A Theory of Minimalist Luxury

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Abstract

In this paper, we incorporate incomplete information about consumer wealth and presence of high-quality counterfeits to investigate when a consumer may engage in moderate, as opposed to, excessive types of conspicuous consumption to signal his or her wealth. Past literature has shown that when wealth is not observable, the wealthy are motivated to outspend the low-types to signal their wealth. We show in a single model, however, that when high-quality counterfeits exist and are visibly indistinguishable from the authentic products, excessive consumption as a signal for wealth may no longer be effective. Instead, the wealthy may purposefully restrain from consumption of luxury goods to separate themselves from the rest, and hence, ownership of fewer items can be more effective in signaling wealth. We refer to this equilibrium as the minimalist luxury consumption equilibrium and describe the conditions under which a minimalist or an excessive consumption equilibrium exists. We draw managerial implications for luxury brands’ pricing strategy under each equilibrium.

Keywords: Luxury brands, Signaling, Conspicuous consumption, Counterfeits, Minimalism as Luxury

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1 Motivation

In “The Theory of Leisure Class,” Veblen (1899) argues that in a world where wealth is unobservable, individuals could engage in excessive consumption of goods to signal their wealth to others. He calls this “conspicuous consumption.” Since Veblen, many studies demonstrated that conspicuous consumption is indeed a prevalent behavior. Today, it is well accepted that luxury brands are purchased not just for functional benefits, but also for symbolic benefits (Belk, 1988). As Arrow and Dasgupta (2009) pointed out, “‘[C]onspicuous consumption’ in Veblen’s account plays an instrumental role in human behavior, namely, as a way of signaling one’s wealth to others in an environment where wealth is not observable. Wealth confers status and esteem; conspicuous consumption is a way of displaying wealth.” (pg. F498). Conspicuous consumption can display wealth because anyone not wealthy cannot afford to mimic. Thus more spending is better for status signaling.

If "more is better," it is puzzling to see a growing trend of wealthy luxury consumers embracing “minimalist luxury,” where they exercise “purposeful limitation” (Fagan, 2017) or “material purge,” (Chayka, 2016) by buying few luxury goods even if they can afford more. A search on Google Trends reveals that the interest in minimalism as luxury has nearly tripled in the past decade, as Figure 1 shows. By doing so, they stand out such that “The richer you are, the less you have” (Chayka, 2016). As Fagan (2017) points out, “It is just another form of conspicuous consumption, a way of saying to the world: ‘Look at me! Look at all of the things I have refused to buy, and the incredibly-expensive, sparse items I have deemed worthy instead!’” In this paper, we will investigate how minimalist luxury can paradoxically become a new form of conspicuous consumption by which the wealthy signal their status.

Moreover, the “more is better” rationale also fails to recognize the presence of high-quality, low-price counterfeits in some product categories. Their presence makes it harder for the wealthy to signal their status through excessive consumption, as mimicking the wealthy through purchasing those counterfeits becomes feasible. The existence of such counterfeits is a reality. In a Bloomberg article titled “Alibaba’s Jack Ma: Better-Than-Ever Fakes Worsen Piracy War,” Jack Ma, Alibaba’s founder and CEO, is quoted as saying that “fake products today […] are [from] exactly the [same] factories, exactly the same raw materials” (Ramli and Chen, 2016). In this paper, we will examine if minimalist luxury can be used by the wealthy as an effective signal for their status in the presence of high-quality, low-price counterfeits.

We develop a parsimonious theoretical model on minimalist luxury with two salient features:
unobservable consumer types and the availability of high-quality, low-price counterfeits. We show that the restraint in consuming luxury goods displayed on the part of the wealthy, whom we shall call the “high-types,” is a viable strategy to signal their types in a product category where high-quality, low-price counterfeits exist. In this minimalist luxury equilibrium, the high-types purchase fewer luxury goods in that product category than the total number of (authentic plus counterfeit) goods the rest or the “low types” do. They also purchase fewer luxury goods in the presence of counterfeits versus in their absence. This is in contrast to the familiar thesis where the high-types always purchase more luxury goods to display their wealth. We focus on identifying the mechanism through which “less is more” and derive the managerial implications of this signaling phenomenon.

Our analysis provides a rational explanation for why we observe the wealthy consumers engage in owning fewer items and why owning fewer items can be a credible signal of wealth. We do so by deriving a minimalist luxury equilibrium in which the high-types consume fewer luxury goods to signal their wealth. The high-types can rely on minimalist consumption compared to the low-types to effectively signal their status. This is because, even though both types value functional as well as symbolic benefits, the high-types value symbolic benefits more and functional benefits less than the low-types. As a result, they are more willing to sacrifice functional benefits for symbolic benefits. This allows the high-types to pursue a strategy characterized by high symbolic benefits and low functional benefits, a strategy that the low-types are not willing to mimic. The separation allows wealthy minimalists to stand out.

