Agency Bidding in Online Advertising

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Abstract

We study the impact of ad agencies on the online advertising market. To increase their clients' payoffs, in a process known as "bid rotation," agencies act as bidding rings where they strategically hold out some ad candidates from publishers' auctions to soften competition. We show that by withholding targeting information from agencies, publishers can dampen the effectiveness of bid rotations and induce some advertisers to directly use the publishers' ad platforms instead of using an agency. Withholding information from agencies, however, may also hurt the publishers by lowering the efficiency of allocation for advertisers who use agencies. The publishers' central trade-off is that disclosing information increases the size of the pie by increasing efficiency, but risks losing a larger share of the pie to the agency. We find that for an intermediate value of information, the publishers "compete" against agencies by withholding information to induce advertisers not to use the agency. On the other hand, if the value of information high, withholding information becomes too costly for publishers; therefore, publishers "cooperate" with agencies by disclosing information.

Keywords: ad agency, bid delegation, bidding collusion, information disclosure

1 Introduction

Online advertising spending in the US is expected to reach $248B in 2022.[1] The growth in size and complexity of this industry, combined with the proliferation of new media types (e.g., mobile, connected TV, and retail media), has motivated advertisers to use intermediaries, known as *digital advertising agencies*, for online advertising. Digital advertising agencies (henceforth, agencies) help advertisers meet their advertising goals by managing campaigns and optimizing bidding and targeting on their behalf. In recent years, agencies have become an integral part of the online advertising ecosystem. The annual revenue of digital agencies in 2022 is projected to exceed $30 billion dollars in the US, a 106% growth in just five years.[2] Moreover, a recent study by Decarolis and Rovigatti (2021) indicates that agencies are involved in as much as 75% of ads sold on Google search.

Advertisers use agencies for multiple reasons. Some agencies provide end-to-end solutions, from product positioning and creative design to bidding and optimization across platforms. Moreover, because agencies serve as advertisers' agents, advertisers are more willing to provide sensitive information such as profit margins and conversion data with agencies than with publishers.[3]

In addition to optimizing bids and targets, agencies act as *bidding rings*; they send only few ad candidates to publishers' auctions to soften competition and increase surplus for advertisers. The agencies' role in facilitating collusion is not new to the literature (Villas-Boas, 1994). However, in the context of online advertising, the collusion extends beyond information sharing. To illustrate, consider an impression opportunity for which three advertisers – A, B, and C – have valuations $3, $2 and $1, respectively. Suppose that A and B use a common agency, whereas C directly bids on the publisher's advertising platform. If the agency submits the bids of A and B, the (second-price) auction clears at a price of $2. However, if the agency only submits A's bid to the publisher's auction, the auction clears at $1, decreasing the publisher's revenue by $1. The drop in publisher's revenue due to collusion translates to higher surplus for the agency (potentially split with advertisers). Such agency-based collusion is common in practice in the online ad industry. Indeed, a recent study estimates that, due

[1] https://forecasts-na1.emarketer.com/584b26021403070290f93a56/5851918a0626310a2c1869c4

[2] www.statista.com/statistics/299099/revenue-of-digital-advertising-agencies-in-the-us

[3] We broadly use the term publisher to refer to any online platform that shows ads. This includes search engines such as Google, social media platforms such as Facebook, e-commerce platforms such as Amazon, as well as content providers such as The New York Times.

to agency's role in promoting collusion, mergers and acquisitions among agencies in 2014-2017 have lowered Google's search ad revenue by 11% (Decarolis and Rovigatti, 2021).[4]

Despite their role in facilitating collusion, agencies can also be a boon to publishers. During the initial growth phase, new publishers rely on agencies for several reasons. First, it is challenging and inefficient for nascent publishers to acquire new advertisers. Working with agencies (who have many advertisers as their clients) could be economical for acquisition purposes. In fact, new publishers work closely with agencies as they develop their advertising platforms. Second, it takes several years for new publishers to develop automated bidding and targeting tools. Advertisers initially have limited access to the publisher as advertisers do not know whom to target, how much to bid, or how to measure their advertising campaigns. Agencies automate this process and help both advertisers and publishers by reducing market friction.

In total, publishers face a dilemma when working with agencies. On the one hand, agencies can act as bidding rings and soften advertiser competition, thereby lowering publishers' revenues. On the other hand, they benefit indirectly from the agencies' service to advertisers that automate and optimize the bidding process. Publishers also benefit from demand expansion through agencies. In this paper, we study how the trade-offs of working with agencies impact publishers' strategies. Specifically, we focus on the implications for the publisher's information disclosure strategy. We examine the conditions under which a publisher shares more or less information about the ad inventory with agencies.

The following example illustrates the significance of information disclosure in a setting where advertisers potentially delegate bidding to agencies. The example highlights a key force in the paper: the publisher's withholding information can *weaken* the agency by making its bidding collusion less efficient.

Example: Consider three advertisers M, F, and U, who respectively produce male, female, and unisex shoes. Consumers query "shoes" and potentially click on shoe ads displayed on the search results. Consumers are more likely to click on gender-matching ads than gender-mismatching ads; e.g., male consumers are more likely to click on M's ad than F's ad. Combining consumers' expected click-through rates (CTRs) with the advertisers' per-click valuations, the auction scores for each consumer-

[4]Bidding rings within agencies are legal because the intermediaries are independent entities from advertisers. However, agency delegation with explicit intent to collude would constitute anti-trust infringement; this is beyond the scope of our paper. See Decarolis and Rovigatti (2021) for details.

advertiser pair can be computed as a product of the advertisers' CTRs and bids (see Table 1).[5]

Advertiser	Bidding Channel	Bid ($)	CTR		Auction Score ($)	
			Male	Female	Male	Female
M	Agency	3	.9	.1	2.7	.3
F	Agency	4	.1	.9	.4	3.6
U	Direct	2	.5	.5	1	1

Table 1: Illustrative Example

Suppose M and F delegate to the agency while U bids directly to the publisher's platform. Suppose a male customer generates an ad impression opportunity. If the publisher discloses gender information, the agency knows that for this ad opportunity M has a higher auction score than F; i.e., the agency knows the strongest ad candidate. Thus, the agency sends M (and *only* M) to the auction, who then competes against the direct-bidder U. Since M's auction score is \$2.7 and U's \$1, M wins the auction and pays to the publisher

$$\text{CTR}_{M,\text{Male}} \times \inf\{b : \text{CTR}_{M,\text{Male}} \times b \geq \text{CTR}_U \times \$2\} = \$1. \tag{1}$$

On the other hand, if the publisher withholds gender information, the agency cannot accurately predict its clients' expected CTRs; i.e., it does not know which advertiser is the strongest candidate. The agency's optimal strategy is to send to the publisher's auction the advertiser with the highest per-click bid, which is F. The publisher then computes F's auction score by accounting for its CTR for a male customer. Since this CTR is low (due to gender-mismatch), F's auction score suffers and F loses to U. U's expected payment to the publisher is

$$\text{CTR}_U \times \inf\{b : \text{CTR}_U \times b \geq \text{CTR}_{F,\text{Male}} \times \$4\} = \$.4. \tag{2}$$

There are two main takeaways from this example. First, the publisher's withholding information can decrease the probability of the agency's bidding collusion winning in the auction – we call this *weakening* the agency. This effect can either induce the agency to send multiple ad candidates to the publisher's auction, or incentivize advertisers to abandon the agency and directly use the publisher's platform. On the other hand, withholding the information can be costly for the publisher: since M,

[5] We use the terms *auction score* and *effective bid* interchangeably.

who generates the highest potential revenue for the male impression, is not sent by the agency to the publisher's auction, the publisher's revenue decreases by 60% from (1) to (2).

In total, it is *a priori* unclear how the publisher should manage information in a setting where the agency strategically manipulates the auction density. This paper seeks (a) to shed light on the nuanced dynamics between the publisher and the agency, and (b) to elucidate the conditions under which the publisher withholds or discloses information. We are interested in the following research questions:

1. Under what market conditions should a publisher share more or less information with advertisers and agencies?
2. What is the agencies' optimal strategy and how does it vary by market conditions?
3. Under what conditions should advertisers delegate bidding to the agency or bid directly on the publisher's advertising platform?

To address these questions, we develop a game theory model with two exogenous parameters as market conditions: *the campaign management cost* and *the value of information*. The campaign management cost is incurred by advertisers that bid directly on the publisher's platform. This cost may be low if, for example, the publisher provides automated campaign management such as targeting and bidding tools (Amaldoss et al., 2016). Instead of bidding directly, advertisers may delegate to the agency at some endogenous fee. In this case, advertisers do not incur campaign management costs.

The second parameter is the value of the information that the publisher can share with bidders (advertisers or agencies); this is captured by the reduction in bidders' uncertainty surrounding their auction scores. For example, in the context of fashion apparel, CTRs may be highly correlated with gender. In this case, the customer's gender is valuable information because the predicted CTRs, and therefore, the effective bids, can vary by gender. This information, or lack thereof, affects the agency's fee, as well as its collusion strategy. For instance, if the agency does not have this information, the cost of collusion would be high for the agency because it does not know which candidate to send to the publisher's auction.

In answering the first question, we find that the presence of agencies qualitatively changes the publisher's information disclosure incentives. When the campaign management cost is low and the information value is high, by withholding information, the publisher can motivate some (but not all) of the advertisers to bid directly to the publisher instead of using the agency. However, the publisher only uses this lever (i.e., information restriction) if the value of information is not too high. If the value of information is high, the negative impact of inefficient allocation on the publisher's revenue dominates the positive impact of mitigating bid collusion. On the other hand, when the value of information is intermediate, the publisher's optimal strategy is to withhold information. Even though disclosing information enhances efficiency, the publisher withholds information to weaken the agency's market power, thereby mitigating the revenue loss due to bid collusion.

In answering the second question, we show that under certain conditions the agency rotates bids to soften competition. If the publisher discloses information, the agency only sends the advertiser with the highest probability of winning to the publisher's auction. However, if the publisher withholds information, the agency's strategy depends on the advertisers' probabilities of winning. If the agency has an advertiser with a sufficiently high probability of winning, the agency only sends that advertiser to the publisher's auction. If all advertisers who use the agency have low probabilities of winning, the agency sends multiple advertisers to the auction. In doing so, the agency maximizes the probability of winning in the auction, at the expense of foregoing collusion surplus. We also show that the agency's fee changes non-monotonically in the campaign management cost and in the value of information due to shifts in the publisher's information disclosure strategy.

In answering the third question, we find that advertisers' decisions to either bid directly or delegate to the agency depend crucially on the publisher's information disclosure strategy. For example, if information is withheld and the campaign management cost is low, then some advertisers bid directly to the publisher in competition against the agency's collusion. This is because the winning probability is higher under direct-bidding than under the agency's bid rotation. On the other hand, if information is disclosed, advertisers delegate to the agency, however low the campaign management cost. Intuitively, information disclosure strengthens agency collusion, which in turn keeps advertisers from defecting and competing against an efficient collusion.

In sum, our paper provides both theoretical and managerial contributions. We demonstrate that insights from the theory literature regarding information disclosure in online advertising, notably the market-thinning effect (e.g., Bergemann and Bonatti, 2011; Levin and Milgrom, 2010; Rafieian and Yoganarasimhan, 2021) can change qualitatively in the presence of intermediaries. Beyond the market-thinning effect, whereby information disclosure thins out auctions and reduces ad revenue, the presence of agencies introduces a new trade-off between efficiency and surplus share. We shed light on a novel incentive for publishers to withhold information from agencies and advertisers; i.e., to weaken the agency and induce advertisers to bid directly to the publisher, thereby mitigating the revenue loss from the agency's bidding collusion.

In addition, our paper provides important insights for managers navigating the increasingly complex online ad system. Understanding the effect of information disclosure on publishers, advertisers, and agencies is particularly important in the context of retail media. Retail platforms such as Walmart and Instacart have enabled advertising on their platforms. The campaign management tools they provide are primitive compared to those offered by more established platforms such as Google and Facebook. Moreover, the information retail platforms have access to are different from, and potentially more valuable than the information search engines and social media platforms have access to. Our results suggest that while retail platforms may benefit from using agencies to expand advertising demand, they should invest in lowering campaign management costs for advertisers, even if most of their demand comes from agencies. By doing so, publishers can maximize the positive effect of withholding information: i.e., weakening the bidding ring and inducing advertisers to bid directly on the platform.

Related Literature

Our paper contributes to the common agency literature. Villas-Boas (1994) analyzes firms' trade-offs of using a common agency in a setting where going through agencies enhances efficiency but creates information spillovers. In our model, the advertisers' trade-offs of using agencies pertain to agencies' bid rotation strategies, as opposed to sharing private information to competitors. Bernheim and Whinston (1985) show that competing firms may collude through a common agency. While we identify a similar effect in the context of online advertising, our core insight pertains to the impact of agencies on the publisher's information disclosure strategies that affect the equilibrium collusion outcome.

Our work also relates to the literature on information disclosure in online advertising (e.g., de Corniere and de Nijs, 2016; Hummel and McAfee, 2016; Rafieian and Yoganarasimhan, 2021; Shi et al., 2022). The general finding is that the publisher's revenue is inverted U-shaped in the level of information disclosed. Sharing information with advertisers increases targeting capabilities, and hence the advertisers' willingness-to-pay for ad inventory (e.g., Ada et al., 2021). However, as more information is shared, advertisers can target more finely, such that fewer advertisers bid on a given impression. Overall, competition softens and the publisher's revenue falls. This is known as the market-thinning effect of information disclosure. While the setting where publishers interact directly with advertisers has been studied extensively – both theoretically (e.g., Bergemann and Bonatti, 2011; Choi and Sayedi, 2019) and empirically (e.g., Rafieian and Yoganarasimhan, 2021; Wu, 2015) – the role of agencies has been largely overlooked in the literature. This paper demonstrates that the existence of agencies has important implications for the publisher's strategy.

