kremer, M., Debo, L., 2016. Inferring quality from wait time. *Management Scienc*e 62, 3023–3038.

Kummer, M., Schulte, P., 2019. When private information settles the bill: Money and privacy in Google’s market for smartphone applications. *Management Science* 65, 3470–3494.

Ichihashi, S. (2020). Online privacy and information disclosure by consumers. *American Economic Review*, 110(2), 569-95.

Koksal, I. 2018. Artificial intelligence may know you better than you know yourself. *Forbes* (Feb 27).

Li, K. J. and Jain, S. 2016. Behavior-based pricing: an analysis of the impact of peer-induced fairness. *Management Science*, 62(9), 2705-2721.

Li, X., Z X, 2022, Superior Knowledge, Price Discrimination, and Customer Inspection, *Marketing Science*

Li, Z., Rantakari, H., Yang, H., 2016. Competitive cheap talk. *Games and Economic Behavior* 96, 65–89.

McDonald, A.M., Cranor, L.F., 2010. Americans’ attitudes about internet behavioral advertising practices, in: Proceedings of the 9th annual ACM workshop on Privacy in the electronic society, pp. 63–72.

Montes, Rodrigo, Wilfried Sand-Zantman, and Tommaso Valletti (2019), “The Value of Personal Information in Online Markets with Endogenous Privacy,” *Management Science*, 65 (3), 1342–62.

Oxera Consulting LLP. 2017. When algorithms set prices: winners and losers.

Discussion paper.

Perez, S. 2019. Target's personalized loyalty program launches nationwide next month. *TechCrunch* (September 10).

Roettgers, J. 2018. Netflix's Secrets to Success: Six Cell Towers, Dubbing and More. *Variety* (March 8).

Schelter, S., Kunegis, J., 2018. On the ubiquity of web tracking: Insights from a billion-page web crawl. *The Journal of Web Science*.

Swedloff, R. 2019. Algorithms and AI are radically changing insurance. Regulators are behind the curve. *Promarket* (June 5).

Townley, C., Morrison, E., & Yeung, K. 2017. Big data and personalized price discrimination in EU competition law. *Yearbook of European Law*, *36*, 683-748.

Van de Waerdt, P.J., 2020. Information asymmetries: recognizing the limits of the GDPR on the data-driven market. *Computer Law & Security Review* 38, 105436.

Wrabetz, J. 2022, What is inferred data and why is it important? *Business Law Today*. https://businesslawtoday.org/2022/08/what-is-inferred-data-why-is-it-important/ accessed in Jan. 2023

Xu, Z. and Dukes, A., 2019. Product line design under preference uncertainty using aggregate consumer data. *Marketing Science*, 38(4), 669–689.

Xu, Z. and Dukes, A., 2021. Personalization, consumer data aggregation, and the role of list price. *Management Science*, 68(2), 960-980.

Yu, H. 2020. GDPR Isn't Enough to Protect Us in an Age of Smart Algorithms (2018), https://theconversation.com/gdpr-isnt-enough-to-protect-us-in-an-age-of-smart◎algorithms-97389, accessed in May 2022.

Zhao, H., 2000. Raising awareness and signaling quality to uninformed consumers: A price-advertising model. *Marketing Science* 19, 390–396.

Zhou, B., Zou, T., 2021. Competing for recommendations: The strategic impact of personalized product recommendations in online marketplaces. Available at SSRN 3761350.

Appendix A:

Table 1: Comparison of personal information protection laws among the U.S., Europe and China

Personal information	GDPR (Europe)	PIPL (China)	CCPA & CCRA (California)
Definition	Information related to any identified or identifiable natural person, such as: (1) name; (2) Identification number; (3) Address data; (4) Online identification; (5) One or more physical, physiological, genetic, spiritual, economic, cultural or social identities (Article 4).	Various kinds of information related to identified or identifiable natural persons recorded electronically or otherwise (Article 4).	Information that directly or indirectly identifies, relates, describes, or can reasonably be associated with a particular consumer or family. For example: (1) identification code; (2) Biometric information; (3) Geographic location data; (4) Professional or employment related information; (5) Protected features; (6) Network activity information; (7) Inference of user portrait; (8) Business in- formation; (9) Educational information. (Article 1798.140)
Restrictions	Personal information (Article 4)	Inferred consumer scores (Article 24)	Fully accessible with consent (Article 1798.140)
Automated processing and decision making	(1) Legislation, rationality and transparency (2) purpose limitation; (3) Minimum data; (4) Accuracy; (5) Storage within a time limit; (6) integrity and confidentiality of data; (7) Accountability. (Article 22)	The use of inferred data shall ensure the transparency of decision-making and the fairness and impartiality of the results, and shall not impose unreasonable differential treatment on individuals in terms of transaction prices. (Article 24 & 73)	1) De-identification information; (2) Aggregated consumer information; (3) Publicly available information; (4) Real information legally obtained and attracting public attention. (Article 1798.140)

Appendix B: Proof of Lemmas and Propositions

Proof of Lemma 1:

The conditions for the acceptance decision are straightforward from $\frac{1}{2} - |r - I| \geq \frac{\rho}{2}$, where $r \in [0,1]$. Under the full information case, I_0 consumers accept, if and only if $\frac{1}{2} - r \geq \frac{\rho}{2}$; $I_{0.5}$ consumers accept, if and only if $\frac{1}{2} - \left|r - \frac{1}{2}\right| \geq \frac{\rho}{2}$; and I_1 consumers accept, if and only if $\frac{1}{2} - (1 - r) \geq \frac{\rho}{2}$. Simple algebra follows. **QED**