Past literature mostly builds on the Veblen (1899)’s thesis that conspicuous consumption is a
form of costly wealth signaling. Bagwell and Bernheim (1996) theoretically derives the condition under which the Veblen effect can arise. A number of papers in economics, marketing, and psychology also expand the number of signals to include non-conformity, brand prominence, product size, and social capital (Bellezza et al., 2014; Han et al., 2010; Berger et al., 2011; Dubois et al., 2012). By incorporating social influence into consumption utility, Amaldoss and Jain (2005a) shows that an upward-sloping demand curve can occur when both snobs and followers derive utility, albeit differently, from others who purchase the same product. Subsequently, they also extend this model to a competitive context to discuss pricing and promotion decisions (Amaldoss and Jain, 2005b). Also in the competitive context, Kuksov and Xie (2012) shows that when consumers value status goods, the dynamics of pricing competition can be quite unintuitive in that for apparent substitute goods, competition can raise rather than decrease market prices. Our objective here is different. Our research extends the original Veblen effect and shows that when high-quality counterfeits exist in a product category, it is possible that minimalist instead of excessive consumption of luxury goods can help the high-types to stand out.

The concept of minimalist luxury is closely related to the idea of “quiet luxury” that has been explored in the behavioral literature (Han et al., 2010; Dubois et al., 2012; Bellezza et al., 2014; Berger and Ward, 2010). The premise of quiet luxury is that the discreet signals of luxury such as smaller logo size, less-striking color schemes and subtle design can be more powerful and more telling about one’s social status and social aspirations. As a result, luxury brands can charge more for “quieter” items. In our research, we go back to the original Veblen’s thesis on excessive consumption and examine if minimalist consumption can also signal wealth. To the extent that minimalist consumption is a discreet signal, our study extends the concept of quiet luxury to include the situation where the high-types purposefully limit their consumption of luxury goods.

Qian (2008, 2014a,b) have significantly advanced the research on the impact of counterfeits. Qian (2008) offers insights about the macroeconomic consequences of the entry by counterfeiters into a country. Qian (2014b) finds that counterfeits have positive impact on sales of luxury goods through their advertising effect in addition to a negative impact due to substitution. A similar conclusion is also drawn in the context of pirated digital goods such as movies and music (Givon et al., 1995; Bai and Waldfogel, 2012; Waldfogel, 2012). Wilcox et al. (2009) study how the usage of counterfeits may change the preference for authentic products. In this paper, we take up the issue of high-quality counterfeits and how their presence poses difficulty for the high-types to use conspicuous consumption as a signal for their status. In that context, we explore theoretically
the possibility of using minimalist consumption as an effective signal for wealth, which has, to our knowledge hitherto been overlooked in the literature.

Another related research is Feltovich et al. (2002), who study in a labor market context the phenomenon of counter-signaling, i.e., refraining from sending a signal. They show that when a sender’s information is noisy, the high-types can choose not to signal to separate themselves from the medium-types and the low-types. Our framework differs from theirs in two distinct ways. First, counter-signaling in their framework refers to the case where the high-types refrain from sending any signals, whereas in our case the high-types refrain from excessive consumption to send a clear signal of their types. Second, the mechanism of counter-signaling in their framework is motivated by the fact that the receiver has prior noisy information about the type of a sender even before observing a signal. In this context, only the medium-types decide to send a costly signal to stand out. As a result, the high-types choose not to send signals and rely on the additional information to stochastically separate from the low-types. Counter-signaling by the high-types arises here as a way to distinguish themselves from the medium-types. With only the high-types and the low-types in our model, we show that the high-types, instead of refraining from sending any signal, send a costly one by sacrificing functional benefits.

Our research uncovers the mechanism through which minimalist luxury can be a signal of wealth. In the process, we generate a number of managerially relevant insights. First, we show that the damage inflicted on a luxury brand by counterfeits comes not only from those who switch to counterfeits, but also from those who never purchase counterfeits. This is because the high-types purposefully refrain from conspicuous consumption as a signal for their status, and hence reduce luxury sales. Second, this reduction in luxury sales is dependent on how much one cares about the functionality and the quality of a product. To reduce the sales distortion, a luxury brand can draw consumer attention, rather surprisingly, more to the functional benefits rather than to the symbolic benefits of its products. Third, the sales distortion also arises from unobservable wealth and the presence of high-quality, low-price counterfeits, and the size of this reduction hinges on both factors. A luxury brand can thus leverage these two factors to minimize such a distortion. Finally, the sales reduction offers a good opportunity for a luxury brand to raise its prices. This means that a growing trend of minimalist luxury will lead to more exclusivity in the luxury market: higher prices and lower sales.