We contribute to the economics literature on bidding rings. Aoyagi (2003) characterizes a grim-trigger-based equilibrium where bidding collusion is enabled by the existence of a "communication center." McAfee and McMillan (1992) analyze collusive mechanisms among bidders in the absence of an intermediary. In contrast to these papers, our key model feature is that agencies facilitate bidding collusion that attenuates auction density (Decarolis and Rovigatti, 2021). Allouah and Besbes (2017) and Decarolis et al. (2020) study a similar setup in which ad sellers react to agency-collusion under complete information. Allouah and Besbes (2017) examine the effects of agency collusion on advertisers' surplus. Decarolis et al. (2020) demonstrates the superiority of the Vickrey-Clark-Groves (VCG) mechanism over the Generalized Second-Price (GSP) mechanism in terms of publisher revenue. Our main interest lies is the publisher's information disclosure strategy, which impacts advertisers' and agencies' knowledge of auction scores.

Broadly, our work contributes to the burgeoning literature on online advertising and advertising auctions. Katona and Sarvary (2010) and Jerath et al. (2011) study advertisers' incentives in obtaining low vs. high positions in search advertising auctions. Sayedi et al. (2014) and Desai et al. (2014) investigate the competition between brand owners and their competitors on brand keywords. Lu et al. (2015) and Shin (2015) study budget constraints and budget allocation across keywords. Similarly, Zia and Rao (2019) investigate the budget allocation problem across search engines. Wilbur and Zhu (2009) find conditions under which allowing click-fraud is a search engine's profit-maximizing strategy. Cao and Ke (2019) and Jerath et al. (2021) study manufacturers' and retailers' cooperation in search advertising its impact on intra- and inter-brand competition. Amaldoss et al. (2015) show how a search engine can increase its profits and improve advertiser welfare by providing first-page bid estimates. Berman and Katona (2013) study the impact of search engine optimization, and Amaldoss et al. (2016) analyze the effect of keyword management costs on advertiser strategies. While our formulation of campaign management costs is similar to Amaldoss et al. (2016), our model considers an alternative channel (i.e., agencies) through which advertisers can participate in the auction. Agencies mitigate campaign management costs and strategically distort auction density; this latter force, absent in Amaldoss et al. (2016), is central to our results. Long et al. (2022) study the informational role of search advertising on the organic rankings of online retail platforms. Sayedi et al. (2018) study advertisers' bidding strategies when publishers allow advertisers to bid for exclusive placement on their websites. Zhu and Wilbur (2011) and Hu et al. (2015) study the trade-offs involved in choosing between "cost-per-click" and "cost-per-action" contracts. Berman (2018) explores the effects of advertisers' attribution models on their bidding behavior and their profits. Despotakis et al. (2021b), Subramanian and Zia (2021), and Gritckevich et al. (2022) examine the impact of ad blockers on the online advertising ecosystem, and Dukes et al. (2022) show how skippable ads affect publishers' and advertisers' strategies and profits. Choi et al. (2022) analyze consumers' privacy choices which affect their targetability along the purchase journey. Kuksov et al. (2017) study firms' incentives to display ads of their competitors on their websites. Choi and Sayedi (2019) study the optimal ad selling mechanism when a publisher does not know, but benefits from learning, the performance of firms' ads. Choi and Mela (2019) study the problem of optimal reserve prices in the context of real-time bidding (RTB), and estimate the advertisers' demand curve as a function of the reserve price. Despotakis et al. (2021a) examine how the transition from waterfalling to header bidding alters the competition between exchanges, and how this change motivates exchanges to move from second- to first-price auctions. Zeithammer (2019) shows that introducing a soft reserve price, a bid level below which a winning bidder pays its own bid instead of the second-highest bid, cannot increase publishers' revenue in RTB auctions when advertisers are symmetric; however, it can when advertisers are asymmetric. Sayedi (2018) analyzes the interaction between selling impressions through RTB and selling through reservation contracts. Choi and Sayedi (2022) study the emergence of private exchanges in the display advertising market and its impact on advertisers' and publishers' strategies. Shin and Yu (2021), Despotakis and Yu (2022), and Ning et al. (2022) explore the strategic implications of consumer inference for advertisers' targeting strategies. While these papers do not distinguish between advertisers and agencies, we explicitly draw this distinction and investigate the impact of agencies on publishers' and advertisers' strategies. Several papers (e.g., Amaldoss et al., 2016; Despotakis et al., 2021a; Gordon et al., 2021; Sayedi, 2018) mention studying the role of agencies as a direction for future research; however, this problem has been understudied in the literature. Shin and Shin (2022) provide an agency-based rationale for the persistence of irrelevant advertising. Our model is different in that we account for bidding collusion facilitated by agencies, whereas agencies in Shin and Shin (2022) are contractually obligated to fulfill their clients' impressions requirements across time.

The rest of the paper is organized as follows. In Section 2, we describe the main model, and in Section 3, we present the analyses for the benchmark (without the agency) and the main model (with agency). In Section 4, we analyze extension models to demonstrate robustness of our key insights. Section 5 concludes. Proofs for the main model and extension models are relegated to the Appendix and Online Appendix, respectively.

2 Model

The game consists of a publisher, an ad agency, and three advertisers. The publisher sells a single ad slot via second-price auction.[6] Its decision variables are two-fold: reserve price R and information regime $I \in \{0, 1\}$, where 0 and 1 denote withholding and disclosing information, respectively. The agency charges an upfront fee F for managing advertisers' bids for the publisher's ad auction.[7]

Advertisers decide whether to delegate bidding to the agency at fee F, or to bid directly through the publisher's platform and incur a campaign management cost of k. Advertisers can abstain from the auction and receive a normalized outside option utility of 0. The campaign management cost $k \in [0, 1/2]$ captures various costs ranging from the costs of identifying ad opportunities to the operational costs of setting up and managing bids on the publisher's platform.[8]

[6] While ad exchanges that connect third-party publishers and advertisers have moved to first-price auctions, publishers primarily use (variations of) second-price auctions when directly selling their own inventory to agencies and advertisers (Despotakis et al., 2021a). For example, Google uses GSP auctions for selling search ad slots and YouTube impressions. Facebook uses VCG mechanism, a generalization of second-price auctions. Walmart recently announced its transition from first-price to second-price auction (`https://bit.ly/3fbywYW`).

[7] For technical reasons, we also assume that the agency charges advertisers an infinitesimal commission rate $\alpha \downarrow 0$ as a percentage of the advertiser's utility. This assumption allows us to align the agency's incentives with the advertiser's; therefore, after charging the fee F, the agency maximizes the advertiser's surplus. In practice, agencies maximize advertisers' surplus for contract renewals. To parsimoniously capture such dynamics in a static model, we create that incentive for the agency by assuming the agency takes an infinitesimal cut from the advertiser's utility.

[8] We rule out the trivial case of $k > 1/2$ where the advertiser does not bid directly even at $R = 0$.

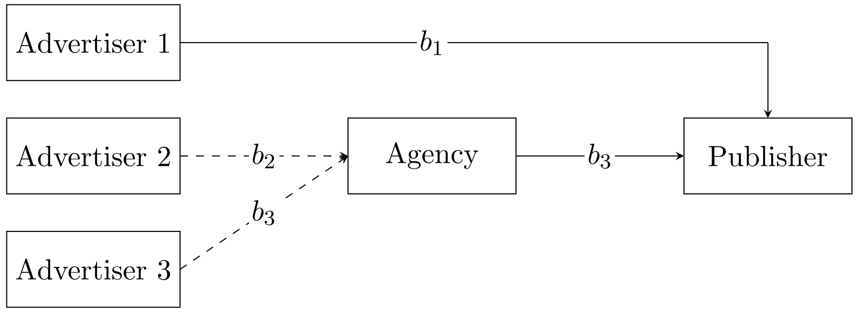

Figure 1: Model Illustration

If more than one advertiser delegates to the agency, the agency decides the subset of advertisers to send to the publisher's ad auction. For instance, if two advertisers delegate to the agency and one advertiser bids directly to the publisher, the agency may send only one or both advertisers to the publisher's auction. If the agency sends only one advertiser, the auction density (i.e., total number of auction participants), including the direct-bidding advertiser, will be two (see Figure 1). The agency facilitates collusion among advertisers by sending only few advertisers to the publisher's auction. This decreases the publisher's auction revenue, while increasing the winning advertiser's expected utility. We assume that if the agency is indifferent between sending few or many advertisers to the auction, the agency breaks the tie by sending few advertisers.[9]

Role of Information: Revealing information about the impression opportunity can have two effects on agencies and advertisers. It can improve the bidders' estimates of (i) the probability of winning in the auction, and (ii) the valuation conditioned on winning. For instance, in a pay-per-click auction where the publisher multiplies bids with expected CTRs, the CTR may be correlated with, say, demographic information and web page content.[10] Sharing information enables bidders to estimate their CTRs more accurately, and thus their probabilities of winning in the auction. Such information can be valuable for agencies in deciding which advertiser to send to the publisher's auction.

Similarly, to the extent that conversion rates (i.e., the probability of conversion conditioned on a click) are correlated with customer or context information, information disclosure impacts advertisers' per-click valuations, and thus their bids in pay-per-click auctions. Information disclosure may have both effects simultaneously: improve bidders' estimates of probability of winning *and* their estimates of

[9] For example, this can be due to the small operational costs of ad serving.

[10] While our model is presented for pay-per-click auctions, the logic extends to pay-per-impression pricing as well. The key model feature that drives our results is neither the CTR nor the pricing model; it is that advertisers and the agency do not fully know the publishers' *ad ranking algorithm*. For example, in pay-per-impression auctions, the publisher sometimes selects advertisers with lower bids because they are deemed more relevant (or less harmful) for consumers.

per-click valuation. In the main model, we focus on the first effect of information sharing to deliver our main insights cleanly; i.e., we assume that information is correlated with CTRs, but does not impact per-click valuation. In Section 4.1, we show that allowing information to impact both the CTRs and per-click valuation does not qualitatively change the core insights.

We assume that advertisers have homogeneous per-click valuations, which we normalize to 1. Advertisers differ in their CTRs, which are independently and identically distributed as

$$c = \begin{cases} \frac{1}{2} - \sigma & \text{with probability } \frac{1}{2}, \\ \frac{1}{2} + \sigma & \text{with probability } \frac{1}{2}, \end{cases} \tag{3}$$

where $0 \leq \sigma \leq 1/2$. Whereas the CTR distribution (3) is common knowledge, the actual CTR realizations are privately known by the publisher (de Corniere and de Nijs, 2016). If the publisher withholds information (i.e., $I = 0$), the advertisers and the agency do not know the advertisers' CTR prior to bidding in the auction. If the publisher discloses information (i.e., $I = 1$), the advertisers and the agency know the CTRs. The publisher's information disclosure can also be interpreted as the publisher (dis)allowing targeted bidding. It is worth noting that the example in the introduction (Table 1) corresponds to $\sigma = .4$ with half of the consumer population being male, and the other half female in our model.

Auction: The auction allocation and payment are determined by the advertisers' effective bids:

$$\text{effective bid} = \text{CTR} \times \text{bid}.$$

The bidder with the highest effective bid wins the auction (provided it exceeds the reserve price), and pays the minimum bid required to win the auction. For instance, suppose $R = .02$, and the CTRs, bids, and effective bids of two advertisers are as given in Table 2.

Advertiser	CTR	Bid ($)	Effective Bid ($)
M	4%	1	.04
F	3%	1	.03

Table 2: Auction Example

Since M has the highest effective bid (which exceeds $R = .02$), M wins and pays per-click

$$\inf\{b : \text{CTR}_M \times b \geq \max\left[\text{CTR}_F \times \text{bid}_F, R\right]\} = \frac{.03}{.04} = .75.$$

Timeline: We assume the following order for the game.

Period 1: the publisher sets the reserve price R and the information disclosure policy I.

Period 2: the agency sets the fee F.

Period 3A: advertisers (i) bid directly to the publisher, (ii) delegate to the agency, or (iii) not bid.

Period 3B: ad impression arrives and advertisers' CTRs are realized; CTRs are (i) privately known by the publisher if $I = 0$, or (ii) common knowledge if $I = 1$.

Period 4: the agency decides the subset of bids to send to the publisher's auction.

Period 5: the auction takes place and players' payoffs are realized.

Note that the information disclosure policy I and reserve price R are set before the advertisers' and the agency's decisions. This is justified by the fact that publishers are often unaware of the agencies' fees when deciding I and R. Furthermore, the publishers' reserve price algorithms and the information disclosed to advertisers and agencies are updated less frequently than advertisers' contract renewal decisions with agencies, and agencies' fee adjustment decisions. On the other hand, agencies can observe reserve price algorithms (e.g., by observing average reserve prices in different segments) and the information disclosure policy of the publisher when setting their fees. We capture this by assuming the publisher sets I and R before the agency sets F.

We also assume that the advertisers' decisions on whether to participate, and whether to use an agency, are made before CTRs are realized. This is consistent with the previous literature on campaign management costs (e.g., Amaldoss et al., 2016) and is motivated by the fact that CTRs often change throughout a campaign. The campaign management cost includes the fixed costs of setting up a campaign (e.g., keyword selection, ad copy design, target selection, and bid amount) amortized over the span of the campaign, as well as the recurring costs (e.g., managing keyword portfolio, tracking click-through rates and conversion rates). These decisions are made at campaign level, before the CTR of each potential keyword in each target segment is realized.

3 Analysis

We begin the analysis with the benchmark case in which the agency does not exist. This benchmark analysis helps elucidate the impact of agencies on the market structure. We solve for pure-strategy subgame perfect Nash equilibrium by backward induction.

3.1 Benchmark: Without Agency

In the absence of the agency, all three advertisers decide whether to pay the campaign management cost k and participate in the publisher's auction. Participating advertisers, after incurring cost k, observe ad impression arrivals and decide how much to bid. The advertisers' decisions are invariant to the publisher's information disclosure. The reason is that the advertisers' decision to join the auction is made prior to CTR realizations (Amaldoss et al., 2016), and their bids are per-click. We state this preliminary result in the following lemma.[11]

Lemma 1. In the absence of the ad agency, the publisher is indifferent between disclosing and withholding information.