Proof of Proposition 1:

If P_0 consumers reject, the reservation value to the firm is $\rho\mathbb{E}[I|P_0] = \rho(1-\beta)/2$. To induce P_0-consumer to accepts a recommendation, the recommendation $r \leq \min\left\{\frac{2-\beta-\rho}{2}, \frac{\beta-\rho}{2(2\beta-1)}\right\}$. When $\rho \leq 1-\beta$, to induce consumers to accept, the highest recommendation the firm can choose is $r = \frac{2-\beta-\rho}{2}$, which is always higher than $\frac{\rho(1-\beta)}{2}$ for $\rho \leq 1$. When $1-\beta < \rho \leq \beta$, the highest recommendation the firm can choose is $r = \frac{\beta-\rho}{2(2\beta-1)}$, which is higher than $\frac{\rho(1-\beta)}{2}$ only when $\rho \leq \frac{1}{3-2\beta}$. For $\rho > \beta$, no recommendation can induce P_0 to accept.

If P_1 consumers reject, the reservation value to the firm is $\rho\mathbb{E}[I|P_{0.5}] = \rho\left(1-\frac{\beta}{2}\right)$. When $\rho \leq 1-\beta$, the highest recommendation the firm can induce acceptance is $r = 1$, which is clearly higher than $\rho\left(1-\frac{\beta}{2}\right)$. When $1-\beta < \rho \leq \beta$, the highest recommendation the firm can induce acceptance is $r = \frac{3\beta-\rho-1}{2(2\beta-1)}$, which is higher than $\rho\left(1-\frac{\beta}{2}\right)$ in this parameter range. For $\rho > \beta$, no recommendation can induce P_1 to accept. **QED**

Proof of Lemma 2:

First, we show that there does not exist a pure separating equilibrium when the firm uses I-segment to personalize recommendation. Suppose in equilibrium $r^*(I_0) \neq r^*(I_{0.5}) \neq r^*(I_1)$. From the requirement of belief consistency, the consumers must infer their types upon observing the recommendation. Suppose that P_0 accepts $r^*(I_{0.5})$ in equilibrium. Then if $r^*(I_{0.5}) > r^*(I_0)$, the firm will always deviate when it identifies

P_0 as I_0, i.e., $r'(I_0) = r^*(I_{0.5})$, since the equilibrium profit with $r^*(I_0)$ is strictly lower regardless of whether P_0 accepts $r^*(I_0)$. But since $r^*(I_{0.5}) < r^*(I_0)$ is never optimal, to ensure that the separating equilibrium exists, P_0 must reject $r^*(I_{0.5})$ in equilibrium. This implies that $P_{0.5}$ must also reject $r^*(I_{0.5})$ by the requirement of sequential rationality. Therefore, whenever $r^*(I_{0.5})$ informs the consumers about their type, they must always reject the recommendation on the separating equilibrium. This strategy cannot be optimal.

Second, we show that there exists a pure pooling equilibrium if and only if $\rho \in \left[4\beta, \frac{1}{2\beta} - (1-\beta)\right]$. Suppose $r^*(I_0) = r^*(I_{0.5}) = r^*(I_1)$, then the consumers cannot update their beliefs. From previous analysis, a P_0 consumer accepts if and only if $r \leq \min\left\{\frac{2-\beta-\rho}{2}, \frac{\beta-\rho}{2(2\beta-1)}\right\}$, and a P_1 consumer accepts if and only if $r \in \left[\frac{1-\beta+\rho}{2}, \frac{3\beta-\rho-1}{2(2\beta-1)}\right]$. We discuss the following regions:

(1) If $\rho \in (0,1-\beta)$, then $\frac{1-\beta+\rho}{2} < \frac{2-\beta-\rho}{2} < \frac{\beta-\rho}{2(2\beta-1)}$. Therefore, the firm's only possible pooling strategy is to set $r^* = \frac{2-\beta-\rho}{2}$. But then there exists an out-of-equilibrium strategy, $r' = 2 - \beta - \frac{3\rho}{2}$, such that $\frac{r'}{2} + \frac{\rho}{4} \leq r^*$.. Therefore, if the firm deviates by recommending r' to I_1, P_1 will be convinced that the recommendation can never be sent to the $I_{0.5}$ segment and thus believe that they are the I_1 segment. And since $\rho \in (0,1-\beta)$ implies that $\rho < \frac{3}{4} - \frac{\beta}{2}$, thus $r' > \frac{1+\rho}{2}$, the convinced I_1 consumers will then accept $r' > r^*$. Therefore, it does not exist a feasible pooling strategy in this condition.

(2) If $\rho \in \left[1-\beta, \frac{1}{2\beta} - (1-\beta)\right]$, then $\frac{1-\beta+\rho}{2} \leq \frac{\beta-\rho}{2(2\beta-1)} \leq \frac{2-\beta-\rho}{2}$. Therefore, the firm's only possible pooling strategy is to set $r^* = \frac{\beta-\rho}{2(2\beta-1)}$. But since $1-\beta \leq \frac{1}{4\beta} \leq \frac{1}{2\beta} - (1-\beta)$, there exists a region $\rho \in \left[1-\beta, \frac{1}{4\beta}\right)$, such that the firm can deviate by recommending I_1 the product $r' = \frac{\beta-\rho}{2\beta-1} - \frac{\rho}{2}$. In this way, the consumer will be convinced of their type since $\frac{r'}{2} + \frac{\rho}{4} \leq r^*$, and accept the product since $r' > \frac{1+\rho}{2}$. Therefore, the feasible pooling equilibrium exists only if $\rho \in \left[\frac{1}{4\beta}, \frac{1}{2\beta} - (1-\beta)\right]$.