In the rest of the paper, we first start by developing a model in Section 2. We provide our key insights in Section 3 and we make a few extensions to our benchmark model to check the robustness of our conclusions in Section 4. Finally, in Section 5 we conclude.
2 Model

We consider a market with two outlets represented by $s \in S = \{A, C\}$ where $(A)$ is a luxury brand with a portfolio of $n \in \mathbb{R}^+ \ (n \geq 1)$ products\(^1\) and $(C)$ is the counterfeiter which sells replicas of each luxury product. Consumers can choose to buy the authentic version or the replica of each one of the $n$ products, or can buy nothing. An authentic product and its replica have similar functions, and thus, a consumer buys either but not both of them. For instance, a Louis Vuitton handbag of a certain design and its replica are used for the same purpose of carrying personal items, so a consumer chooses between the authentic version or the replica of the product. Consumers in the market know the outlets where they can buy a fake product or an authentic one. For instance, they may go to the flagship store to buy the authentic version and buy the fake on the street. As a result, even when consumers cross-shop from both outlets, they own no more than $n$ products in total. This helps us to consider up to $n$ products and significantly reduces the solution space we consider. Luxury goods are sold all at price $p_A$ and counterfeits at $p_C$, where $p_A > p_C$. In the benchmark model, we assume that the price of counterfeit goods is negligible by setting $p_C = 0$. This assumption simplifies the expressions we derived in equilibrium. We show in Section 4 that when $p_C > 0$, the qualitative insights remain identical as long as $p_C$ is not too high.

Consumer utility depends on two factors, which we refer to as the functional and symbolic benefits respectively. Consumers are heterogeneous in two dimensions. First, they care either highly or little about the symbolic value of a product relative to its functional value, calibrated by the parameter $\alpha \ (\in \{\alpha_H, \alpha_L\})$. Second, consumers have high or low wealth levels and hence different budgets on luxury goods purchase, denoted by $B \ (\in \{B_H, B_L\})$. Jointly, the two dimensions of heterogeneity $(\alpha, B)$ indicate the type of a consumer, which is her private information and is unknown to others. The share of type $(\alpha, B)$ consumers in the market is denoted by $\rho(\alpha, B)$. We normalize the total market size to be 1 such that $\sum \rho(\alpha, B) = 1$.

To quickly get to our analytical results, in the benchmark model, we assume that there are only two types of consumers in the market: high-types $(\alpha_H, B_H)$ and low-types $(\alpha_L, B_L)$. The high-types can spend more luxury goods and also care more about the symbolic benefits of a product than the functional value whereas the opposite holds for the low-types. Note that, in reality, there may exist high-wealth consumers who care more about functional benefits and low-wealth consumers who care more about the symbolic benefits of a luxury product. Later we will consider a market with these

\(^1\)For technical convenience, we assume that the size of product portfolio is a continuous variable instead of integers. Note that this assumption is not consequential to our qualitative results.
consumers as well in Section 4. The addition of these two segments of consumers adds complexity to our analysis, but does not alter our qualitative conclusions.

In many markets around the world, the quality of counterfeits are high, and in some product categories, a consumer may not be able to distinguish the authentic product from the fake one as we have noted in the introduction. This is when counterfeits can become a serious threat, and therefore, we will focus our attention on this case by assuming luxury products and counterfeits are not distinguishable solely based on their visible attributes to a consumer, although quality differences exist. Therefore, whether the product is fake or authentic is the private information of the consumer who owns a product, and others cannot tell. Nevertheless, others make an inference about a consumer’s type based on the number of products she displays or shows off.

Counterfeits and authentic goods have objectively different qualities, \( q_A \) and \( q_C \), respectively, which determine the functional benefits to a consumer. Specifically, let

\[
U(k_A, k_C) = q_A k_A + q_C k_C + X
\]

indicate the total functional benefits from owning \( k_A \) authentic and \( k_C \) fake products and \( X \) numeraire goods where the functional benefits of each numeraire good is normalized to 1. We assume that \( q_A > q_C > 1 \). We also rule out the trivial case where low-types can always afford all available authentic products by assuming that \( n (q_A - q_C) > B_L \). Consumers also gain utility when they are perceived as a high-type, which we refer to as the symbolic benefit of luxury consumption. Putting these factors together, a consumer’s decision is to maximize a weighted sum of the functional and symbolic benefits subject to her budget constraint as given below,

\[
V(\alpha, B, k_A, k_C|\mu) = (1 - \alpha) \underbrace{U(k_A, k_C)}_{\text{Functional benefits}} + \alpha \underbrace{P_{\mu(k=k_A+k_C)}(B = B_H)}_{\text{Symbolic benefits}},
\]

s.t. \( p_A k_A + p_C k_C + X \leq B, \)

\[ k_A + k_C \leq n, \]

where \( P_{\mu(k)}(B = B_H) \in [0,1] \) refers to the probability that the consumer will successfully signal being a high-type through ownership of \( k \) products and \( \mu (k = k_A + k_C) \) represents the belief function of others about the probability that a consumer who owns \( k \) items in total is a high-type.
We explain the formation of this belief function in section 3.1. A consumer gains zero symbolic benefits if she is perceived as a low-type and otherwise obtains a non-zero benefit. Since wealth is unobservable to others, this creates the desire to signal wealth through consumption.