Interestingly, the informational invariance in Lemma 1 breaks down in the presence of an agency. When there exists an agency that strategically represents advertisers in the ad auction, the publisher's information disclosure materially impacts the equilibrium. The reason is that, even though advertisers continue to bid truthfully, the CTR information impacts the agency's bid rotation strategy. This in turn affects advertisers' trade-offs in deciding between bidding directly to the publisher (as they do in the case without the agency) and delegating to the agency.

3.2 With Agency

We turn to the main analysis where advertisers have the option to bid indirectly through an agency instead of bidding directly through the publisher. We solve for the two subgames in which the publisher withholds and discloses information.

[11] The indifference in Lemma 1 is partially due to the simplifying assumption in the main model that advertisers' per-click valuations are not affected by the information. In Section 4.1, we show that while the result of Lemma 1 changes when the publisher's information affects advertisers' per-click valuations, the key forces and the main results of the paper continue to hold.

3.2.1 Without CTR Information

The agency earns fee F per advertiser who delegates to the agency, as well as commission α on the advertiser's auction surplus. Because of commission α, which we assume to be infinitesimal, the agency's profit-maximizing strategy (after receiving the fee F) is to maximize the advertisers' total surplus. The agency achieves this by sending only one advertiser to the auction, which lowers the clearing price for the winning advertiser. Furthermore, in the subgame where the publisher withholds information, the advertisers are *ex ante* symmetric; therefore, the agency randomly chooses one of the advertisers. We state this result in the following lemma.[12]

Lemma 2 (Random Bid Rotation)**.** If information is withheld, the agency rotates bids randomly; i.e., the agency randomly selects one of the advertisers it represents and sends only its bid to the publisher's ad auction.

We illustrate here the mechanism behind bid rotation. Suppose the reserve price is R and the agency decides between sending two vs. one advertiser(s) to the auction. Consider the case where it sends both advertisers. Because advertisers bid truthfully, an advertiser's surplus is positive if and only if its CTR is higher than its competitors'. Therefore, the agency's expected payoff is

$$\pi_{ag}^{(2)} = \alpha \cdot 2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2} + \sigma - \max\left[\frac{1}{2} - \sigma, R\right]\right) = \frac{\alpha}{2} \left(\frac{1}{2} + \sigma - \max\left[\frac{1}{2} - \sigma, R\right]\right),$$

where the superscript (2) on the left-hand side denotes the number of advertisers the agency sends to the auction.

On the other hand, if the agency randomly picks one advertiser and sends it to the auction, then the agency's expected payoff is

$$\pi_{ag}^{(1)} = \alpha \cdot \left(\frac{1}{2}\left(\frac{1}{2} + \sigma - R\right) + \frac{1}{2}\left(\frac{1}{2} - \sigma - R\right)\mathbb{I}_{\{R \leq 1/2 - \sigma\}}\right) = \alpha \cdot \begin{cases} \frac{1}{2} - R & \text{if } R \leq \frac{1}{2} - \sigma, \\ \frac{\frac{1}{2} + \sigma - R}{2} & \text{if } R > \frac{1}{2} - \sigma, \end{cases}$$

where $\mathbb{I}_{\{.\}}$ is an indicator variable. It can be shown that $\pi_{ag}^{(1)} \geq \pi_{ag}^{(2)}$, which, coupled with the tie-breaking rule mentioned above, implies that the agency adopts random bid rotation. Thus, the agency

[12] In Section 4.2, we generalize this lemma to show that, if the publisher can disclose information partially, then under certain conditions the agency sends all advertisers to the publisher's auction (i.e., foregoes bid rotation).

softens bidding competition by lowering the number of advertisers participating in the auction.

Given the agency's bid rotation strategy, we analyze the advertisers' bidding channel choice (i.e., delegate bid to the agency or bid directly). From the advertisers' perspectives, delegating to the agency is a double-edged sword. On the one hand, bid rotation softens bid competition because only few advertisers are sent to the publisher's auction; this decreases the ad price. On the other hand, bid rotation may reduce the probability of an advertiser winning the auction because the random selection process may exclude the advertiser, even if its CTR is high.

Overall, in deciding whether or not to delegate to the agency, advertisers weigh these two countervailing effects of bid delegation, as well as the campaign management cost of direct bidding. The following proposition summarizes the advertisers' equilibrium channel choices and the consequent market structure when the publisher withholds information.

Proposition 1 (Market Structure Without Information)**.** If information is withheld, the market structure is as follows:

1. if $k \leq \tilde{k}$ and $\sigma > 3k$, then two advertisers delegate to the agency and one advertiser bids directly to the publisher, such that the auction density is two;
2. otherwise, at least one advertiser delegates to the agency and none of the advertisers bids directly to the publisher, such that the auction density is one,

where

$$\tilde{k} = \frac{1}{12} \cdot \begin{cases} 1 & \text{if } \sigma \leq \frac{1}{4}, \\ \frac{1}{2} + \sigma & \text{otherwise.} \end{cases} \tag{4}$$

Proposition 1 highlights two distinct market structure outcomes: an outcome in which an advertiser bids directly to the publisher in competition against a collusion (see shaded region in Figure 2), and another in which advertisers only participate in the auction through the agency (see unshaded regions in Figure 2). The outcomes are shaped by the interplay of positive and negative effects of agency delegation. First, delegation mitigates advertisers' campaign management costs; therefore, large k motivates advertisers to delegate to the agency.

The second and third effects relate to the agency's bid rotation strategy described above. Agency delegation can benefit advertisers by softening bidding competition. However, it also implies that

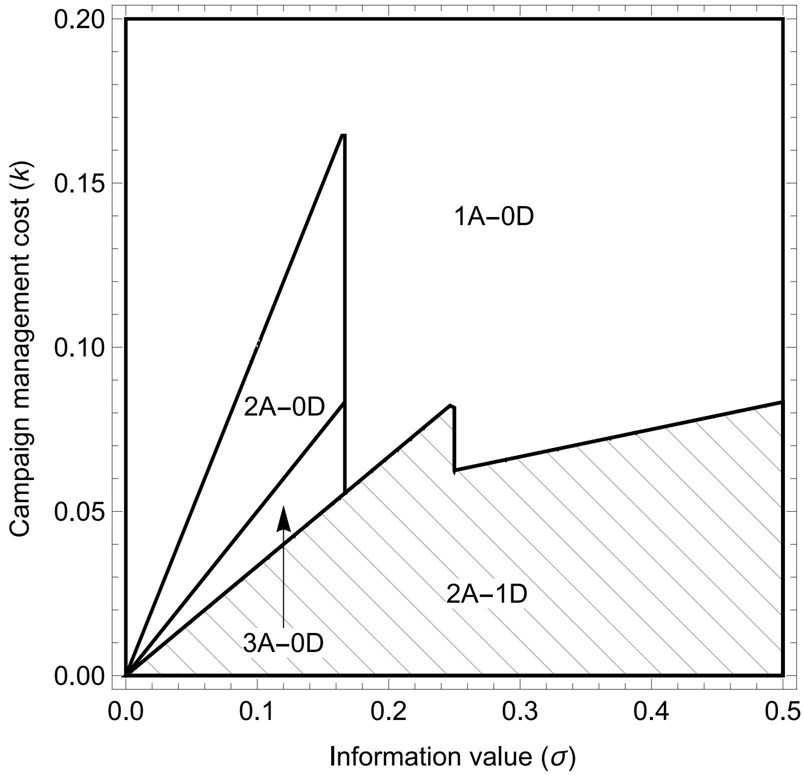

Figure 2: Market Structure With Agency Without Information. Regime notation: the numbers preceding A and D denote the number of advertisers bidding through the agency and directly, respectively.

an advertiser, even if its CTR is high, may be excluded due to the random bid rotation scheme and not win the ad auction. Therefore, if k is small and σ large, the collusion partially breaks down as one advertiser bids directly and competes against the two-advertiser collusion. The reason is that small k increases the appeal of direct bidding, and large σ increases advertiser differentiation. In this case, the negative effect of bid rotation – reducing advertiser's probability of winning – outweighs the positive effect – softening bid competition, such that one advertiser leaves the collusion and bids directly through the publisher.

3.2.2 With CTR Information

If the agency is endowed with advertisers' CTR information, it can decide which advertiser to send to the auction based on this information. As we summarize in the following lemma, the agency exploits advertisers' CTR information by cherry-picking only the advertiser with the highest chance of winning in the auction.

Lemma 3 (Selective Bid Rotation)**.** If information is disclosed, then the agency rotates bids selectively; i.e., the agency selects the advertiser with the highest CTR and sends only its bid to the publisher's ad auction.

The intuition of Lemma 3 mirrors that of random bid rotation discussed in Lemma 2. Because the agency receives commission from advertisers, the agency maximizes the winning advertiser's surplus by softening competition through bid rotation. Unlike the case without information, however, if information is disclosed, the agency can *cherry-pick* the advertiser with the highest CTR; this contrasts sharply with the *random* bid rotation strategy under no information.

Selective bid rotation has important implications for the advertisers and the publisher. Relative to the case of random bid rotation, advertisers expect a higher surplus from agency delegation. If the publisher discloses information, advertisers always delegate to the agency and do not bid directly, however small the campaign management cost. The following proposition states this finding.

Proposition 2 (Market Structure With Information)**.** If the publisher discloses information, then all three advertisers delegate to the agency such that the auction density is one.

The intuition behind Proposition 2 is as follows. Without information, the agency randomly rotates bids such that an advertiser may be excluded from the auction even if it has high CTR. This effect dampens advertisers' incentive to delegate. With information, however, this effect is mitigated because the agency can select advertisers with high CTR. Thus, advertisers expect, conditional on their CTR being high, a higher probability of winning *with* information than *without*. Selective bid rotation strengthens the collusion, which discourages advertisers who delegate to the agency from defecting. In total, when information is disclosed, all advertisers delegate to the agency.

Importantly, the publisher's information disclosure impacts efficiency through the informational effect on the agency's bid rotation strategy. In contrast to selective bid rotation, where advertisers with the highest CTR are sent to the auction, under random bid rotation, an advertiser with high CTR may not be sent to the auction. Therefore, withholding information decreases efficiency. The following proposition characterizes the efficiency loss associated with withholding information.

Proposition 3 (Information Disclosure and Efficiency)**.** Publisher's withholding information leads to efficiency loss, which amplifies with σ.

Proposition 3 highlights the downside risk for the publisher of withholding information. Limiting

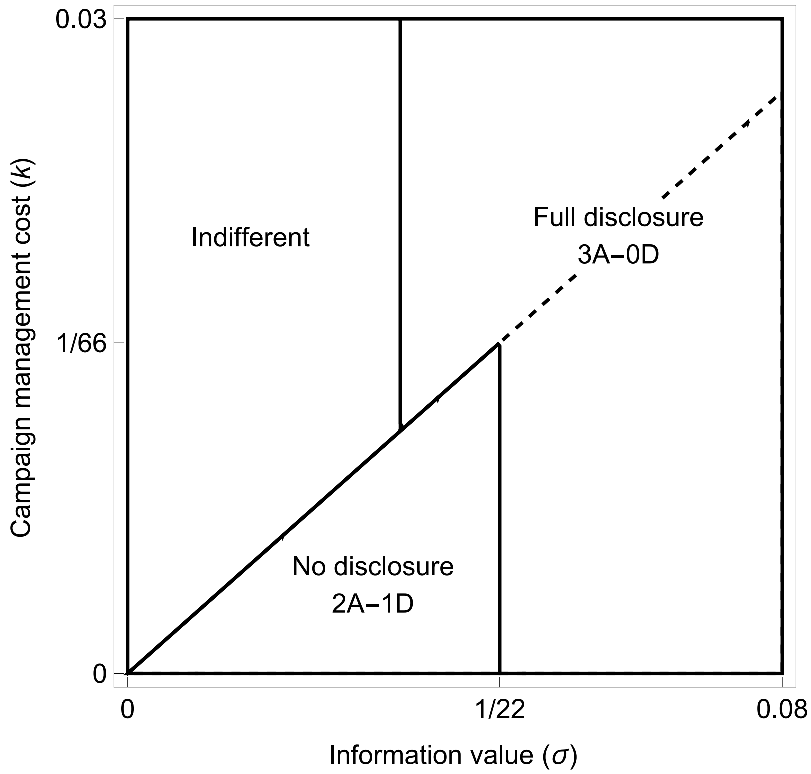

Figure 3: Publisher's Information Disclosure Strategy

information reduces the agency's capacity to rotate bids efficiently such that the advertiser with the highest CTR may not participate in the auction. Therefore, withholding information reduces efficiency. Based on this logic, one may intuit that the publisher should always disclose information; i.e., enlarge the pie by sharing information and then extract value via reserve prices. Interestingly, we show that this logic is only partially correct. The presence of agencies creates nuanced strategic dynamics that incentivizes the publisher to share more or less information.

3.3 Publisher's Information Disclosure Strategy

Compared to the benchmark case without the agency where the publisher was indifferent between disclosing and withholding information (see Lemma 1), the presence of agencies qualitatively alters the publisher's disclosure incentives. In equilibrium, the publisher may either disclose information, withhold information, or remain indifferent, depending on the parameter ranges (see Figure 3). The following proposition summarizes the publisher's equilibrium disclosure strategy.

Proposition 4 (Publisher's Information Disclosure Strategy)**.** In the presence of ad agency, the publisher's information disclosure strategy is as follows:

- if $k < 1/66$ and $3k < \sigma \leq 1/22$, the publisher withholds information;
- if $\sigma \leq \min[3k, 1/30]$, the publisher is indifferent; and
- otherwise, the publisher discloses information.

The publisher's disclosure strategy in Proposition 4 is based on two countervailing forces. On one hand, disclosing information enables the agency to rotate bids selectively (vs. randomly). This ensures that high-CTR advertisers win the auction, and thereby enhances efficiency.

On the other hand, disclosing information makes agency delegation more appealing for advertisers. Due to selective bid rotation, a high-CTR advertiser is more likely to be sent to the auction than when information is withheld. As more advertisers delegate to the agency, the agency takes a larger cut of the pie under information disclosure, leaving a smaller share for the publisher. In sum, information disclosure may enlarge the pie, which may increase the publisher's profit; however, it may also reduce the publisher's share of the pie, as it empowers the agency.