(3) If $\rho \in \left(\frac{1}{2\beta} - (1-\beta), \frac{1}{2}\right]$, then $\frac{1-\beta+\rho}{2} > \min\left\{\frac{2-\beta-\rho}{2},\ \frac{\beta-\rho}{2(2\beta-1)}\right\}$, thus there does not exist a recommendation such that both P_0 and $P_{0.5}$ will accept. This implies that any pooling strategy under this condition will induce either I_0 or I_1 to reject, which cannot be optimal.

In summary, the pooling equilibrium exists if $\rho \in \left[\frac{1}{4\beta}, \frac{1}{2\beta} - (1-\beta)\right]$. **QED**

Proof of Proposition 2:

We examine two possible semi-separating strategies. First, $r^*(I_0) = r^*(I_{0.5}) \neq r^*(I_1)$. Since the strategy is pooling for a P_0 consumer, who accepts if and only if $r \leq \min\left\{\frac{2-\beta-\rho}{2},\ \frac{\beta-\rho}{2(2\beta-1)}\right\}$, we must have $r^*(I_0) = r^*(I_{0.5}) \leq \min\left\{\frac{2-\beta-\rho}{2},\ \frac{\beta-\rho}{2(2\beta-1)}\right\}$ to ensure that I_0 consumers accept the recommendation, otherwise the firm will always deviate for this segment.

If $\rho < 1-\beta$, then $\frac{1}{2} < \frac{2-\beta-\rho}{2} < \frac{\beta-\rho}{2(2\beta-1)}$, and $\frac{2-\beta-\rho}{2} < 1 - \frac{\rho}{2}$ Therefore, $r^*(I_0) = r^*(I_{0.5}) = \frac{2-\beta-\rho}{2}$ can induce acceptance from both the P_0 consumers with the belief $\mu(r^*(I_{0.5})) = 1-\beta$ and the P_1 consumers with the belief $\mu(r^*(I_{0.5})) = 0$. It remains to examine whether $r^*(I_1) = \min\{2-\beta-\frac{3\rho}{2}, 1\}$ can credibly signal the I_1 consumers of their type. Suppose that the firm deviates by recommending $r^*(I_1)$ to the I_0 segment, then the P_0 consumers will reject, and if the $P_{0.5}$ consumers accepts, the deviation profit is $\frac{r^*(I_1)}{2} + \frac{\rho}{4}$ under $P_{0.5}$'s most favorable belief. Since $r^*(I_1) \leq 2-\beta-\frac{3\rho}{2}$, we must have $\frac{r^*(I_1)}{2} + \frac{\rho}{4} \leq \frac{2-\beta-\rho}{2} = r^*(I_{0.5})$. Therefore, the firm has no incentives to deviate. In addition, since $1-\beta \leq \frac{3}{4} - \frac{\beta}{2}$, $\rho < 1-\beta$ implies that $\rho < \frac{3}{4} - \frac{\beta}{2}$, which is equivalent to $2-\beta-\frac{3\rho}{2} > \frac{1+\rho}{2}$. And since $1 > \frac{1+\rho}{2}$, $r^*(I_1) = \min\left\{2-\beta-\frac{3\rho}{2}, 1\right\} > \frac{1+\rho}{2}$. Therefore, $P_{0.5}$ consumers accept $r^*(I_1)$ with the updated belief $\mu(r^*(I_1)) = 1$.

If $\rho \in \left[1-\beta, \frac{1}{4\beta}\right]$, then $\frac{\rho}{2} \leq \frac{\beta-\rho}{2(2\beta-1)} \leq \frac{2-\beta-\rho}{2} \leq \frac{1}{2}$. Therefore, $r^*(I_0) = r^*(I_{0.5}) = \frac{\beta-\rho}{2(2\beta-1)}$ can induce acceptance from both the P_0 consumers with the belief $\mu(r^*(I_{0.5})) = 1-\beta$ and the $P_{0.5}$ consumers with the belief $\mu(r^*(I_{0.5})) = 0$. Then if $r^*(I_1) = \frac{\beta-\rho}{2\beta-1} - \frac{\rho}{2}$, then it credibly convince the I_1 consumers that it will never be sent to the I_0 segment, since $\frac{r^*(I_1)}{2} + \frac{\rho}{4} \leq \frac{\beta-\rho}{2(2\beta-1)} = r^*(I_{0.5})$. And I_1 consumers should accept the recommendation with the updated belief $\mu(r^*(I_1)) = 1$, since $r^*(I_1) \geq \frac{1+\rho}{2}$.

If $\rho \in \left(\frac{1}{4\beta}, \frac{1}{2}\right]$, there does not exist a semi-separating equilibrium for $r^*(I_0) = r^*(I_{0.5}) \neq r^*(I_1)$. This is because from the previous analysis, for any $r^*(I_1)$ that convinces I_1 consumers of their type, $\frac{r^*(I_1)}{2} + \frac{\rho}{4} \leq \frac{\beta-\rho}{2(2\beta-1)}$, the consumers cannot accept the recommendation since $r^*(I_1) < \frac{1+\rho}{2}$. Therefore, the firm will always deviate from this strategy.