The timing of the game is as follows. First, the luxury brand owns a portfolio of $n$ products and chooses a price of $p_A$ for its products. The portfolio and prices are publicly observed by the counterfeiter and all consumers. The counterfeiter provides a replica of all products in the portfolio at the competitive price of $p_C$. Then, conditional on how much they care about signaling their status and their wealth, consumers decide how many authentic and counterfeit items to buy, respectively denoted as $k_A$ and $k_C$, from all available products, i.e., $k_A + k_C \leq n$. Finally, consumers purchase and consume the products. The public observes the total number of products a consumer owns $(k_A + k_C)$, and makes an inference about the consumer’s type. Consumers gain functional benefits and symbolic benefits if they are perceived as high-types.

Figure 2: Timing of the game

<table>
<thead>
<tr>
<th>A luxury Brand</th>
<th>owns a portfolio of $n$ products and chooses a price $p_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Counterfeiter</td>
<td>provides a replica of all products in the portfolio at the price of $p_C$</td>
</tr>
<tr>
<td>Consumers</td>
<td>observe the prices $p_A$ and $p_C$</td>
</tr>
<tr>
<td>Consumers</td>
<td>privately shop for $k_A$ authentic and $k_C$ counterfeit items from each outlet</td>
</tr>
<tr>
<td>Consumers</td>
<td>display a total of $(k_A + k_C)$ different products they own to the public</td>
</tr>
<tr>
<td>The public</td>
<td>observes the total number of products a consumer owns</td>
</tr>
<tr>
<td>The public</td>
<td>makes an inference about the consumer’s types</td>
</tr>
</tbody>
</table>

3 Analysis

3.1 Analysis of Consumer’s Decision

In this section, we formally define the equilibrium concept in our model. Recall that in the benchmark model, there are two types of consumers: high-types and low-types. Each consumer purchases $k_A$ and $k_C$ products from the authentic and counterfeit outlets, respectively, such that the total number of purchased items does not exceed $n$, i.e. $(k_A, k_C) \in \mathcal{L} := \{k_A \in \mathbb{R}_{\geq 0}, k_C \in \mathbb{R}_{\geq 0} : k_A + k_C \leq n\}$. We focus on the pure strategy perfect Bayesian equilibrium (PBE) profile $(k_A^*, k_C^*) : \{(\alpha_H, B_H), (\alpha_L, B_L)\} \rightarrow \mathcal{L}$ that satisfies the following three criteria:

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2By using the same prices here, we simplify our analysis by taking out price distribution as a factor for our equilibrium. We will discuss in section 5 in what sense our conclusions can still hold even without this simplification.
1. Given the public’s belief $\mu$, each consumer $(\alpha, B)$ chooses the number of authentic and counterfeit products to buy to maximize her utility subject to her budget constraint:

$$(k^*_A(\alpha, B), k^*_C(\alpha, B)) \in \arg \max_{k_A, k_C} V(\alpha, B, k_A, k_C|\mu)$$

$$s.t. p_A k_A + p_C k_C + X \leq B$$

$$k_A + k_C \leq n.$$ 

2. For any number of products $k \in [0, n]$ that a consumer purchases, there exists a public belief $\mu : [0, n] \mapsto [0, 1]$ corresponding to the probability that a consumer who owns $k (= k_A + k_C)$ items in total is a high-type:

$$\mathbb{P}^{\mu(k)}(B = B_H) = \mu(k).$$

3. In every on-the-equilibrium path, beliefs are consistent with the equilibrium strategies according to Bayes’ rule. Specifically, the belief $\mu$ formed upon observing $k$ items satisfies the following two conditions:

$$\mu(k) = \begin{cases} 
\frac{\rho(\alpha_H, B_H)}{\rho(\alpha_H, B_H) + \rho(\alpha_L, B_L) \mathbb{1}[k = k^*_A(\alpha_L, B_L) + k^*_C(\alpha_L, B_L)]} & \text{if } k = k^*_A(\alpha_H, B_H) + k^*_C(\alpha_H, B_H) \\
\frac{\rho(\alpha_H, B_H) \mathbb{1}[k = k^*_A(\alpha_H, B_H) + k^*_C(\alpha_H, B_H)]}{\rho(\alpha_H, B_H) \mathbb{1}[k = k^*_A(\alpha_H, B_H) + k^*_C(\alpha_H, B_H)] + \rho(\alpha_L, B_L)} & \text{if } k = k^*_A(\alpha_L, B_L) + k^*_C(\alpha_L, B_L) 
\end{cases}$$

where $k^*_A(\alpha_H, B_H)$ and $k^*_A(\alpha_L, B_L)$ are the equilibrium number of authentic products that the high-types and low-types own respectively. Similarly, $k^*_C(\alpha_H, B_H)$ and $k^*_C(\alpha_L, B_L)$ indicate the number of counterfeits owned by each type in equilibrium. In the expressions above, $\mathbb{1}[]$ is an indicator function that equals one if statement in the bracket is true and zero otherwise.