Proposition 4 shows that the publisher withholds information if and only if k is small and σ is intermediate. Small k ensures that direct bidding is not too costly such that some advertisers consider bidding directly. In addition, σ must be intermediate: large enough that the direct-bidder's surplus sufficiently outweighs the campaign management cost, and simultaneously, small enough that the efficiency loss from withholding information is not too costly for the publisher (see Proposition 3). Limiting information can be viewed as the publisher adopting a competitive stance against the agency to take a larger share of the pie at the expense of reducing the pie size. The publisher reduces the appeal of agency-based collusion for advertisers, incentivizing them to bid directly instead; this increases auction density, and hence the publisher's revenue.

If the information value is large, limiting information significantly shrinks the pie compared to the scenario in which information is disclosed. Therefore, even though disclosing information reduces the publisher's share of the pie by promoting agency-collusion, the efficiency gains is sufficiently large that the publisher discloses information. In other words, the publisher adopts a cooperative stance with the agency to increase the pie size at the expense of compromising its share of the pie.

Finally, if σ is small, the publisher remains indifferent even in the presence of agencies. To understand this, note first that for small σ, the agency sends one advertiser to the auction (randomly without information, and selectively with information). Second, small σ implies low differentiation among advertisers such that the extractable auction surplus is small. This motivates the publisher to primarily consider advertising volume over margin when setting the reserve price: it sets low reserve price (i.e., $R = 1/2 - \sigma$) such that regardless of the advertisers' CTR, the auction always clears at price $1/2 - \sigma$. Thus, if σ is small, the publisher's revenue is invariant to information disclosure.

4 Extensions

In this section, we test the robustness of the key insights – i.e., the effect of publisher's information disclosure on agency bidding ring and advertisers' strategies – by analyzing alternative model specifications. In Section 4.1, we analyze a scenario in which the publisher's information helps bidders infer not only advertisers' expected CTRs, but also their heterogeneous per-click valuations. In Section 4.2, we extend the main model by allowing the publisher to partially disclose information. We discuss the robustness of the results as well as shed light on novel insights stemming from the model variations.

4.1 Heterogeneous Ad Valuations

In the main model, we assumed that advertisers' per-click valuations were fixed and common knowledge. We also assumed that the publisher's information affected only the advertisers' expected CTR, and not their valuations. In this section, we relax both of these assumptions by allowing the publisher's information to reveal advertisers' CTRs as well as their per-click valuations.

Let the per-click value of Advertiser $i \in \{1, 2, 3\}$ be identically and independently distributed as

$$v_i = \begin{cases} 1 & \text{with probability } \beta, \\ 0 & \text{with probability } 1 - \beta, \end{cases}$$

where $\beta \in [0, 1]$. If the publisher withholds information, advertisers share the same expected per-click valuation β. On the other hand, if the advertiser discloses information, advertisers learn their valuations v upon arrival of the ad opportunity.

The analysis for the no-information subgame is analogous to the main model. The advertisers and the agency strategize based on expected per-click valuation of $\beta \cdot 1$ instead of 1. The analysis for the subgame in which the publisher discloses information is more nuanced. We provide here a sketch of the analysis to highlight the key ideas, and relegate the details to the Online Appendix.

We characterize the advertisers' expected payoffs under the two most important regimes: the regime in which three advertisers delegate to the agency (i.e., 3A-0D), and that in which two advertisers delegate to the agency and one advertiser bids directly (i.e., 2A-1D).

Under 3A-0D, an advertiser obtains positive surplus if and only if either of the following conditions hold. It has high c and v (which occurs w.p. $1/2 \cdot \beta$), and

1. both competing advertisers have high c and v (which occurs w.p. $(\beta/2)^2$), in which case the advertiser is sent to the auction w.p. 1/3, due to the agency's random bid rotation; or
2. only one competing advertiser has high c and v (which occurs w.p. $2 \cdot \beta/2 \cdot (1-\beta/2)$), in which case it is sent to the auction w.p. 1/2, due to bid rotation; or
3. none of the competing advertisers has high c and v (which occurs w.p. $(1-\beta/2)^2$), in which case it is sent to the auction.

In these cases, the advertiser's surplus is $1/2 + \sigma - R$.

Another condition under which an advertiser obtains positive surplus under 3A-0D is if it has low c but high v (which occurs w.p. $1/2 \cdot \beta$), and

1. both competing advertisers have low c but high v (which occurs w.p. $(\beta/2)^2$), in which case the advertiser is sent to the auction w.p. 1/3, due to bid rotation; or
2. one competing advertiser has low c but high v and the other has low v (which occurs w.p. $2 \cdot \beta/2 \cdot (1-\beta)$), in which case it is sent to the auction w.p. 1/2, due to bid rotation; or
3. both competing advertisers have low v (which occurs w.p. $(1-\beta)^2$), in which case it is sent to the auction.

In these cases, the advertiser's surplus is $\max[1/2 - \sigma - R, 0]$.[13] Therefore, the advertisers' payoffs

[13] Note that the low-CTR advertiser's positive surplus is partially driven by the independence assumption between CTR and per-click valuation. While the two variables may exhibit positive correlation in practice, the positive surplus would persist insofar as the correlation is less than perfect. To establish robustness in the most basic setup, we abstract away from non-zero correlations in the extension.

excluding the agency fee F are[14]

$$\pi_A(\text{3A-0D}) = \frac{\beta}{2}\left(\left(\frac{\beta}{2}\right)^2 \frac{1}{3} + 2\frac{\beta}{2}\left(1-\frac{\beta}{2}\right)\frac{1}{2} + \left(1-\frac{\beta}{2}\right)^2\right)\left(\frac{1}{2}+\sigma-R\right)$$
$$+ \frac{\beta}{2}\left(\left(\frac{\beta}{2}\right)^2 \frac{1}{3} + 2\frac{\beta}{2}(1-\beta)\frac{1}{2} + (1-\beta)^2\right)\max\left[\frac{1}{2}-\sigma-R, 0\right].$$

Similarly, we can obtain advertisers' payoffs under the regime 2A-1D excluding the agency fee F and campaign management cost k:

$$\pi_A(\text{2A-1D}) = \frac{\beta}{2}\left(\frac{\beta}{2}\frac{1}{2} + \left(1-\frac{\beta}{2}\right)\right)\left(\frac{\beta}{2}\left(\frac{1}{2}+\sigma-\max\left[\frac{1}{2}-\sigma, R\right]\right) + (1-\beta)\left(\frac{1}{2}+\sigma-R\right)\right)$$
$$+ \frac{\beta}{2}\left(\frac{\beta}{2}\frac{1}{2} + (1-\beta)\right)(1-\beta)\max\left[\frac{1}{2}-\sigma-R, 0\right]$$

for the advertiser going through the agency, and

$$\pi_D(\text{2A-1D}) = \frac{\beta}{2}\left(\left(\left(\frac{\beta}{2}\right)^2 + 2\frac{\beta}{2}(1-\beta)\right)\left(\frac{1}{2}+\sigma-\max\left[\frac{1}{2}-\sigma, R\right]\right) + (1-\beta)^2\left(\frac{1}{2}+\sigma-R\right)\right)$$
$$+ \frac{\beta}{2}(1-\beta)^2\max\left[\frac{1}{2}-\sigma-R, 0\right]$$

for the advertiser bidding directly on the publisher.

Given the advertisers' utilities, we can solve the agency's optimal fee. If the publisher discloses information, the agency's profit-maximizing fee that induces the regime 3A-0D is

$$F^*(\text{3A-0D}) = \max\left\{F : \pi_A(\text{3A-0D}) - F \geq \max\left[\pi_D(\text{2A-1D}) - k, 0\right]\right\}.$$

Because the agency collects $F^*(\text{3A-0D})$ from all three advertisers that delegate to the agency, the agency's optimal profit under 3A-0D is

$$\pi_{\text{ag}}(\text{3A-0D}) = 3F^*(\text{3A-0D})$$
$$= \begin{cases} \frac{1}{8}\beta(12-(6-\beta)\beta)\left(\frac{1}{2}+\sigma-R\right) & \text{if } 8k \geq (2-\beta)^2\beta\left(\frac{1}{2}+\sigma-R\right), \\ \frac{1}{4}(3-\beta)\beta^2\left(\frac{1}{2}+\sigma-R\right) + 3k & \text{otherwise.} \end{cases}$$

[14]It is worth noting that in the main model, two competing advertisers could not obtain positive surplus if the advertiser realizes a low CTR: either its competitor has high CTR and outranks it, or the competitor also has low CTR and the random winner pays full valuation due to tied ranking. Under heterogeneous valuation, even a low-CTR advertiser may obtain positive surplus.

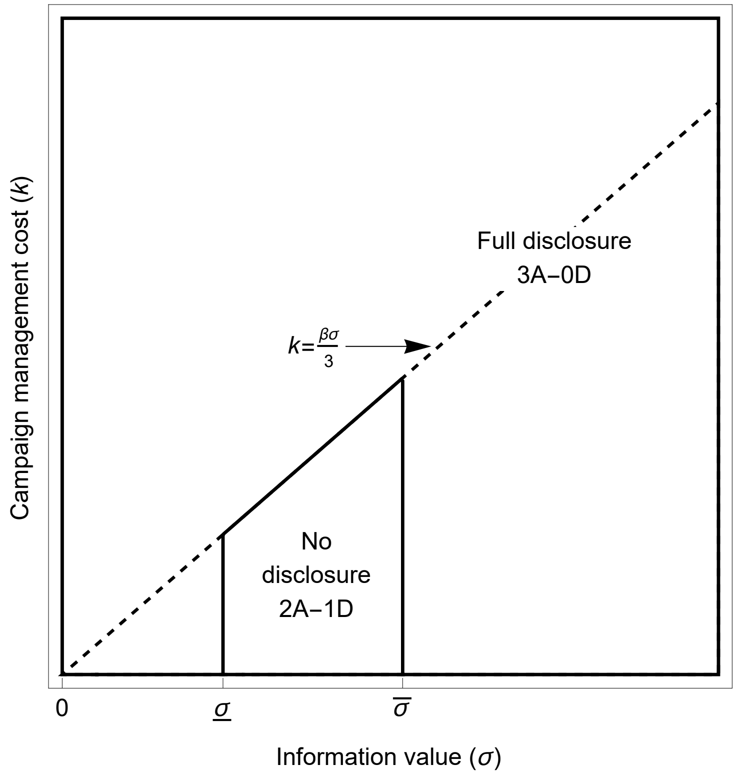

Figure 4: Publisher's Information Disclosure Strategy With Heterogeneous Valuations

In a similar manner, we can compute the agency's optimal profits under other possible regimes for any given reserve price. The publisher then sets the optimal reserve price anticipating the agency's subgame optimal fees, and finally decides whether to disclose or withhold information. The following proposition shows that the publisher's disclosure strategy under heterogeneous valuation is qualitatively similar to that from the main model.

Proposition 5 (Publisher's Information Disclosure Strategy Under Heterogeneous Ad Valuations)**.** Suppose advertisers have heterogeneous ad valuations which can be learned if the publisher discloses information. Let the thresholds $\tilde{\beta}, \underline{\sigma}$, and $\overline{\sigma}$ be as defined in the proof. The publisher withholds information if (i) $k \leq \frac{\beta\sigma}{3}$, (ii) $\tilde{\beta} < \beta$, and (iii) $\underline{\sigma} < \sigma < \overline{\sigma}$; it discloses information otherwise.

As illustrated in Figure 4, the publisher's information disclosure strategy qualitatively mirrors that from the main model. If the campaign management cost is low and the information value intermediate, the publisher withholds information to weaken the agency and induce one advertiser to abandon the agency-coalition and bid directly. That is, the publisher discloses less information to appropriate a larger share of the pie, at the expense of shrinking the pie.

Compared to the main model, the publisher discloses information for a larger parametric region. For example, if the information value is small, then in the main model, the publisher was either indifferent or withheld information. In contrast, when advertisers' valuations are revealed by the publisher's information, the publisher discloses information. The intuition is that with heterogeneous valuation, the publisher's information carries more weight in efficiently ranking the auction participants. Information not only affects the advertisers' expected CTR, but also their per-click valuations. Therefore, the efficiency loss that the publisher internalizes from withholding information is larger compared to the main model. We summarize this finding in the following corollary.

Corollary 1. *The parametric region in which the publisher withholds information increases in β.*

In total, heterogeneous valuations provide additional incentive for the publisher to disclose information compared to the common valuations case. However, the key forces that shape the publisher's trade-offs, and hence the equilibrium outcomes, are preserved. The publisher weighs the efficiency loss against the benefit of weakening the bidding ring to determine its optimal information disclosure strategy. Similar to the main model, the publisher withholds information when campaign management cost is low and the information value intermediate.

4.2 Partial Information Disclosure

The main model restricted attention to an all-or-nothing information space where the publisher either completely withholds or discloses information. While the stylized assumption helped sharpen insights, one may wonder to what extent the results carry over to a setting where publishers can partially disclose information. In practice, publishers may bundle consumers' demographic information (e.g., age or zip-code) to only reveal certain signals to advertisers. In this section, we assess the robustness of the publisher's information disclosure strategy by examining a continuous information space.

We assume that the publisher can send a signal to advertisers about their expected CTR with varying degrees of accuracy. Specifically, the publisher sends signal s about the advertiser's CTR c with accuracy $\rho \in [1/2, 1]$, where $\rho = \mathbb{P}\{s = c \,|\, c\}$ and $c \in \{1/2 - \sigma, 1/2 + \sigma\}$. Thus, the larger is ρ, the more precise information the publisher provides for advertisers and agencies to estimate c. The publisher sets ρ (simultaneously as it sets reserve price R) to maximize its expected profit. The remaining model specifications remain unchanged. The cases of $\rho = 1/2$ and $\rho = 1$ are equivalent to information withholding and disclosing from the main model, respectively.