Next, we examine the strategy $r^*(I_0) \neq r^*(I_{0.5}) = r^*(I_1)$. The strategy is pooling for a $P_{0.5}$ consumer, who accepts if and only if $r \in \left[\frac{1-\beta+\rho}{2}, \min\left\{\frac{3\beta-\rho-1}{2(2\beta-1)}, 1\right\}\right]$. Note that $\frac{3\beta-\rho-1}{2(2\beta-1)} \leq 1$ if only if $\rho \geq 1-\beta$. Therefore, the firm can set $r^*(I_{0.5}) = r^*(I_1) = \min\left\{\frac{3\beta-\rho-1}{2(2\beta-1)}, 1\right\}$ to ensure that $P_{0.5}$ consumers with the belief $\mu(r^*(I_{0.5})) = 1-\beta$ accept the recommendation. Since on the equilibrium, the P_0 consumers must update the belief upon observing $r^*(I_0)$ as $\mu(r^*(I_0)) = 0$. Thus, we can set $r^*(I_0) = \frac{1-\rho}{2}$ to ensure that the consumers accept $r^*(I_0)$ with the updated belief. For the equilibrium to survive the belief refinement, we must ensure that the firm will never deviate by sending $r^*(I_{0.5})$ to I_0. Therefore, the P_0 consumers must always reject $r^*(I_{0.5})$ in equilibrium so that the deviation incentives are absent. This implies that $r^*(I_{0.5})$ must be outside of the acceptance region under the updated belief $I_{0.5}$, which is $r \in \left[\frac{\rho}{2}, 1-\frac{\rho}{2}\right]$. Clearly, $\frac{3\beta-\rho-1}{2(2\beta-1)} > 1-\frac{\rho}{2}$, since $\rho < \frac{1}{2}$. Therefore, the semi-separating exists.

Finally, we derive the optimal strategy by making static comparisons. If $\rho < 1-\beta$, the firm can set $r^*(I_0) = r^*(I_{0.5}) = \frac{2-\beta-\rho}{2}$ and $r^*(I_1) = \min\{2-\beta-\frac{3\rho}{2}, 1\}$. Since all consumers accept, the expected profit is $\Pi_a^* = \frac{1}{4}\left[(1+\beta)(2-\beta-\rho) + 2(1-\beta)\min\{2-\beta-\frac{3\rho}{2}, 1\}\right]$. Alternatively, the firm can set $r^*(I_0) = \frac{1-\rho}{2}$, and $r^*(I_{0.5}) = r^*(I_1) = 1$, and only the P_0 consumers in the $I_{0.5}$ segment will reject. The expected profit is $\Pi_b^* = \frac{1}{4}[\beta(1-\rho) + (1-\beta)\rho + 2]$. Simple algebra yields that $\frac{1}{4}\left[(1+\beta)(2-\beta-\rho) + 2(1-\beta)\left(2-\beta-\frac{3\rho}{2}\right)\right] > \Pi_b^*$ if and only if $\rho < \frac{4-6\beta+\beta^2}{5-4\beta}$, and $\frac{1}{4}[(1+\beta)(2-\beta-\rho) + 2(1-\beta)] > \Pi_b^*$ and only if $\rho < \frac{2-2\beta-\beta^2}{2-\beta}$. In summary, the firm should use the strategy $r^*(I_0) = r^*(I_{0.5}) = \frac{2-\beta-\rho}{2}$ and $r^*(I_1) = \min\{2-\beta-\frac{3\rho}{2}, 1\}$ if $\rho <$

$\min\left\{\frac{2-2\beta-\beta^2}{2-\beta}, \frac{4-6\beta+\beta^2}{5-4\beta}\right\}$, and the strategy $r^*(I_0) = \frac{1-\rho}{2}$, and $r^*(I_{0.5}) = r^*(I_1) = 1$, if $\rho \in \left[\min\left\{\frac{2-2\beta-\beta^2}{2-\beta}, \frac{4-6\beta+\beta^2}{5-4\beta}\right\}, 1-\rho\right]$.

If $\rho \in \left[1-\beta, \frac{1}{4\beta}\right]$, then the firm can set $r^*(I_0) = r^*(I_{0.5}) = \frac{\beta-\rho}{2(2\beta-1)}$ and $r^*(I_1) = \frac{\beta-\rho}{2\beta-1} - \frac{\rho}{2}$. Since all consumers accept, the expected profit is $\Pi_a^* = \frac{1}{4}\left[(1+\beta)\frac{\beta-\rho}{(2\beta-1)} + 2\frac{\beta-\rho}{2\beta-1} - \rho\right]$. Alternatively, the firm can set $r^*(I_0) = \frac{1-\rho}{2}$, and $r^*(I_{0.5}) = r^*(I_1) = \frac{3\beta-\rho-1}{2(2\beta-1)}$, and only the P_0 consumers in the $I_{0.5}$ segment will reject. The expected profit is $\Pi_b^* = \frac{1}{4}\left[\beta(1-\rho) + (1-\beta)\rho + \frac{3\beta-\rho-1}{(2\beta-1)}\right]$. Since $\rho < \frac{1}{4\beta}$, Π_b^* is strictly lower than Π_a^*. Therefore, the firm should use $r^*(I_0) = r^*(I_{0.5}) = \frac{\beta-\rho}{2(2\beta-1)}$ and $r^*(I_1) = \frac{\beta-\rho}{2\beta-1} - \frac{\rho}{2}$.