Finally, the profit of a luxury brand is given by the total revenue from both high-types and low-types:

$$\pi = \rho(\alpha_H, B_H) k^*_A(\alpha_H, B_H) p_A + \rho(\alpha_L, B_L) k^*_A(\alpha_L, B_L) p_A.$$ 

We define an equilibrium as “profit-maximizing” if no other equilibrium produces strictly higher profits for the luxury brand. A luxury brand selects the profit-maximizing equilibrium by choosing its price optimally. It is well-known that signaling games suffer from multiplicity of equilibria
(Feltovich et al., 2002). The Intuitive Criterion is commonly used to eliminate equilibria which are less intuitive or implausible (Cho and Kreps, 1987). In the analysis that follows, we assume that if a luxury brand is indifferent between two profit-maximizing equilibria, it will choose the one that is Pareto dominant and satisfies the Intuitive Criterion. Moreover, to deliver equilibria of a viable luxury market, we assume that the high-types care sufficiently more about their symbolic benefits relative to functional benefits than the low-types do, or \( \frac{\alpha_L}{1-\alpha_L} \leq \frac{q}{q_A} \). We proceed to derive the equilibria of our model.

### 3.2 Preliminary Analysis: Observable Wealth and Absence of Counterfeits

Here we explore to what extent the high-types distort their consumption to stand out and how the availability of counterfeits may impact the sales of the authentic brand. To do all of these, we first establish three benchmark cases. The first case is where a consumer’s purchasing decision only affects her functional benefits, but not her symbolic benefits, which is given in a world of complete information where wealth is observable, and counterfeits are not available. The second case is where a consumer’s purchasing decision affects her functional benefits as well as her symbolic benefits, but counterfeits are not available. The third case is where a consumer’s purchasing decision only affects her functional benefits, but not her symbolic benefits due to complete information, and counterfeits are available. Lemmas below summarize consumer behavior in each case. These analyses will pave the way for analyzing our model of incomplete information where counterfeits are available.

**Lemma 1. (Observable Types and No Counterfeits)** If consumer types are observable to all and counterfeits are not available, then the high-types buy all available authentic luxury goods such that \( k^*_A(\alpha_H, B_H) = n \). The low-types purchase authentic goods in the amount of \( k^*_L(\alpha_L, B_L) = \max \left\{ \frac{B_L}{q_A}, \left( \frac{B_L}{B_H} \right) n \right\} \). The optimal price of the luxury brand is given by \( p^*_A = \min \left\{ q_A, \frac{B_H}{n} \right\} \).

The proof is provided in the Appendix. In this equilibrium, consumers need not signal their types and they each purchase only authentic goods to maximize their own functional benefits constrained only by their own budget. In this market, a luxury brand sells more to the high-types than to the low-types, and it can always exhaust the low-types’ budget, but can only do so for the high-types if its products have sufficiently high-quality.

**Lemma 2. (Unobservable Wealth and No Counterfeits)** If consumer types are unobserved by others, and counterfeits are not available,
(i) Case 1 \( (B_H \leq \rho(\alpha_L, B_L) B_L + nq_A \text{ or } B_L \geq \left( \frac{\alpha_L}{1-\alpha_L} \right) \rho(\alpha_H, B_H) ) : \)

The high-types buy all available authentic luxury goods such that \( k^*_h (\alpha_H, B_H) = n \), and the low-types purchase authentic goods in the amount of \( k^*_l (\alpha_L, B_L) = \max \left\{ B_L, \left( \frac{B_L}{B_H} \right) n \right\} . \) The luxury brand sets price optimally at \( p^*_A = \min \left\{ q_A, \frac{B_H}{n} \right\} . \)

(ii) Case 2 \( (B_H > \rho(\alpha_L, B_L) B_L + nq_A \text{ and } B_L < \left( \frac{\alpha_L}{1-\alpha_L} \right) \rho(\alpha_H, B_H) ) : \)

The high-types buy authentic goods in the amount of \( k^*_h (\alpha_H, B_H) = \min \left\{ \frac{B_H - \alpha_L}{q_A} n, B_L \right\} \) and the low-types do not buy any luxury goods, i.e., \( k^*_l (\alpha_L, B_L) = 0 \). The luxury brand sets price optimally at \( p^*_A = \max \left\{ q_A \left( \frac{B_H}{B_H - \alpha_L} \right), \left( \frac{\alpha_L}{1-\alpha_L} \right) \frac{B_H}{n} + q_A \right\} . \)