That the agency can now strategize conditional on signal s for each advertiser it represents creates interesting nuances in the analysis. We highlight here only the key intuition and relegate details of the analysis to the Online Appendix.

Example: Suppose two advertisers delegate to the agency (and no advertiser bids directly). There are two cases to consider: (i) at least one of the two advertisers has high-CTR signal, and (ii) both advertisers have low-CTR signals. In the first case, the agency's profit-maximizing strategy is to rotate bids, as in the main model: it only sends one high-CTR-signal advertiser to the auction.

In the second case, however, the agency's optimal strategy is to forego the collusion profit and send both of the low-CTR-signal advertisers to the auction. To see this, assume that $R \geq 1/2 - \sigma$ (which holds in equilibrium) and compare the agency's expected profit when it sends one vs. two low-CTR-signal advertisers to the auction.

- Suppose the agency sends only one low-CTR-signal advertiser to the auction. The auction-participating advertiser's surplus is positive if and only if its low-CTR signal is erroneous (i.e., its true CTR is high). Therefore, the agency's expected payoff is

$$\pi_{\text{ag}}^{(1)} = \underbrace{\alpha}_{\text{commission}} \cdot \underbrace{(1-\rho)}_{\mathbb{P}\{c=1/2+\sigma | s=1/2-\sigma\}} \cdot \underbrace{\left(\frac{1}{2} + \sigma - R\right)}_{\text{winning advertiser's exp. surplus}} \tag{5}$$

- Suppose the agency sends both low-CTR-signal advertiser to the auction. Each advertiser's surplus is positive if and only if its low-CTR signal is erroneous while its competitor's is correct. Therefore, the agency's expected payoff is

$$\pi_{\text{ag}}^{(2)} = \alpha \cdot 2(1-\rho)\rho \left(\frac{1}{2} + \sigma - \max\left[\frac{1}{2} - \sigma, R\right]\right) = \alpha \cdot 2(1-\rho)\rho\left(\frac{1}{2} + \sigma - R\right). \tag{6}$$

Comparing the agency's payoffs (5) and (6), we see that $\pi_{\text{ag}}^{(2)} \geq \pi_{\text{ag}}^{(1)}$ for all $\rho \in [1/2, 1]$ with equality holding only at the endpoints $\rho \in \{1/2, 1\}$.[15]

[15] At $\rho = 1/2$, the distinction between low-CTR and high-CTR signals becomes immaterial. At $\rho = 1$, the agency is indifferent between sending 1 and 2 low-CTR-signal advertisers. Note that we break this tie in the main model by

In sum, if the agency estimates that both advertisers it represents have low CTRs, it obtains a higher payoff by sending both advertisers to the auction. Intuitively, even if sending multiple advertisers to the auction intensifies competition, the fact that the advertisers likely have low CTRs motivates the agency to increase the probability of winning by sending multiple ad candidates. The following lemma formalizes this result in the context of three advertisers.

Lemma 4 (Multiple Ad Candidates)**.** Suppose that three advertisers delegate to the agency. If the agency estimates at least one of the advertisers it represents to have a high CTR, then it sends that advertiser to the auction. On the other hand, if the agency estimates all advertisers to have low CTRs, then the number of low-CTR-signal advertisers it sends to the auction is

$$n = \begin{cases} 2 & \text{if } \frac{1}{2} \leq \rho \leq \frac{2}{3}, \\ 3 & \text{if } \frac{2}{3} < \rho \leq 1. \end{cases}$$

Lemma 4 states that the more precise the signal, the more low-CTR-signal advertisers the agency sends to the auction. This is because larger ρ implies a lower chance of winning for the low-CTR-signal advertisers.

The agency's strategy in Lemma 4 has important implications for the publisher's information disclosure strategy. Recall from the main model that the publisher had incentive to obscure information to induce advertisers to abandon the agency-ring and bid directly (see Figure 3). Lemma 4 sheds light on a countervailing force: by disclosing more precise information, the publisher can induce the agency to break its own collusion and increase the auction density. In the event that all advertisers' CTRs are estimated to be low, the agency sends multiple advertisers to the auction, thereby increasing its probability of winning in the publisher's auction. As we demonstrate below, under certain conditions, this new force induces the publisher to disclose more information than it does in the main model.

We numerically solve for the publisher's optimal disclosure level ρ^*. As illustrated in Figure 5, the qualitative insights regarding the publisher's information disclosure strategy carry over. If campaign management cost is low and information value intermediate, the publisher "competes" against the agency by withholding information. In doing so, the publisher induces an advertiser to abandon the agency-coalition and bid directly; the publisher thereby obtains a larger share of the pie, at the

assuming that the agency chooses fewer advertisers.

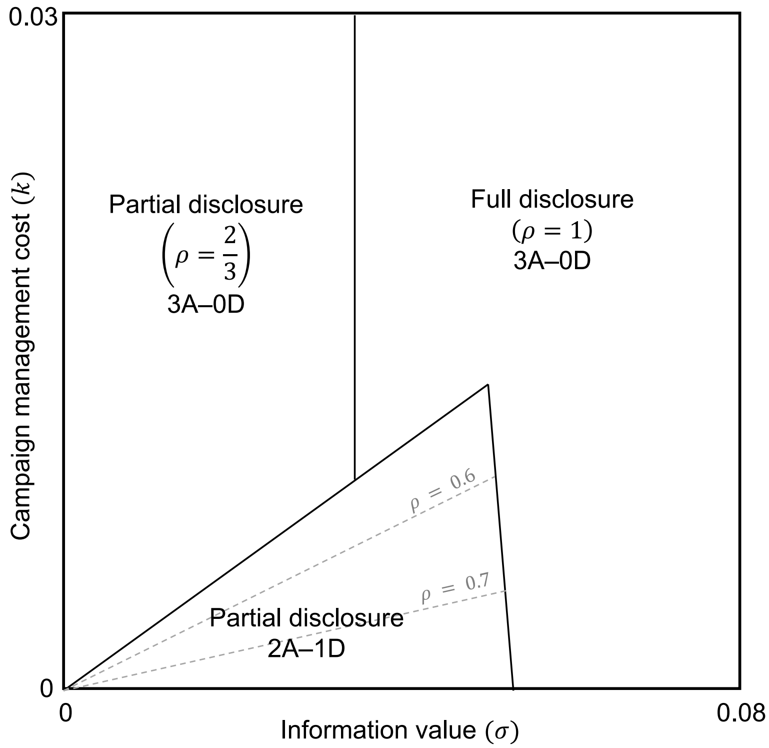

Figure 5: Publisher's Information Disclosure Strategy on Continuous Scale. The gray dotted lines denote contours.

expense of shrinking the overall pie.

We obtain two interesting insights from the continuous signal extension. First, the publisher's information disclosure level first decreases then increases in the campaign management cost (k). Intuitively, as k increases, advertisers have a higher willingness-to-pay for agencies; therefore, to break the agency-collusion, the publisher proportionately weakens the agency by withholding more information. As k increases further, however, advertisers choose to outsource to the agency to mitigate this cost *even if* the publisher withholds information and the bidding ring is inefficient. In this case, the publisher "cooperates" with the agency by disclosing information; i.e., the publisher enlarges the pie at the expense of conceding a larger share of the pie to the agency.

The second novel insight relates to the agency's strategy in Lemma 4. As discussed above, the possibility of inducing the agency to break its own collusion incentivizes the publisher to disclose more information. For instance, if the information value is small, the publisher sets $\rho^* = 2/3$: information is precise enough to induce the agency to send all low-CTR-signal advertisers to the auction, but simultaneously obscure enough to ensure that some of the low-CTR-signal advertisers that the agency sends in fact have high CTRs.

5 Conclusion

The question of information disclosure by publishers has been studied in the literature; however, previous works overlook the strategic role played by agencies. Our results show that the presence of agencies has important implications for the publishers' trade-offs associated with information disclosure. Unlike the no-agency models where information disclosure can thin out the auction market, in the presence of agencies, information disclosure can thicken the market. Our results have implications for managers of advertising firms, agencies, and publishers. New publishers with high campaign management costs for advertisers (e.g., because the publisher has limited automated campaign optimization tools) should disclose more information to the agencies (e.g., through APIs). Even though disclosing information facilitates collusion for agencies, which generally decreases the publisher's revenue, it helps agencies create more value to advertisers. This in turn incentivizes more advertisers to indirectly advertise on the publisher through the agency. Thus, the publisher *cooperates* with the agency by disclosing information to maximize the total surplus created in the market, at the expense of relinquishing a larger cut of the surplus to the agency. The publisher would rely on other strategic levers such as reserve prices to extract surplus.

On the other hand, an established publisher with low campaign management costs for advertisers (e.g., by offering sophisticated automation tools for campaign optimization) may want to withhold information from agencies. In doing so, the publisher limits the value that the agency creates for advertiser. As a result, either (a) the agency sends multiple ad candidates to the publisher's auctions to maintain a sufficiently high probability of winning in the auction, which would mitigate bidding collusion, or (b) some advertisers directly bid on the publisher's advertising platform without going through the agency. In summary, as publishers grow and facilitate campaign management for advertisers, it is critical that they revisit their relationships with agencies, and re-assess the extent of information shared with agencies.

Our results indicate that when the publisher is new, the agency provides value to advertisers by mitigating campaign management costs, and facilitating collusion among advertisers. Since agencies create more value for advertisers in this setting, they can set higher fees and extract more surplus. However, as the publisher becomes more established and the campaign management cost declines, the publisher may withhold information from the agency to lower the advertisers' benefit of partaking in the collusion. In response, agencies lower their fees, and expect some advertisers to switch to bidding directly on the publisher's advertising platform.

From the advertisers' perspective, working with agencies has several advantages over direct bidding. Agency delegation mitigates campaign management costs, which may be significant when the publisher is new. Furthermore, advertisers benefit from agency-based collusion by being part of the bidding ring. However, our results show that when the publisher withholds valuable information from agencies, some advertisers may switch from agency delegation to bidding directly to the publisher's platform. Therefore, when advertising on new publishers, it is optimal for advertisers to bid through an agency. As the platform grows and offers more campaign optimization tools, advertisers should consider bidding directly into the publisher's advertising platform. In equilibrium, some advertisers will continue working with agencies while others switch to direct bidding on the publisher's platform.

Finally, our research contributes to the discussion of privacy regulations. To the extent that privacy regulations limit the amount of information publishers are allowed to disclose, our results suggest that new publishers may be disproportionately hurt by privacy regulations relative to their established counterparts. The reason is that for established publishers with low campaign management cost, restricting information could be a boon in terms of inducing advertisers to bid directly, thereby mitigating the negative effects of agency-collusion. On the other hand, new publishers with high campaign management costs do not have this option. Limiting information results in greater efficiency losses for new publishers. Overall, our model highlights the importance of considering unintended anti-competitive effects of privacy regulations that may disadvantage new players in the industry.

Declarations

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Appendix

A1 Proofs

A1.1 Proof of Lemma 1

Proof. Because the advertisers' per-click valuations are unaffected by the CTR information, the advertisers' bidding strategies in the second-price auctions are unaffected by the publisher's information disclosure strategy. Moreover, because advertisers decide whether to bid at cost k prior to observing CTR realizations, CTR information does not affect the advertisers' participation decision either. ■

A1.2 Proof of Lemma 2

Proof. First, consider the case where two advertisers delegate to the agency. If the agency randomly sends one advertiser to the auction (say, Advertiser 1), the agency's payoff from the auction is

$$\begin{aligned}\pi_{\text{ag}}^{(1)} &= \alpha\left(\mathbb{P}\{c_1 = c_H\}(c_H - R) + \mathbb{P}\{c_1 = c_L\}\max[c_L - R, 0]\right)\\ &= \alpha\left(\frac{1}{2}\left(\frac{1}{2} + \sigma - R\right) + \frac{1}{2}\max\left[\frac{1}{2} - \sigma - R, 0\right]\right).\end{aligned}$$

On the other hand, if it sends two advertisers to the auction (say, Advertisers 1 and 2), the agency's payoff is

$$\pi_{\text{ag}}^{(2)} = \alpha\,\mathbb{P}\{c_1 \neq c_2\}(c_H - \max[c_L, R]) = \alpha \cdot 2\left(\frac{1}{2}\right)^2\left(\frac{1}{2} + \sigma - \max\left[\frac{1}{2} - \sigma, R\right]\right).$$

Because $\frac{1}{2} + \sigma - R \geq \frac{1}{2} + \sigma - \max\left[\frac{1}{2} - \sigma, R\right]$, it follows that $\pi_{\text{ag}}^{(1)} \geq \pi_{\text{ag}}^{(2)}$.

Second, consider the case where three advertisers delegate to the agency. If the agency sends three advertisers to the auction, the agency's payoff is

$$\pi_{\text{ag}}^{(3)} = \alpha\,\mathbb{P}\{\text{exactly one } c_H\}(c_H - \max[c_L, R]) = \alpha \cdot 3\left(\frac{1}{2}\right)^3\left(\frac{1}{2} + \sigma - \max\left[\frac{1}{2} - \sigma, R\right]\right),$$

from which it follows that $\pi_{\text{ag}}^{(1)} \geq \pi_{\text{ag}}^{(3)}$. Therefore, sending one advertiser dominates. ■

A1.3 Proof of Proposition 1

Proof. Suppose the publisher withholds information ($I = 0$). We derive (i) the advertisers' expected payoffs under each regime, (ii) the agency's optimal fee and associated market structure, and (iii) the publisher's optimal reserve price.