If $\rho \in \left[\frac{1}{4\beta}, \frac{1}{2\beta} - (1-\beta)\right]$, then we need to compare the semi-separating strategy with the pooling strategy in Lemma 4. The firm's expected profit from pooling is $\Pi_a^* = \frac{\beta-\rho}{2(2\beta-1)}$, and from previous analysis, $\Pi_b^* = \frac{1}{4}\left[\beta(1-\rho) + (1-\beta)\rho + \frac{3\beta-\rho-1}{(2\beta-1)}\right]$. Since $\rho > \frac{1}{4\beta}$, it is clear that $\Pi_b^* > \Pi_a^*$. Thus, the pooling strategy is dominated. **QED**

Proof of Lemma 3:

First, consider the separating strategy for P_0 consumers. On the equilibrium, $\mu(r(P_0, B_0)) = 0$ and $\mu(r(P_0, B_1)) = 1$. We show that P_0 must reject $r(P_0, I_{0.5})$ on the equilibrium by contradiction. Suppose that the consumers accept $r(P_0, I_{0.5})$, we must have $r(P_0, I_{0.5}) \in \left[\frac{\rho}{2}, \frac{2-\rho}{2}\right]$, thus the firm profits at least $\frac{\rho}{2}$ by offering $r(P_0, I_{0.5})$. Since $\mu(r(P_0, I_0)) = 0$, then the firm profits at most $r(P_0, I_0)$, provided that the consumer accepts. This is because if the consumer rejects, then the firm obtain zero profit. If $\rho \geq \frac{1}{2}$, then $r(P_0, I_{0.5}) \geq \frac{\rho}{2} \geq \frac{1-\rho}{2} = r(P_0, I_0)$, thus the firm always has incentives to deviate by recommending $r(P_0, I_{0.5})$ to the (P_0, I_0) segment. Contradiction! Since the consumers always reject $r(P_0, I_{0.5})$, the firm may recommend $r(P_0, I_{0.5}) = 1$. Since the firm obtains $\frac{\rho}{2}$ from the $(P_0, I_{0.5})$ segment, it has no incentives to trick them into accepting $r(P_0, I_0) \leq \frac{\rho}{2}$, so that the equilibrium is sequentially rational.

If $\rho < \frac{1}{2}$, then $\frac{\rho}{2} < \frac{1-\rho}{2}$, thus the firm always has incentives to recommend $r(P_0, I_0)$ to the $(P_0, I_{0.5})$ segment. To ensure that $\mu(r(P_0, I_0)) = 0$, the firm must then reduce $r(P_0, I_0)$ to $\frac{\rho}{2}$. This is, however, not optimal, since for any $r' \in \left(\frac{\rho}{2}, \frac{1-\rho}{2}\right]$, both (P_0, I_0) and $(P_0, I_{0.5})$ segments would accept under the full information. Therefore, even under the least-favorable beliefs, the firm always has incentives to deviate from the separating equilibrium if $\rho < \frac{1}{2}$.

Similarly, we can show that $P_{0.5}$ must reject $r(P_{0.5}, I_1)$ on the separating equilibrium. Suppose that $P_{0.5}$ accepts $r(P_{0.5}, I_1)$ under the equilibrium belief $\mu\big(r(P_{0.5}, I_1)\big) = 1$, then we must have $r(P_{0.5}, I_1) \geq \frac{1+\rho}{2}$. If $\rho \geq \frac{2}{3}$, then $\frac{1+\rho}{2} > \rho \geq \frac{2-\rho}{2} = r(P_{0.5}, I_{0.5})$, thus the firm always has incentives to deviate by recommending $r(P_{0.5}, I_1)$ to the $(P_{0.5}, I_{0.5})$ segment. Therefore, $P_{0.5}$ must reject $r(P_{0.5}, I_1)$, in which the firm may recommend $r(P_{0.5}, I_1) = \frac{\rho}{2} - \varepsilon$. Again, the firm obtains ρ from the $(P_{0.5}, I_1)$ segment, it has no incentives to trick them into accepting $r(P_{0.5}, I_{0.5}) = \frac{2-\rho}{2} \leq \rho$. But if $\rho < \frac{2}{3}$, any separating strategy cannot be optimal, because $r' = \frac{2-\rho}{2}$ is accepted by both segments under the least-favorable belief.

Finally, we show that the separating equilibrium for P_0 survives D1. For a deviation r' to be profitable under the D1-belief, we must have $\mu(r') = 1$ and $r' > \frac{\rho}{2}$, otherwise if $\mu(r') = 0$, the firm does not have any incentive to deviate from $r(P_0, I_0) = \frac{1-\rho}{2}$, which is the highest recommendation that the consumer may accept under this belief. However, since for any arbitrary μ that enables $\Pi' > \Pi^*(P_0, I_{0.5}) = \frac{\rho}{2}$, we must also have $\Pi' \geq \Pi^*(P_0, I_0) = \frac{1-\rho}{2}$, it contradicts with the assumption of the D1-belief $\mu(r') = 1$. Therefore, the separating equilibrium survives D1. The case for $P_{0.5}$ is similar. **QED**

Proof of Lemma 4:

We first show the case for P_0. Note that if $\rho \leq \frac{1}{2}$, then $\min\left\{\frac{2-\beta-\rho}{2}, \frac{\beta-\rho}{2(2\beta-1)}\right\} \geq \frac{\rho}{2}$. Suppose there exists a deviation $r' > \min\left\{\frac{2-\beta-\rho}{2}, \frac{\beta-\rho}{2(2\beta-1)}\right\}$, such that for some out-of-equilibrium belief μ' that consumers accept r'. However, whenever the consumers accept, $\Pi' > \Pi^* = \min\left\{\frac{2-\beta-\rho}{2}, \frac{\beta-\rho}{2(2\beta-1)}\right\}$, and whenever the consumers reject, $\Pi' \leq \frac{\rho}{2} \leq \min\left\{\frac{2-\beta-\rho}{2}, \frac{\beta-\rho}{2(2\beta-1)}\right\}$. Therefore, there does not exist a profitable deviation under D1-belief in any behavioral segment.

However, if $\rho > \frac{1}{2}$, then $\min\left\{\frac{2-\beta-\rho}{2}, \frac{\beta-\rho}{2(2\beta-1)}\right\} < \frac{\rho}{2}$. Clearly, any deviation $r' > \frac{1+\rho}{2} > \min\left\{\frac{2-\beta-\rho}{2}, \frac{\beta-\rho}{2(2\beta-1)}\right\}$ is profitable for the segment (P_0,B_0) only in the belief set μ' that enables the consumers to accept. But for the segment (P_0,B_1), the deviation is profitable even under the belief that induces the consumers to reject. Therefore, the D1-belief should be updated to $\mu(r') = 1$, under which the consumer accepts r'. Therefore, the pooling equilibrium fails D1 if $\rho > \frac{1}{2}$.

The case for $P_{0.5}$ is similar. Note that $\frac{3\beta-1}{4\beta-1} < \beta$, thus if $\rho \leq \frac{3\beta-1}{4\beta-1}$, then $\left[\frac{1-\beta+\rho}{2}, \frac{3\beta-\rho-1}{2(2\beta-1)}\right]$ exists, thus the consumers accept $r = \frac{3\beta-\rho-1}{2(2\beta-1)}$ under the equilibrium belief $\mu(r) = 1 - \beta$. In addition, $\rho \leq \frac{3\beta-1}{4\beta-1}$, if and only if $\frac{3\beta-\rho-1}{2(2\beta-1)} \geq \rho$, which implies that the firm must be worse-off than the equilibrium when ($P_{0.5}$,I_1) rejects. Then any belief set that sustains a profitable deviation must be the same between ($P_{0.5}$,$I_{0.5}$) and ($P_{0.5}$,I_1). Therefore, the pooling equilibrium survives D1. However, if $\rho \in \left(\frac{3\beta-1}{4\beta-1}, \frac{3\beta-1}{2\beta}\right]$, then $\frac{\rho}{2} \leq \frac{3\beta-\rho-1}{2(2\beta-1)} < \rho < \frac{1+\rho}{2}$. Consider a deviation $r' = \frac{1+\rho}{2}$, then it is profitable for ($P_{0.5}$,I_1) regardless of whether they accept. But since for ($P_{0.5}$,$I_{0.5}$) the deviation is only profitable under the belief set that the consumers accept r', the firm always has stronger incentives to deviate with $r' = \frac{1+\rho}{2}$ to ($P_{0.5}$,I_1) than to ($P_{0.5}$,$I_{0.5}$). By the definition of D1, the consumer belief should be updated to $\mu(r') = 1$, under which the consumer accepts $r' = \frac{1+\rho}{2}$, and the firm is better off than the equilibrium. Therefore, the pooling equilibrium fails D1. If $\rho \in \left(\frac{3\beta-1}{2\beta}, 1\right)$, then $\frac{3\beta-\rho-1}{2(2\beta-1)} < \frac{\rho}{2}$, and the pooling is dominated by $r' = 0$, when all consumers reject. **QED**

Proof of Proposition 3:

We first show that the mixed strategy is impossible for P_0. Since the consumers are either I_0 and $I_{0.5}$, we must have $r_1 a = r_0$, thus the firm's expected profit is $\Pi^{M*} = r_0 + \frac{(1-\beta)\rho}{2}\left(1 - \frac{r_0}{r_1}\right)$, thus, it is optimal to set r_0 and $r_1 > \frac{1}{2}$ as large as possible. Note that maximum recommendation that (P_0, I_0) accepts under the informed belief is

$r_0 \leq \frac{1-\rho}{2}$, and the indifference acceptance condition indicates that $r_1 = \frac{1+\mu-\rho}{2}$. Clearly, for any $m > 0$ on the semi-separating equilibrium, there must exist a deviation $m' \in (0, m)$, such that the $\mu' > \mu$, and thus $r_1' > r_1$ is profitable. But if $m = 0$, the mixed strategy is reduced to a separating strategy, which cannot sustain the belief refinement. Therefore, there does not exist a semi-separating equilibrium for P_0.