The proof is provided in the Appendix. In Case 1, when the budget difference between the high-types and the low-types is not very large, the luxury brand has the incentives to sell to both types, and each type purchases according to the functional benefits and budget as if the types are observable. No purchase distortion is necessary for the high-types to stand out in this case, as functional utilities alone will drive the high-types to purchase more luxury goods than what the low-types could afford. In Case 2, the budget difference is sufficiently large such that the luxury brand will focus on selling only to the high-types with high prices. To extract more rents from the high-types who value symbolic benefits, the luxury brand charges a prohibitively high price to deter the low-types from purchasing any luxury goods. In this case, the high-types will purchase more than what maximizes their functional benefits and what the low-types could afford to mimic. In fact, given the prices that the luxury brand charges, both the high-types and the low-types should purchase zero luxury goods setting aside the symbolic benefits.

Comparing Lemma 2 with Lemma 1, we see that unobservable consumers’ types are actually a blessing to the luxury brand. This is reflected in the fact that the luxury brand makes as much profits in both cases when the budget for the high-types is sufficiently low. When the budget is sufficiently high, the luxury brand charges higher prices and makes more profits when the types are unobservable. Thus Lemma 1 and Lemma 2 together show that consumer’s signaling benefits the luxury brand when counterfeits are not available.

Similarly, we can isolate the effect of counterfeits in a luxury market through the following lemma.

**Lemma 3. (Observable Wealth with Counterfeits)** If consumer types are observable to all, and counterfeits are also available, then the high-types buy all available authentic luxury goods and no counterfeits. The low-types purchase a mixture of authentic and counterfeit goods. Specifically,
\[
k^*_A(\alpha_H, B_H) = n, k^*_C(\alpha_H, B_H) = 0,
\]
\[
k^*_A(\alpha_L, B_L) = \max \left\{ \frac{B_L}{q_A - q_C} \left( \frac{B_L}{B_H} \right)^n \right\}, k^*_C(\alpha_L, B_L) = \min \left\{ n - \frac{B_L}{q_A - q_C} \left( 1 - \frac{B_L}{B_H} \right)^n \right\},
\]
where the optimal price of a luxury brand is given by \( p^*_A = \min \left\{ q_A - q_C, \frac{B_H}{n} \right\} \).

The proof for Lemma 3 is in the Appendix. Comparing this lemma with both Lemma 1 and Lemma 2, we see that the availability of counterfeits will reduce a luxury brand’s sales, prices, and profitability as expected, and it is an austere harm for the luxury brand. This effect is well-known in the literature (Grossman and Shapiro, 1988) and it is the chief motivation for a luxury brand to combat counterfeits in practice.

### 3.3 Distortion in Consumption with Counterfeits and Unobservable Wealth

It is important to notice a general insight from all three lemmas from the previous subsection: when counterfeits are not available, or when it is available but consumer types are observable, the high-types always purchase more authentic luxury goods than the combined total purchases of authentic and counterfeit goods by the low-types. This is hardly surprising, given the conventional wisdom that high-types always engage in excessive consumption of luxury goods to signal their status. The following proposition will show, however, that the high-types may break from that set behavior when counterfeits are available and their types are unobservable.

**Proposition 1. (Minimalist Luxury Equilibrium)** The “minimalist luxury” equilibrium emerges where the high-types refrain from buying authentic products to signal their types such that the low-types purchase more authentic and counterfeit goods than the high-types, or

\[
k^*_A(\alpha_H, B_H) + k^*_C(\alpha_H, B_H) < k^*_A(\alpha_L, B_L) + k^*_C(\alpha_L, B_L).
\]

Thus, the low-types, instead of the high-types, may appear to engage in excessive consumption of luxury goods. Specifically, in equilibrium, we have
where the firm chooses price optimally at

$$p^*_A = \begin{cases} 
\frac{B_H}{n - \left(\frac{\alpha_L}{1 - \alpha_L}\right) q_C} & \text{if } B_H < (q_A - q_C) \left(n - \left(\frac{\alpha_L}{1 - \alpha_L}\right) \frac{1}{q_C}\right) \\
q_A - q_C & \text{if } B_H \geq (q_A - q_C) \left(n - \left(\frac{\alpha_L}{1 - \alpha_L}\right) \frac{1}{q_C}\right)
\end{cases}$$

Proposition 1 provides a sharp contrast to the conventional wisdom in that less is actually more for the high-types to signal their types. In this equilibrium, the high-types stand out not because they consume more authentic goods, but because they consume less than what is available and also fewer in total number than the low-types’ purchases. Thus by being minimalists, the high-types separate themselves in a way that low-types are not willing to mimic. The separation is achieved here by the high-types sacrificing functional benefits for the sake of symbolic benefits, a trade-off that is too costly for the low-types to make. To see this more clearly, in this equilibrium the high-types can clearly increase their total functional benefits by adding more counterfeit goods that are available in the market at zero price. However, if the high-types were to do so, the low-types would find it easier to mimic the high-types. Here by buying fewer, the high-types stand out by leveraging the fact that they are more willing than the low-types to sacrifice their functional benefits in pursuit of symbolic benefits. Said differently, the mechanism of high-types’ signaling is not based on excessive expenditure that has been hitherto studied extensively, but on sacrificing functional benefits.