1. Advertisers' utilities in each subgame
    - 3A-0D: After paying agency fee F, an advertiser receives positive auction surplus only if it is randomly chosen by the agency (which occurs w.p. $1/3$). Conditional on being chosen, if the advertiser's CTR is high (which occurs w.p. $1/2$), its payoff is $c_H(1-R/c_H) = 1/2+\sigma-R$; and if its CTR is low (which occurs w.p. $1/2$), its payoff is $c_L(1-R/c_L)\mathbb{I}_{\{R\leq c_L\}} = \max[1/2-$

$\sigma - R, 0]$. Therefore, each advertiser's expected payoff in the 3A-0D regime *excluding the agency fee* F is

$$\pi_A^{I=0}(\text{3A-0D}) = \frac{1}{3}\left(\frac{1}{2}\left(\frac{1}{2} + \sigma - R\right) + \frac{1}{2}\max\left[\frac{1}{2} - \sigma - R, 0\right]\right). \tag{A1}$$

- 2A-1D: We derive the payoffs for an advertiser in the two-advertiser coalition, and for the advertiser bidding directly. The advertiser going through the agency receives positive auction surplus only if it is randomly chosen by the agency (which occurs w.p. 1/2) and the direct bidding competitor has low CTR (which occurs w.p. 1/2). Conditional on being chosen, if the advertiser's CTR is high, its payoff is $c_H(1 - \max[c_L, R]/c_H)(1/2) = (1/2 + \sigma - \max[1/2 - \sigma, R])/2$, where the division by 2 is due to the necessary condition that the direct bidding competitor have low CTR; and if its CTR is low, its payoff is 0. Therefore, the expected payoff of the advertiser going through the agency in the 2A-1D regime *excluding the agency fee* F is

$$\pi_A^{I=0}(\text{2A-1D}) = \frac{1}{2}\frac{1}{2}\frac{1}{2}\left(\frac{1}{2} + \sigma - \max\left[\frac{1}{2} - \sigma, R\right]\right). \tag{A2}$$

  The direct bidding advertiser's payoff is positive only if its CTR is high, and the randomly chosen competing advertiser (who goes through the agency) has low CTR. Therefore, the expected payoff of the advertiser bidding directly in the 2A-1D regime *excluding the campaign management cost* k is

$$\pi_D^{I=0}(\text{2A-1D}) = \frac{1}{2}\frac{1}{2}\left(\frac{1}{2} + \sigma - \max\left[\frac{1}{2} - \sigma, R\right]\right). \tag{A3}$$

Following the reasoning above, we obtain the advertisers' payoffs for the remaining regimes:

$$\pi_A^{I=0}(\text{1A-2D}) = \frac{1}{2}\frac{1}{2}\frac{1}{2}\left(\frac{1}{2} + \sigma - \max\left[\frac{1}{2} - \sigma, R\right]\right) \tag{A4}$$

$$\pi_D^{I=0}(\text{1A-2D}) = \frac{1}{2}\frac{1}{2}\frac{1}{2}\left(\frac{1}{2} + \sigma - \max\left[\frac{1}{2} - \sigma, R\right]\right) \tag{A5}$$

$$\pi_A^{I=0}(\text{2A-0D}) = \frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2} + \sigma - R\right) + \frac{1}{2}\max\left[\frac{1}{2} - \sigma - R, 0\right]\right) \tag{A6}$$

$$\pi_A^{I=0}(\text{1A-1D}) = \frac{1}{2}\frac{1}{2}\left(\frac{1}{2} + \sigma - \max\left[\frac{1}{2} - \sigma, R\right]\right) \tag{A7}$$

$$\pi_D^{I=0}(\text{1A-1D}) = \frac{1}{2}\frac{1}{2}\left(\frac{1}{2} + \sigma - \max\left[\frac{1}{2} - \sigma, R\right]\right) \tag{A8}$$

$$\pi_A^{I=0}(\text{1A-0D}) = \frac{1}{2}\left(\frac{1}{2} + \sigma - R\right) + \frac{1}{2}\max\left[\frac{1}{2} - \sigma - R, 0\right] \tag{A9}$$

2. Agency's optimal fee under each regime. Note that inducing a regime requires the agency satisfy the advertisers' individual rationality (IR) and incentive compatibility (IC) constraints. Since the agency's payoff monotonically increases in F, F will be set such that the tighter of the two constraints binds. If both constraints cannot be jointly satisfied by any $F \geq 0$, then we let the agency's payoff be equal to $-\infty$.
   (a) 3A-0D: the agency maximizes $\pi_{\text{ag}}^{I=0}(\text{3A-0D}) = 3F$ subject to

$\pi_A^{I=0}(\text{3A-0D}) - F \geq \max\left[\pi_D^{I=0}(\text{2A-1D}) - k, 0\right]$. Therefore,

$$\pi_{\text{ag}}^{I=0}(\text{3A-0D}) = 3 \cdot \left(\pi_A^{I=0}(\text{3A-0D}) - \max\left[\pi_D^{I=0}(\text{2A-1D}) - k, 0\right]\right).$$

(b) 2A-1D: the agency maximizes $\pi_{\text{ag}}^{I=0}(\text{2A-1D}) = 2F$ subject to

$$\pi_A^{I=0}(\text{2A-1D}) - F \geq \max\left[\pi_D^{I=0}(\text{1A-2D}) - k, 0\right],$$
$$\pi_D^{I=0}(\text{2A-1D}) - k \geq \max\left[\pi_A^{I=0}(\text{3A-0D}) - F, 0\right],$$

which can be expressed in terms of F as $F \leq \pi_A^{I=0}(\text{2A-1D}) - \max\left[\pi_D^{I=0}(\text{1A-2D}) - k, 0\right]$, and $F \geq \pi_A^{I=0}(\text{3A-0D}) - \left(\pi_D^{I=0}(\text{2A-1D}) - k\right)$, and $\pi_D^{I=0}(\text{2A-1D}) - k \geq 0$. Therefore, $\pi_{\text{ag}}^{I=0}(\text{2A-1D})$ equals $2 \cdot \left(\pi_A^{I=0}(\text{2A-1D}) - \max\left[\pi_D^{I=0}(\text{1A-2D}) - k, 0\right]\right)$ if
(i) $\pi_A^{I=0}(\text{3A-0D}) - \left(\pi_D^{I=0}(\text{2A-1D}) - k\right) \leq \pi_A^{I=0}(\text{2A-1D}) - \max\left[\pi_D^{I=0}(\text{1A-2D}) - k, 0\right]$ and (ii) $\pi_D^{I=0}(\text{2A-1D}) - k \geq 0$; and it equals $-\infty$ otherwise.

(c) 1A-2D: impossible because constraints cannot be jointly satisfied for any $F \geq 0$. We need

$$\pi_A^{I=0}(\text{1A-2D}) - F \geq \max\left[\pi_D^{I=0}(\text{0A-3D}) - k, 0\right],$$

which can be re-expressed, using the equalities $\pi_A^{I=0}(\text{1A-2D}) = \pi_D^{I=0}(\text{1A-2D})$ and $\pi_D^{I=0}(\text{0A-3D}) = \pi_D^{I=0}(\text{1A-2D})$, as

$$\pi_D^{I=0}(\text{1A-2D}) - F \geq \max\left[\pi_D^{I=0}(\text{1A-2D}) - k, 0\right],$$

from which it follows that $F \leq k$. The second non-deviation constraint is

$$\pi_D^{I=0}(\text{1A-2D}) - k \geq \max\left[\pi_A^{I=0}(\text{2A-1D}) - F, 0\right],$$

from which we obtain

$$k \leq \pi_D^{I=0}(\text{1A-2D}) - \max\left[\pi_A^{I=0}(\text{2A-1D}) - F, 0\right].$$

Combining the two inequalities yields

$$F \leq k \leq \pi_D^{I=0}(\text{1A-2D}) - \max\left[\pi_A^{I=0}(\text{2A-1D}) - F, 0\right],$$

which implies $F \leq k \leq F$, because $\pi_D^{I=0}(\text{1A-2D}) = \pi_A^{I=0}(\text{2A-1D})$. Therefore, the constraints can only be jointly satisfied if $F = k$, a zero-measure parameter space which we ignore.

(d) 2A-0D: the agency maximizes $\pi_{\text{ag}}^{I=0}(\text{2A-0D}) = 2F$ subject to $\pi_A^{I=0}(\text{2A-0D}) - F \geq \max\left[\pi_D^{I=0}(\text{1A-1D}) - k, 0\right]$, $0 \geq \max\left[\pi_A^{I=0}(\text{3A-0D}) - F, \pi_A^{I=0}(\text{2A-1D}) - k\right]$, which can be expressed in terms of F as $F \leq \pi_A^{I=0}(\text{2A-0D}) - \max\left[\pi_D^{I=0}(\text{1A-1D}) - k, 0\right]$, and $F \geq \pi_A^{I=0}(\text{3A-0D})$, and $\pi_D^{I=0}(\text{2A-1D}) - k \leq 0$. Therefore, $\pi_{\text{ag}}^{I=0}(\text{2A-0D})$ equals $2 \cdot \left(\pi_A^{I=0}(\text{2A-0D}) - \max\left[\pi_D^{I=0}(\text{1A-1D}) - k, 0\right]\right)$ if
(i) $\pi_A^{I=0}(\text{3A-0D}) \leq \pi_A^{I=0}(\text{2A-0D}) - \max\left[\pi_D^{I=0}(\text{1A-1D}) - k, 0\right]$ and (ii) $\pi_D^{I=0}(\text{2A-1D}) - k \leq 0$; and it equals $-\infty$ otherwise.

(e) 1A-1D: impossible because constraints cannot be jointly satisfied for any $F \geq 0$. We need

$$\pi_A^{I=0}(\text{1A-1D}) - F \geq \max\left[\pi_D^{I=0}(\text{0A-2D}) - k, 0\right],$$

which can be re-expressed, using the equalities $\pi_A^{I=0}(\text{1A-1D}) = \pi_D^{I=0}(\text{1A-1D})$ and $\pi_D^{I=0}(\text{0A-2D}) = \pi_D^{I=0}(\text{1A-1D})$, as $\pi_D^{I=0}(\text{1A-1D}) - F \geq \max\left[\pi_D^{I=0}(\text{1A-1D}) - k, 0\right]$, from which it follows that $F \leq k$. The second non-deviation constraint is $\pi_D^{I=0}(\text{1A-1D}) - k \geq$

$\max\left[\pi_A^{I=0}(\text{2A-0D}) - F, 0\right]$, from which we obtain $k \leq \pi_D^{I=0}(\text{1A-1D}) - \max\left[\pi_A^{I=0}(\text{2A-0D}) - F, 0\right]$. Combining the two inequalities yields $F \leq k \leq \pi_D^{I=0}(\text{1A-1D}) - \max\left[\pi_A^{I=0}(\text{2A-0D}) - F, 0\right]$, which implies $F \leq k \leq F$,. because $\pi_D^{I=0}(\text{1A-2D}) = \pi_A^{I=0}(\text{2A-1D})$. Therefore, the constraints can only be jointly satisfied if $F = k$, a zero-measure parameter space which we ignore.

(f) 1A-0D: the agency maximizes $\pi_{\text{ag}}^{I=0}(\text{1A-0D})$, which equals $\pi_A^{I=0}(\text{1A-0D}) - \max\left[\pi_D^{I=0}(\text{0A-1D}) - k, 0\right]$ (where $\pi_D^{I=0}(\text{0A-1D}) = \pi_D^{I=0}(\text{1A-0D})$) if (i) $\pi_A^{I=0}(\text{2A-0D}) \leq \pi_A^{I=0}(\text{1A-0D}) - \max\left[\pi_D^{I=0}(\text{0A-1D}) - k, 0\right]$ and (ii) $\pi_D^{I=0}(\text{1A-1D}) - k \leq 0$; and it equals $-\infty$ otherwise.

Next, we derive the agency's optimal regime given R. Note that the publisher will never set $R < 1/2 - \sigma$, so it suffices to consider the agency's optimal regimes for $1/2 - \sigma \leq R \leq 1/2 + \sigma$. Using the tie-breaking rule that if the agency is indifferent between inducing regimes 1A-0D and either 2A-0D and 3A-0D, it chooses 1A-0D; and if it is indifferent between 2A-0D and 3A-0D, it chooses 2A-0D, we obtain the following subgame equilibrium market structure:

(a) 2A-1D if $0 \leq k \leq (1/2 + \sigma - R)/6$;
(b) 3A-0D if $(1/2 + \sigma - R)/6 < k \leq (1/2 + \sigma - R)/4$;
(c) 2A-0D if $(1/2 + \sigma - R)/4 < k \leq (1/2 + \sigma - R)/2$; and
(d) 1A-0D if $(1/2 + \sigma - R)/2 < k$.

3. Finally, we solve for the publisher's optimal reserve price. We first derive the publisher's payoff under each regime as a function of R:

- 2A-1D: From the publisher's perspective, this regime is equivalent to a setting where two advertisers compete for the ad slot. Thus,

$$\begin{aligned}\pi_P^{I=0}(\text{2A-1D}) &= \mathbb{P}\{c_1 = c_2 = c_H\}c_H + \mathbb{P}\{c_1 \neq c_2\}\max\left[c_L, R\right] + \mathbb{P}\{c_1 = c_2 = c_L\}c_L \mathbb{I}_{\{R \leq c_L\}} \\ &= \frac{1}{4}\left(\frac{1}{2} + \sigma\right) + 2\frac{1}{4}\max\left[\frac{1}{2} - \sigma, R\right] + \frac{1}{4}\left(\frac{1}{2} - \sigma\right)\mathbb{I}_{\{R \leq 1/2 - \sigma\}}.\end{aligned}$$

- 3A-0D, 2A-0D, 1A-0D: From the publisher's perspective, this regime is equivalent to a setting where only advertiser bids (due to agency's bid rotation) for the ad slot. Thus,

$$\begin{aligned}\pi_P^{I=0}(\text{3A-0D}) = \pi_P^{I=0}(\text{2A-0D}) = \pi_P^{I=0}(\text{1A-0D}) &= \mathbb{P}\{c = c_H\}R + \mathbb{P}\{c = c_L\}R\,\mathbb{I}_{\{R \leq c_L\}} \\ &= \frac{R}{2}\left(1 + \mathbb{I}_{\{R \leq 1/2 - \sigma\}}\right).\end{aligned}$$

Therefore, the publisher's optimal R is obtained as follows.