By contrast, for $P_{0.5}$, we need to have $r_1 a + \frac{\rho}{2}(1-a) = r_0$. But since r_1 must be greater than $\frac{\rho}{2}$, the firm's incentive of minimizing a is equivalent of maximizing r_1, which is bounded by 1. Therefore, there must exist a condition under which the mixed strategy is feasible. Since $P_{0.5}$ is either $I_{0.5}$ or I_1, we have $r_0 \leq \frac{2-\rho}{2}$, and $r_1 \leq 1$. By the indifference acceptance condition, $r_1 = \frac{\rho+3\mu-2}{2(2\mu-1)}$, the optimal mixing probability satisfies $\frac{1-\beta}{1-\beta+\beta m} = \mu = \rho$, which implies $m^* = \frac{(1-\beta)(1-\rho)}{\beta\rho}$. Since $m^* \in (0,1)$, this condition requires that $\rho > 1-\beta$. By the indifference condition that $r_1 a + \frac{\rho}{2}(1-a) = r_0$, we have $a^* = 1 - \frac{\rho}{2-\rho}$.

Finally, we compare the mixed strategy with the separating strategy and the pooling strategy specified in Lemma 3 and 4. Note that $\Pi^{M*} = r_0^* + \frac{(1-\beta)\rho(1-a^*)}{2} = \frac{(2-\rho)^2+\rho^2(1-\beta)}{2(2-\rho)}$, but the expected profit from separating strategy is $\Pi^{S*} = \beta r(P_1, B_0) + (1-\beta)\rho = \frac{(2-\rho)\beta+2\rho(1-\beta)}{2}$, thus $\Pi^{M*} - \Pi^{S*} = \frac{2(1-\rho)^2(1-\beta)}{2-\rho} > 0$. Therefore, the mixed strategy dominates the separating strategy. Next consider the pooling strategy $\Pi^{P*} = \frac{3\beta-\rho-1}{2(2\beta-1)}$. Thus $\Pi^{M*} - \Pi^{P*} = \frac{G(\rho)}{2(2\beta-1)(2-\beta)}$, where $G(\rho) = \beta^2(3-2\rho^2) + \beta(1-9\rho+5\rho^2) + 2(3\rho-\rho^2-1)$. Note that $\frac{\partial}{\partial\rho}G(\rho) = 2(2-\beta)(2\beta-1)\rho - 3 < 0$, thus $\Pi^{M*} - \Pi^{P*} \geq \frac{G(\rho=1)}{2(2\beta-1)(2-\beta)} = \frac{1-\beta}{2(2\beta-1)} > 0$. Therefore, the mixed strategy dominates the pooling strategy. **QED**

Proof of Proposition 4:

The static comparison is straight forward from simple algebra. First, note that when $\rho < \min\left\{\frac{2-2\beta-\beta^2}{2-\beta}, \frac{4-6\beta+\beta^2}{5-\beta}\right\} < 1-\beta$, $\mathrm{E}\Pi^{\mathrm{I}} = \frac{1}{4}\left[(1+\beta)(2-\beta-\rho) + 2(1-\beta)\min\{2-\beta-\frac{3\rho}{2}, 1\}\right] \leq \frac{1}{4}[4-(\rho+\beta)(1+\beta)] < \frac{1}{4}[4-(\rho+\beta)] = \mathrm{E}\Pi^{\mathrm{P}} = \mathrm{E}\Pi^{\mathrm{BP}}$. Therefore, the personal-based and four-segment strategy dominate in this region.

Third, when $\rho \in \left[\min\left\{\frac{2-2\beta-\beta^2}{2-\beta}, \frac{4-6\beta+\beta^2}{5-\beta}\right\}, 1-\beta\right)$, $E\Pi^{I} = \frac{1}{4}[\beta(1-\rho) + (1-\beta)\rho + 2]$. Simple algebra yields that $E\Pi^{P} = E\Pi^{PI} = \frac{1}{4}[4-(\rho+\beta)] > E\Pi^{I}$. Therefore, the personal-based and four-segment strategy dominate in this region.

Third, we show that $E\Pi^{PI}$ dominates when $\rho > \frac{5-\sqrt{1+16\beta}}{6-4\beta}$. In the region of $\rho \in \left[\frac{1}{4\beta}, 1\right)$, we must have $E\Pi^{I} = \frac{1}{4}\left[\beta(1-\rho) + (1-\beta)\rho + \frac{3\beta-\rho-1}{(2\beta-1)}\right]$ and $E\Pi^{PI} = \frac{\rho^2(2\beta-1)(1-\beta)+(2-\rho)(5\beta-2\rho\beta-2)}{4(2-\rho)(2\beta-1)}$. Therefore, $E\Pi^{I} - E\Pi^{PI} = \frac{1-\beta}{2-\rho}(1-\rho)(3\rho-2) < 0$, since $\rho < \frac{1}{2}$. Similarly, in this region, $E\Pi^{PI} > E\Pi^{P}$ is equivalent to $\rho > \frac{5-\sqrt{1+16\beta}}{6-4\beta}$. Therefore, $E\Pi^{PI} > \max\{E\Pi^{I}, E\Pi^{P}\}$, if and only if $\rho > \frac{5-\sqrt{1+16\beta}}{6-4\beta}$.

Since $\frac{5-\sqrt{1+16\beta}}{6-4\beta} > \frac{1}{4\beta}$, it remains to examine the case $\rho \in \left[1-\beta, \frac{1}{4\beta}\right]$. Since $E\Pi^{P} - E\Pi^{I} = \frac{\beta+\beta^2-1+2\beta\rho(1-\beta)}{4(2\beta-1)} \geq \frac{\beta^3-(1-\beta)^3}{4(2\beta-1)} \geq 0$, as $\rho \geq 1-\beta$. Therefore, the personal-based discrimination dominates in this region. **QED**

Proof of Proposition 5:

We make static comparison using simple algebra to characterize the boundary conditions among the data usage rules. Clearly, CS^{PI} is the lowest. It suffices to examine CS^{P} and CS^{I}.