With the benchmarks introduced in Lemma 2 and Lemma 3, we can now isolate the impact of the availability of counterfeits and the observability of consumer types on the sales of the authentic luxury goods. Let $\Delta k^O_A$ be the difference in authentic purchases by the high-types when consumer types are observable vs. not. Similarly, let $\Delta k^C_A$ be the difference in authentic purchases by the high-types when counterfeits are available vs. not. We have the following proposition.

**Proposition 2.** (Consumption Distortion of Minimalist Luxury) When consumer types are un-
observable, high-types sacrifice their functional benefits by consuming $\Delta k^O_A = \left(1 - \frac{\alpha_L}{1 - \alpha_L}\right) \frac{1}{q_C}$ fewer authentic luxury goods to signal their status, compared to the case when consumer types are observable. The availability of counterfeits reduce the sales of luxury goods to high-types by $\Delta k^C_A > 0$, where

$$\Delta k^C_A = \begin{cases} 
\left(1 - \frac{\alpha_L}{1 - \alpha_L}\right) \frac{1}{q_C} & B_H \leq \frac{\rho(\alpha_L, B_L)}{\rho(\alpha_H, B_H)} B_L + nq_A \text{ or } B_H > \frac{\alpha_L}{1 - \alpha_L} + nq_A \\
\frac{B_H - nq_A}{q_A} & B_H \leq \frac{\rho(\alpha_L, B_L)}{\rho(\alpha_H, B_H)} B_L + nq_A < B_H \leq \frac{\alpha_L}{1 - \alpha_L} + nq_A
\end{cases}$$

Proposition 2 captures the distortions that the unobservability of consumer types and the availability of counterfeits create in the luxury market. When consumer types are not observable versus when they are, the high-types always purchase fewer luxury goods. This is another way in which the high-types become “minimalists.” To signal their types, the high-types are not only less ostentatious than the low-types in the total number of luxury goods, authentic or fake, they purchase, but also more self-restrained in consumption than if their types were observable.

In our minimalist luxury equilibrium, as stated in Proposition 2, a luxury brand’s sales to the high-types will always fall due to the availability of counterfeits. This is in addition to the sales decrease coming from the low-types switching to counterfeits. This additional source of harm has two significant managerial implications. First, the conventional wisdom in combating counterfeits is to focus on counterfeit users either through law enforcement or through penalizing them. Our analysis shows that while this managerial focus can be intuitive, it may not be the most effective way to boost the sales of authentic goods. The reason is that even when counterfeit sales cut into those of authentic goods, the most harm to a luxury brand may come from the purchasers of authentic goods who purposefully reduce their consumption to signal their types, and they never purchase any counterfeits. In other words, it could very well be the high-types who require more managerial focus. Second, by identifying this new source of harm, additional managerial prescriptions are available. Our analysis suggests that promoting functionality of the luxury goods can also be a good way to reduce the distortion. To see this, in Proposition 2, we have $\frac{\partial \Delta k^O_A}{\partial (1 - \alpha_L)} < 0$, and $\frac{\partial \Delta k^C_A}{\partial (1 - \alpha_L)} < 0$. All else equal, this means that a higher weight on functionality by the low-types (smaller $\alpha_L$) will reduce the high-type distortion as a high-type does not need to sacrifice so much consumption of authentic goods in order to make it functionally costly for the low-types to mimic. Similarly, the quality improvement in counterfeits (a higher $q_C$), as well as in authentic goods (a higher $q_A$) when the low-types do not care much about symbolic value, can all reduce the distortions. This is because quality
improvements increase the functional benefits of the underlying products so that less sacrifice is needed for signaling purposes.

We note that a luxury brand can never be better off in terms of higher profitability when counterfeits become available. This is because the availability of counterfeits limits how high a price a luxury brand can charge for their product before the high-types switch to acquire counterfeits. This reduces its ability to claim consumer surplus. However, surprisingly, the brand is not always worse off. This is because, although the availability of counterfeits encourages the low-types to acquire more functional benefits from counterfeits, the high-types will want to sacrifice such benefits to stand out by purchasing fewer authentic goods. The luxury brand can accommodate both types by charging high prices to maintain its profitability as long as the budget for high-types is not too large, as Figure 3 illustrates.