- $1/2 - \sigma \leq 1/2 + \sigma - 6k$. Publisher considers $R = 1/2 - \sigma$ to induce 2A-1D for profit

$$\pi_P^{I=0}(2A - 1D)|_{R=1/2-\sigma} = \frac{1}{4}\left(\frac{1}{2} + \sigma\right) + \frac{3}{4}\left(\frac{1}{2} - \sigma\right) = \frac{1 - \sigma}{2},$$

or $R = 1/2 + \sigma - 6k$ to induce 2A-1D for profit

$$\pi_P^{I=0}(2A - 1D)|_{R=1/2+\sigma-6k} = \frac{1}{4}\left(\frac{1}{2} + \sigma\right) + \frac{1}{2}\left(\frac{1}{2} + \sigma - 6k\right) = \frac{3}{4}\left(\frac{1}{2} + \sigma\right) - 3k,$$

or $R = 1/2 + \sigma$ to induce 0D for profit

$$\pi_P^{I=0}(0D)|_{R=1/2+\sigma} = \frac{1}{2}\left(\frac{1}{2} + \sigma\right).$$

- $1/2 + \sigma - 6k < 1/2 - \sigma$. Publisher considers $R = 1/2 - \sigma$ to induce 0D for profit

$$\pi_P^{I=0}|_{R=1/2-\sigma} = 1/2 - \sigma,$$

or $R = 1/2 + \sigma$ to induce 0D for profit

$$\pi_P^{I=0}|_{R=1/2+\sigma} = \frac{1}{2}\left(\frac{1}{2} + \sigma\right).$$

The publisher's maximum profit under $I = 0$ is

$$\pi_P^{I=0} = \begin{cases} \max\left[\frac{1-\sigma}{2}, \frac{3}{4}\left(\frac{1}{2}+\sigma\right) - 3k, \frac{1}{2}\left(\frac{1}{2}+\sigma\right)\right] & \text{if } 3k \leq \sigma, \\ \max\left[1/2 - \sigma, \frac{1}{2}\left(\frac{1}{2}+\sigma\right)\right] & \text{otherwise.} \end{cases} \tag{A10}$$

Solving for optimal R embedded in max operator yields conditions stated in the proposition. ■

A1.4 Proof of Lemma 3

Proof. Sending the advertiser with the highest CTR maximizes the agency's probability of winning as well as its commission-based surplus. If the agency sends multiple advertisers, it only increases competition and hence reduces the auction surplus. ■

A1.5 Proof of Proposition 2

Proof. The steps closely mirror the proof of Proposition 1.

1. Advertisers' payoffs in each subgame
   (a) 3A-0D: After paying agency fee F, Advertiser 1 receives positive auction surplus only if it is selectively chosen by the agency; this occurs with probability:

$$\mathbb{P}\{c_1 = c_H\}\left(\mathbb{P}\{c_2 = c_3 = c_H\} \cdot \frac{1}{3} + \mathbb{P}\{c_2 \neq c_3\} \cdot \frac{1}{2} + \mathbb{P}\{c_2 = c_3 = c_L\} \cdot 1\right)$$
$$+ \mathbb{P}\{c_1 = c_L\}\mathbb{P}\{c_2 = c_3 = c_L\} \cdot \frac{1}{3},$$

where the probability 1/3 inside the bracket is the probability that Advertiser 1 is chosen by the agency when two competing advertisers also have high CTR, the probability 1/2 that when only one competing advertisers has high CTR, and the probability 1 that when all competing advertisers have low CTR. Finally, Advertiser 1 can be chosen w.p. 1/3 if all advertisers have low CTR. The probability of being chosen simplifies to

$$\frac{1}{2}\left(\frac{1}{4}\frac{1}{3} + 2\frac{1}{4}\frac{1}{2} + \frac{1}{4}\right) + \frac{1}{2}\frac{1}{4}\frac{1}{3} = \frac{1}{3}.$$

Conditional on being chosen, if the advertiser's CTR is high, its payoff is $c_H(1 - R/c_H) = 1/2 + \sigma - R$ if its CTR is high; and if its CTR is low, its payoff is $c_L(1 - R/c_L)\mathbb{I}_{\{R \leq c_L\}} = \max[1/2 - \sigma - R, 0]$. Therefore, each advertiser's expected payoff in the 3A-0D regime is

$$\pi_A^{I=1}(\text{3A-0D}) = \frac{1}{2}\left(\frac{1}{4}\frac{1}{3} + 2\frac{1}{4}\frac{1}{2} + \frac{1}{4}\right)\left(\frac{1}{2} + \sigma - R\right) + \frac{1}{2}\frac{1}{4}\frac{1}{3}\max\left[\frac{1}{2} - \sigma - R, 0\right]. \tag{A11}$$

(b) 2A-1D: Assume without loss of generality that Advertisers 1 and 2 go through the agency, and Advertiser 3 bids directly. Advertiser 1 receives positive auction surplus only if it is chosen by the agency, which, following the reasoning above, occurs with probability

$$\mathbb{P}\{c_1 = c_H\}\left(\mathbb{P}\{c_2 = c_H\} \cdot \frac{1}{2} + \mathbb{P}\{c_2 = c_L\} \cdot 1\right).$$

Another necessary condition for Advertiser 1 to gain positive surplus is that the direct bidding competitor has low CTR. Conditional on being chosen, if the advertiser's CTR is high, its payoff is $c_H(1 - \max[c_L, R]/c_H)(1/2) = (1/2 + \sigma - \max[1/2 - \sigma, R])/2$, where the division by 2 is due to the necessary condition that the direct bidding competitor have low CTR; and if its CTR is low, its payoff is 0. Thus, the expected payoff of the advertiser going through the agency in the 2A-1D regime *excluding the agency fee* F is

$$\pi_A^{I=1}(\text{2A-1D}) = \frac{1}{2}\frac{1}{2}\left(\frac{1}{2}\frac{1}{2} + \frac{1}{2}\right)\left(\frac{1}{2} + \sigma - \max\left[\frac{1}{2} - \sigma, R\right]\right). \tag{A12}$$

The direct bidding advertiser's payoff is positive only if its CTR is high, and the competing advertiser – selectively chosen by the agency – has low CTR. Therefore, the expected payoff of the advertiser bidding directly in the 2A-1D regime *excluding the campaign management cost* k is

$$\pi_D^{I=1}(\text{2A-1D}) = \frac{1}{2}\frac{1}{2}\frac{1}{2}\left(\frac{1}{2} + \sigma - \max\left[\frac{1}{2} - \sigma, R\right]\right). \tag{A13}$$

Following the reasoning above, we obtain the advertisers' payoffs for the different regimes:

$$\pi_A^{I=1}(\text{1A-2D}) = \pi_A^{I=0}(\text{1A-2D}) \text{ and } \pi_D^{I=1}(\text{1A-2D}) = \pi_D^{I=0}(\text{1A-2D}) \tag{A14}$$

$$\pi_A^{I=1}(\text{2A-0D}) = \frac{1}{2}\left(\frac{1}{2}\frac{1}{2} + \frac{1}{2}\right)\left(\frac{1}{2} + \sigma - R\right) + \frac{1}{2}\frac{1}{2}\frac{1}{2}\max\left[\frac{1}{2} - \sigma - R, 0\right] \tag{A15}$$

$$\pi_A^{I=1}(\text{1A-1D}) = \pi_A^{I=0}(\text{1A-1D}) \text{ and } \pi_D^{I=1}(\text{1A-1D}) = \pi_D^{I=0}(\text{1A-1D}) \tag{A16}$$

$$\pi_A^{I=1}(\text{1A-0D}) = \pi_A^{I=0}(\text{1A-0D}). \tag{A17}$$

2. Agency's optimal fee under each regime. Note that inducing a regime requires the agency satisfy the advertisers' IR and IC constraints. Since agency's payoff monotonically increases in F, F will be set such that the tighter of the two constraints binds. If both constraints cannot be jointly satisfied by any $F \geq 0$, we let the agency's payoff be equal to $-\infty$. The non-deviation conditions are analogous to the case without information; the profit expressions can be obtained by simply substituting $I = 0$ with $I = 1$. Impossibility of 1A-2D and 1A-1D continues to hold. Agency's optimal regime given R. Note that the publisher will never set $R < 1/2 - \sigma$, so it suffices to consider the agency's optimal regimes for $1/2 - \sigma \leq R \leq 1/2 + \sigma$.
Algebraic manipulations show that inducing 3A-0D yields highest payoff for the agency for all $R \in [1/2 - \sigma, 1/2 + \sigma]$ when information is disclosed.
3. Optimal R. Since agency induces 3A-0D for all $R \in [1/2 - \sigma, 1/2 + \sigma]$, publisher's profit is

$$\begin{aligned}\pi_P^{I=1}(\text{3A-0D}) &= \mathbb{P}\{\text{at least 1 } c_H\}R + \mathbb{P}\{\text{exactly one } c_H\}\max[c_L, R] + \mathbb{P}\{\text{all } c_L\}R\mathbb{I}_{\{R \leq c_L\}} \\ &= \frac{7}{8}R + \frac{1}{8}R\mathbb{I}_{\{R \leq 1/2 - \sigma\}}.\end{aligned}$$

The publisher considers the two end-points $R = 1/2 - \sigma$ and $R = 1/2 + \sigma$ as optimal reserve price candidates. The publisher's optimal profit under $I = 1$ is

$$\pi_P^{I=1} = \max\left[\frac{1}{2} - \sigma, \frac{7}{8}\left(\frac{1}{2} + \sigma\right)\right]. \tag{A18}$$

■

A1.6 Proof of Proposition 3

Proof. Since the market structure with information is 3A-0D, the total efficiency is

$$E_{I=1} = \frac{7}{8}\left(\frac{1}{2} + \sigma\right) + \frac{1}{8}\left(\frac{1}{2} - \sigma\right).$$

In the case where information is withheld, there are two market structures to consider. If the market structure without information is 2A-1D, the auction density is two. Therefore, total efficiency is

$$E_{I=0} = \frac{3}{4}\left(\frac{1}{2} + \sigma\right) + \frac{1}{4}\left(\frac{1}{2} - \sigma\right) - k.$$

If the market structure without information is either 1A-0D, 2A-0D, or 3A-0D, the auction density is one. Therefore, total efficiency is

$$E_{I=0} = \frac{1}{2}\left(\frac{1}{2} + \sigma\right) + \frac{1}{2}\left(\frac{1}{2} - \sigma\right).$$

The change in efficiency from withholding information is

$$\Delta_E \equiv E_{I=0} - E_{I=1} = \begin{cases} -\frac{1}{16} - \frac{1}{4}\sigma - k & \text{if 2A-1D,} \\ -\frac{3}{4}\sigma & \text{if 1A-0D, 2A-0D, 3A-0D.} \end{cases}$$

In either case, we have $\Delta_E < 0$ and $d|\Delta_E|/d\sigma > 0$.

■

A1.7 Proof of Proposition 4

Proof. The result follows immediately from comparing the subgame optimal profits $\pi_P^{I=0}$ in (A10) and $\pi_P^{I=1}$ in (A18). ■

Online Appendix

OA1 Proofs of Extension Results

OA1.1 Proof of Proposition 5

Proof. We first solve for the publisher's optimal expected profit under information withholding and then under information disclosure.

Suppose the publisher withholds information. Under information withholding, the agency does not know the advertisers' CTR nor their valuations, and therefore, continues to randomly rotate bids as it does in the main model.

Under 3A-0D, advertisers obtain positive surplus iff either

1. it has high CTR and v, and
   (a) both competing advertisers have high CTR and v, in which case it is sent to the auction with probability 1/3 for surplus $1/2+\sigma-R$,
   (b) only one competing advertiser has high CTR and v, in which case it is sent to the auction with probability 1/2 for surplus $1/2+\sigma-R$,
   (c) none of the competing advertiser has high CTR and v, in which case it is sent to the auction for surplus $1/2+\sigma-R$; or
2. it has low CTR but high v, and
   (a) both competing advertisers have low CTR but high v, in which case it is sent to the auction with probability 1/3 for surplus $\max[1/2-\sigma-R,0]$,
   (b) one competing advertiser has low CTR but high v, and the other has v, in which case it is sent to the auction with probability 1/2 for surplus $\max[1/2-\sigma-R,0]$,
   (c) both competing advertisers have low v, in which case it is sent to the auction for surplus $\max[1/2-\sigma-R,0]$.