First, when $\rho \geq \frac{1}{4\beta} \geq 1-\beta$, we have $CS^{P} = CS^{I} = CS^{PI} = \frac{\rho}{2}$. Considering the results in Proposition 5, this implies that BP-discrimination is Pareto-efficient if and only if $\rho \geq \frac{5-\sqrt{1+16\beta}}{6-4\beta}$, since $\frac{5-\sqrt{1+16\beta}}{6-4\beta} \geq \frac{1}{4\beta}$ for any $\beta \in \left[\frac{1}{2}, 1\right]$. In addition, P-discrimination is Pareto-efficient if and only if $\rho \in \left[\frac{1}{4\beta}, \frac{5-\sqrt{1+16\beta}}{6-4\beta}\right)$.

Second, if $\rho \in \left(1-\beta, \frac{1}{4\beta}\right)$, $CS^{I} = \frac{1}{4(2\beta-1)}[(2-\beta)-(1-\beta^2)(1+2\rho)] > CS^{P} = \frac{\rho}{2}$. Therefore, I-discrimination maximizes the consumer surplus.

Third, if $\rho \in \left(\frac{2}{3}(1-\beta), 1-\beta\right)$, then if $2 < 5\beta^2$, $\min\left\{\frac{2-2\beta-\beta^2}{2-\beta}, \frac{4-6\beta+\beta^2}{5-4\beta}\right\} < \frac{2}{3}(1-\beta)$, and $CS^{I} = \frac{1-\beta+\rho}{4}$. If $2 \geq 5\beta^2$, then there exists a region $\rho \in \left[\frac{2}{3}(1-\beta), \min\left\{\frac{2-2\beta-\beta^2}{2-\beta}, \frac{4-6\beta+\beta^2}{5-4\beta}\right\}\right]$, in which $CS^{I} = \frac{1}{4}[\rho+\beta^2+\beta\rho+(1-\beta)(3-2\beta-3\rho)]$.

Note that $\mathrm{CS^I} - \frac{2\rho+\beta(1-2\rho)}{4} = \frac{3(1-\beta)^2+2\rho(3\beta-2)}{4}$. If $3\beta - 2 \geq 0$, then since $\rho > \frac{2}{3}(1-\beta)$, $\mathrm{CS^I} - \frac{2\rho+\beta(1-2\rho)}{4} > \frac{1-\beta}{4}\left(\frac{1}{3}+\beta\right) > 0$. If $3\beta - 2 < 0$, then since $\rho < (1-\beta)$, we also have $\mathrm{CS^I} - \frac{2\rho+\beta(1-2\rho)}{4} > \frac{1-\beta}{4}(3\beta-1) > 0$. In addition, $\mathrm{CS^I} - \frac{2\rho+\beta\rho}{4} = \frac{3(1-\beta)(1-\beta-\rho)+\beta-\rho}{4} > 0$, since $\rho < (1-\beta)$ and $\rho < \frac{1}{2} \leq \beta$. Therefore, $\mathrm{CS^I} > \max\left\{\frac{2\rho+\beta(1-2\rho)}{4}, \frac{2\rho+\beta\rho}{4}\right\}$.

Forth, if $\rho \leq \frac{2}{3}(1-\beta)$, then $\min\{1, 3-2\beta-3\rho\} = 1$. And if $2 \geq 5\beta^2$, then $\min\left\{\frac{2-2\beta-\beta^2}{2-\beta}, \frac{4-6\beta+\beta^2}{5-4\beta}\right\} \geq \frac{2}{3}(1-\beta)$. In this case, if $\rho < \min\left\{\frac{2-2\beta-\beta^2}{2-\beta}, \frac{4-6\beta+\beta^2}{5-4\beta}\right\}$, we must have $\mathrm{CS^I} = \frac{1}{4}[\rho+\beta^2+\beta\rho+(1-\beta)] > \frac{1-2\beta(1-\beta-\rho)}{4}$, and $\mathrm{CS^I} > \frac{1-\beta+\rho}{4} = \mathrm{CS^P}$. Therefore, I-discrimination maximizes consumer-surplus. But if $2 < 5\beta^2$, then there exists a region $\rho \in \left[\min\left\{\frac{2-2\beta-\beta^2}{2-\beta}, \frac{4-6\beta+\beta^2}{5-4\beta}\right\}, \frac{2}{3}(1-\beta)\right]$, $\mathrm{CS^P} = \mathrm{CS^I} = \frac{1-\beta+\rho}{4}$.

In summary, for any $\rho < \min\left\{\frac{2-2\beta-\beta^2}{2-\beta}, \frac{4-6\beta+\beta^2}{5-4\beta}\right\}$, and $\rho \in \left(1-\beta, \frac{1}{4\beta}\right)$, I-discrimination maximizes consumer-surplus; and if $\rho \in \left[\frac{1}{4\beta}, 1\right)$, consumers are indifferent on average. It remains to check whether any data usage rule is Pareto-efficient if $\rho < \frac{1}{4\beta}$. It is clear that I-discrimination cannot be Pareto-efficient since it is never firm-preferred. In addition, both the four-segment and P-discrimination are not consumer beneficial in this region, thus no data usage is Pareto-efficient. **QED**