Therefore, the advertisers' payoffs *excluding the fee and campaign management cost* are

$$\pi_A(\text{3A-0D}) = \frac{\beta}{2}\left(\left(\frac{\beta}{2}\right)^2\frac{1}{3}+2\frac{\beta}{2}\left(1-\frac{\beta}{2}\right)\frac{1}{2}+\left(1-\frac{\beta}{2}\right)^2\right)\left(\frac{1}{2}+\sigma-R\right)$$
$$+\frac{\beta}{2}\left(\left(\frac{\beta}{2}\right)^2\frac{1}{3}+2\frac{\beta}{2}(1-\beta)\frac{1}{2}+(1-\beta)^2\right)\max\left[\frac{1}{2}-\sigma-R,0\right].$$

Similarly, we can obtain

$$\pi_A(\text{2A-1D}) = \frac{\beta}{2}\left(\frac{\beta}{2}\frac{1}{2}+\left(1-\frac{\beta}{2}\right)\right)\left(\frac{\beta}{2}\left(\frac{1}{2}+\sigma-\max\left[\frac{1}{2}-\sigma,R\right]\right)+(1-\beta)\left(\frac{1}{2}+\sigma-R\right)\right)$$
$$+\frac{\beta}{2}\left(\frac{\beta}{2}\frac{1}{2}+(1-\beta)\right)(1-\beta)\max\left[\frac{1}{2}-\sigma-R,0\right]$$

and

$$\pi_D(\text{2A-1D}) = \frac{\beta}{2}\left(\left(\left(\frac{\beta}{2}\right)^2+2\frac{\beta}{2}(1-\beta)\right)\left(\frac{1}{2}+\sigma-\max\left[\frac{1}{2}-\sigma,R\right]\right)+(1-\beta)^2\left(\frac{1}{2}+\sigma-R\right)\right)$$
$$+\frac{\beta}{2}(1-\beta)^2\max\left[\frac{1}{2}-\sigma-R,0\right].$$

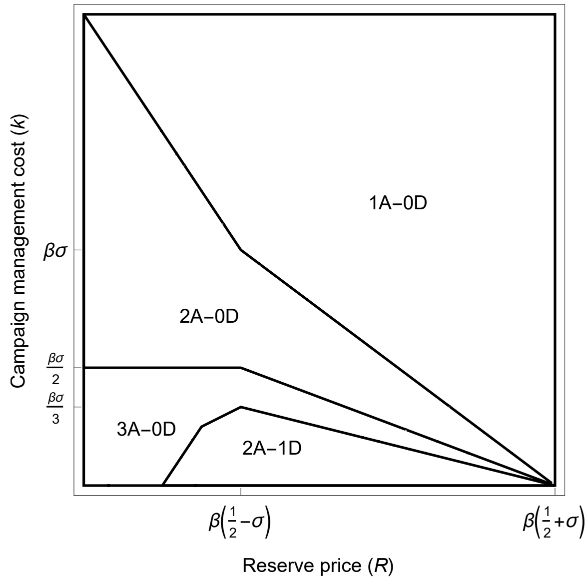

Figure OA1: Agency's Optimal Market Structure Under No Information

The agency's expected profit for each regime is the maximum fee that it can charge to induce the regime as the advertisers' subgame equilibrium choices, provided feasibility (i.e., advertisers' non-deviation). For example, the agency's expected profit under 3A-0D is

$$\pi_{\text{ag}}(\text{3A-0D}) = 3 \cdot \max\left\{F : \pi_A(\text{3A-0D}) - F \geq \max\left[\pi_D(\text{2A-1D}) - k, 0\right]\right\},$$

where the advertisers' payoffs $\pi_A(\text{3A-0D})$ and $\pi_D(\text{2A-1D})$ are as given above. Similarly, the agency's expected profit under 2A-1D is

$$\pi_{\text{ag}}(\text{2A-1D}) = 2 \cdot \max\left\{F : \pi_A(\text{2A-1D}) - F \geq \max\left[\pi_D(\text{1A-2D}) - k, 0\right], \pi_D(\text{2A-1D}) - k \geq \max\left[\pi_A(\text{3A-0D}) - F, 0\right]\right\}$$

For any reserve price R, the agency induces the regime that maximizes its expected profit. It can be shown that the agency's optimal regimes are as shown in Figure OA1.

Since setting reserve price strictly less than $\beta(1/2 - \sigma)$ is dominated for the publisher, it suffices to consider the agency's optimal strategies for $R \in [\beta(1/2 - \sigma), \beta(1/2 + \sigma)]$. Thus, the agency induces

$$\begin{cases} \text{2A-1D} & \text{if } 0 \leq k < -(R - \beta(1/2 + \sigma))/6, \\ \text{3A-0D} & \text{if } -(R - \beta(1/2 + \sigma))/6 < k \leq -(R - \beta(1/2 + \sigma))/4, \\ \text{2A-0D} & \text{if } -(R - \beta(1/2 + \sigma))/4 < k \leq -(R - \beta(1/2 + \sigma))/2, \\ \text{1A-0D} & \text{if } -(R - \beta(1/2 + \sigma))/2 < k. \end{cases}$$

Next, we solve for the publisher's optimal reserve price. Due to bid rotation, the publisher's expected profits under 3A-0D, 2A-0D, and 1A-0D are

$$\pi_P(\text{3A-0D}) = \pi_P(\text{2A-0D}) = \pi_P(\text{1A-0D}) = \frac{1}{2}R + \frac{1}{2}R \cdot \mathbb{I}_{\{R \leq \beta(1/2-\sigma)\}}$$

The publisher's expected profit under 2A-1D is

$$\pi_P(\text{2A-1D}) = \frac{1}{4}\left(\beta(1/2+\sigma) + 2\max[\beta(1/2-\sigma), R] + R \cdot \mathbb{I}_{\{R \le \beta(1/2-\sigma)\}}\right).$$

Therefore, the candidates for optimal R are

$$\begin{cases} \beta(1/2-\sigma), \beta(1/2+\sigma) - 6k, \text{ or } \beta(1/2+\sigma) & \text{if } 0 \le k < \beta\sigma/3, \\ \beta(1/2-\sigma) \text{ or } \beta(1/2+\sigma) & \text{if } \beta\sigma/3 < k. \end{cases}$$

The publisher sets R that induces 2A-1D iff

$$\frac{3k}{\beta} < \sigma \text{ and } k \le \tilde{k}', \tag{OA1}$$

where

$$\tilde{k}' = \frac{\beta}{12} \cdot \begin{cases} 1 & \text{if } \sigma \le \frac{1}{4}, \\ \frac{1}{2} + \sigma & \text{if } \sigma > \frac{1}{4}. \end{cases}$$

Consider the case when the publisher discloses information. Similar to the proof of Proposition 2, the advertisers' expected profits can be derived from the following observations regarding the two most representative regimes.

1. Under 3A-0D, the advertiser going through the agency obtains positive surplus iff either:
   (a) its CTR and valuation are both high, or
   (b) its CTR is low but valuation high, its competitors' CTR or valuation is low, and $R \le 1/2 - \sigma$.
2. Under 2A-1D, the advertiser going through the agency obtains positive surplus iff either
   (a) its CTR and valuation are high, whereas the direct-bidding competitor's CTR or valuation is low, or
   (b) its CTR is low but valuation high, its agency-bidding competitor's CTR or valuation is low, its direct-bidding competitor's valuation is low, and $R \le 1/2 - \sigma$.

It can then be shown that the publisher's expected profit is highest under 3A-0D.

It follows that withholding information yields the publisher a higher profit than disclosing information only if (OA1) holds. Otherwise, even under information withholding, the publisher induces none of the advertisers to bid directly; therefore, disclosing information cannot yield a lower profit in these parameter regions.

Comparing the subgame optimal profits $\pi_P(\text{2A-1D})$ under information withholding and $\pi_P(\text{3A-0D})$ under information disclosure, we obtain that withholding information yields higher profit iff

$$k \le \frac{\beta\sigma}{3} \text{ and } \underline{\sigma} < \sigma < \overline{\sigma},$$

where

$$\underline{\sigma} = \frac{(2-\beta)(1-\beta)}{5-2\beta(3-\beta)} \text{ and } \overline{\sigma} = \frac{6}{16-\beta(6-\beta)} - \frac{1}{2}. \tag{OA2}$$

The latter condition holds only if $\beta > \tilde{\beta}$, where is the unique root of

$$\tilde{\beta} = \left\{ \beta \in [0,1] : \frac{(2-\beta)(1-\beta)}{5-2\beta(3-\beta)} = \frac{6}{16-\beta(6-\beta)} - \frac{1}{2} \right\}. \tag{OA3}$$

Note that $\tilde{\beta}$ exists uniquely by the Intermediate Value Theorem because the difference

$$\frac{(2-\beta)(1-\beta)}{5-2\beta(3-\beta)} - \left(\frac{6}{16-\beta(6-\beta)} - \frac{1}{2} \right)$$

is strictly decreasing in β with opposite valences at endpoints $\beta = 0$ and $\beta = 1$. ■

OA1.2 Proof of Corollary 1

Proof. We first derive the area A of the parametric region for which the publisher withholds information. The region is a trapezoid, whose area can be expressed using the definitions of $\underline{\sigma}$ and $\overline{\sigma}$ in (OA2):

$$A = \frac{1}{2} \cdot \left(\frac{\beta \underline{\sigma}}{3} + \frac{\beta \overline{\sigma}}{3} \right) \cdot (\overline{\sigma} - \underline{\sigma}) = - \frac{\beta(44-\beta(66-23\beta))(84-\beta(174-\beta(121-4(9-\beta)\beta)))}{24(16-(6-\beta)\beta)^2(5-2(3-\beta)\beta)^2}. \tag{OA4}$$

For all $\beta < \tilde{\beta}$, where $\tilde{\beta}$ is as defined in (OA3), there is no parametric region for which the publisher withholds information. For all $\beta \geq \tilde{\beta}$, (OA4) increases in β. ■

OA1.3 Proof of Lemma 4

Proof. See Section OA2 below. ■

OA2 Analysis for Partial Information Disclosure Extension

We provide here the analysis for partial information disclosure extension. We focus on the 3A-0D subgame in which all three advertisers go through the agency. The analyses for the other subgames are analogous and omitted.

Since the agency can decide how many advertisers to send to the auction conditional on the signal realizations, we distinguish three different cases.

1. Suppose only one advertiser has high-CTR-signal. The agency's payoffs when it sends only the high-CTR-signal advertiser, when it sends a high-CTR-signal advertiser and a low-CTR-signal advertiser, and when it sends all advertisers, respectively, are as follows (we economize notation by letting $c_H = 1/2 + \sigma$ and $c_L = 1/2 - \sigma$

$$\begin{aligned}\pi_{\text{ag}}(1H) &= \mathbb{P}\left\{c_H | s = c_H\right\}(c_H - R) + \mathbb{P}\left\{c_L | s = c_H\right\} \max\left[c_L - R, 0\right] \\ &= \rho(c_H - R) + (1-\rho)\max[c_L - R, 0],\end{aligned}$$

$$\begin{aligned}\pi_{\text{ag}}(1H1L) &= \left(\mathbb{P}\{c_H|s=c_H\}\mathbb{P}\{c_L|s=c_L\} + \mathbb{P}\{c_H|s=c_L\}\mathbb{P}\{c_L|s=c_H\}\right)(c_H - \max[c_L, R]) \\ &= \left(\rho^2 + (1-\rho)^2\right)(c_H - \max[c_L, R])\end{aligned}$$

and

$$\pi_{\text{ag}}(1H2L) = \left(\mathbb{P}\{c_H|s=c_H\}\mathbb{P}\{c_L|s=c_L\}^2 + 2\mathbb{P}\{c_L|s=c_H\}\mathbb{P}\{c_H|s=c_L\}\mathbb{P}\{c_L|s=c_L\}\right)(c_H - \max[c_L, R]),$$

which simplifies to $\left(\rho^3 + 2\rho(1-\rho)^2\right)(c_H - \max[c_L, R])$.

2. Suppose only two advertisers have high-CTR-signals.

$$\pi_{\text{ag}}(1H) = \text{same as above}$$

$$\begin{aligned}\pi_{\text{ag}}(2H) &= 2\mathbb{P}\{c_H|s=c_H\}\mathbb{P}\{c_L|s=c_H\}(c_H - \max[c_L, R]) \\ &= 2\rho(1-\rho)(c_H - \max[c_L, R])\end{aligned}$$

and

$$\pi_{\text{ag}}(2H1L) = \left(2\mathbb{P}\{c_H|s=c_H\}\mathbb{P}\{c_L|s=c_H\}\mathbb{P}\{c_L|s=c_L\} + \mathbb{P}\{c_L|s=c_H\}^2\mathbb{P}\{c_H|s=c_L\}\right)(c_H - \max[c_L, R]),$$

which simplifies to $\left(2\rho^2(1-\rho) + \rho(1-\rho)^2\right)(c_H - \max[c_L, R])$.

3. Suppose three advertisers have high-CTR-signals. $\pi_{\text{ag}}(1H)$ and $\pi_{\text{ag}}(2H)$ are same as above.

$$\begin{aligned}\pi_{\text{ag}}(3H) &= 3\mathbb{P}\{c_H|s=c_H\}\mathbb{P}\{c_L|s=c_H\}^2(c_H - \max[c_L, R]) \\ &= \rho(1-\rho)^2(c_H - \max[c_L, R])\end{aligned}$$

In all cases above in which at least one advertiser has high-CTR signal, it is most profitable for the agency to send only one high-CTR-signal advertiser to the auction.

Next, suppose three advertisers have low-CTR signals. If the agency sends three advertisers to the auction, its payoff is

$$\begin{aligned}\pi_{\text{ag}} &= 3\mathbb{P}\{c_H|s=c_L\}\left(1 - \mathbb{P}\{c_H|s=c_L\}\right)^2(c_H - \max[c_L, R]) \\ &= 3(1-\rho)\rho^2(c_H - \max[c_L, R]).\end{aligned}$$

If it sends two advertisers, its payoff is

$$\begin{aligned}\pi_{\text{ag}} &= 2\mathbb{P}\{c_H|s=c_L\}\left(1 - \mathbb{P}\{c_H|s=c_L\}\right)(c_H - \max[c_L, R]) \\ &= 2(1-\rho)\rho(c_H - \max[c_L, R]).\end{aligned}$$

If it sends one advertiser, its payoff is

$$\begin{aligned}\pi_{\text{ag}} &= \mathbb{P}\{c_H|s=c_L\}(c_H - R) + \mathbb{P}\{c_L|s=c_L\}\max[c_L - R, 0] \\ &= (1-\rho)(c_H - R) + \rho\max[c_L - R, 0].\end{aligned}$$

Therefore, for $R \in [c_L, c_H]$, when all advertisers have low-CTR signals, the agency sends only two advertisers if $\rho \leq 2/3$, and three advertisers otherwise.

Given the agency's strategies, we can compute the advertisers' expected profits under each regime.

For example, under 3A-0D,

$$\pi_A(\text{3A-0D}) = \frac{1}{2}\frac{1}{4}\left(\frac{1}{3} + 2\frac{1}{2} + 1\right)(\rho(1/2 + \sigma - R) + (1-\rho)\max[1/2 - \sigma - R, 0]) \\ + \frac{1}{8}(1-\rho)\rho(1/2 + \sigma - \max[1/2 - \sigma, R]) \cdot \begin{cases} \frac{2}{3} & \text{if } 1/2 \leq \rho \leq 2/3, \\ \rho & \text{if } 2/3 < \rho \leq 1. \end{cases}$$

The agency then sets the fee to induce the profit-maximizing regime (e.g., 3A-0D vs. 2A-1D) under the advertisers' non-deviation conditions.

Anticipating the agency's subgame equilibrium strategies, the publisher sets the profit-maximizing disclosure level ρ and reserve price R. Due to the high polynomial order of profit function, we solve for the optimal ρ and R numerically using Mathematica's NMaximize function.