![Figure 3: Distortion of Profitability](image)
We can similarly investigate how observability of consumer types and the availability of counterfeit goods may impact a luxury brand’s pricing decisions. Let \( \Delta p^O_A \) be the difference in the optimal price of a luxury brand when consumer types are observable vs. not. Similarly, let \( \Delta p^C_A \) be the difference in the optimal price of a luxury brand when counterfeits are available vs. not. Our analysis is summarized in the following proposition.

**Proposition 3. (Price Distortion of Minimalist Luxury)** When counterfeits are available, if the consumer types are unobservable, a luxury brand charges a price higher than if the types are observable by \( \Delta p^O_A \geq 0 \), where

\[
\Delta p^O_A = \begin{cases} 
\min \left\{ \frac{B_H}{n - \left( \frac{\alpha_L}{\alpha_H} \right) \frac{1}{q_C}}, q_A - q_C \right\} - \frac{B_H}{n} & B_H < n(q_A - q_C) \\
0 & B_H \geq n(q_A - q_C)
\end{cases}
\]

The availability of counterfeits distorts the price of a luxury brand by \( \Delta p^C_A \), where

\[
\Delta p^C_A = \begin{cases} 
\min \left\{ \frac{B_H}{n - \left( \frac{\alpha_L}{\alpha_H} \right) \frac{1}{q_C}}, q_A - q_C \right\} - \frac{B_H}{n} & B_H < nq_A \\
-q_C & nq_A \leq B_H \leq \frac{\rho(\alpha_L, B_L)}{\rho(\alpha_H, B_H)} B_L + nq_A \\
-q_C - \max \left\{ \left( \frac{\alpha_L}{B_H - \frac{\alpha_L}{1-\alpha_L}} \right) q_A, \left( \frac{\alpha_L}{1-\alpha_L} \right) \frac{1}{n} \right\} & \frac{\rho(\alpha_L, B_L)}{\rho(\alpha_H, B_H)} B_L + nq_A < B_H
\end{cases}
\]

The price is distorted upward (\( \Delta p^C_A > 0 \)) iff the budget of high-types is sufficiently small (\( B_H < n(q_A - q_C) \)), and otherwise the price is distorted downward (\( \Delta p^C_A < 0 \)).

Proposition 3 is illustrated in Figure 4. Interestingly, the unobservability of consumer types will make a luxury brand worse off in terms of profitability when counterfeit goods are available, in contrast to the previous discussion where counterfeits are not available. Here the luxury brand is worse off, not because the low-types spend less on authentic goods, or the high-types acquire counterfeits. It is because the high-types switch from excessive to minimalist consumption all for the purpose of signaling. Thus in the market with counterfeits, it is in a brand’s interest to make consumer types observable, while the opposite is true when counterfeits are not available.

The availability of counterfeits can lower a luxury brand’s prices as expected when the budget for the high-types is sufficiently large, as raising prices can encourage the high-types to acquire counterfeits. However, a luxury brand’s prices can also be higher relative to the benchmark case.
of no counterfeits if the budget is sufficiently low. This is because the availability of high-quality counterfeits motivates the low-types to pursue more functional benefits and hence the high-types pursue a “minimalist luxury” equilibrium by purchasing less luxury goods to stand out. Thus, for the high-types, the willingness-to-pay for each one of the luxury products has increased. By charging a higher price, a brand manager can seize upon this opportunity and extract more rent from the high-types.

![Counterfeits are Available](image1)

![Wealth is unobservable](image2)

Figure 4: Distortion of Prices
4 Extensions

In the benchmark analysis, in order to facilitate a quick access to our main conclusions, we have simplified our model by assuming that there are only high- and low-types and that the price of counterfeits is zero. In this section, we will relax these two assumptions to show, first of all, that our main conclusions are robust and, secondly, that additional insights are generated.

4.1 Four types of consumers

In addition to the two types of consumers, high- and low-types, we have modeled before, we now add two other types of consumers: those who are wealthy but do not care much about their symbolic benefits and those who are not wealthy but care a lot about their symbolic benefits (Han et al., 2010). To start, the share of four types of consumers in the population are respectively denoted by \( \rho(\alpha, B) > 0, \alpha \in \{\alpha_H, \alpha_L\}, B \in \{B_H, B_L\} \). The results of our analysis based on this full model are summarized in the proposition below (see appendix for details).

Proposition 4. (Partial Pooling Equilibria with Four Consumer Types) In a market with four types \((\alpha, B) \in \{\alpha_H, \alpha_L\} \times \{B_H, B_L\}\), minimalist luxury emerge if there are proportionately more consumers who care highly about their symbolic benefits among the wealthy relative to those who lack wealth, i.e., \( \frac{\rho(\alpha_H, B_H)}{\rho(\alpha_L, B_H)} > \frac{\rho(\alpha_H, B_L)}{\rho(\alpha_L, B_L)} \). In this equilibrium, all those who care little about their symbolic benefits purchase \( n \) items in total consisting of the following authentic and counterfeit goods:

\[
\begin{align*}
k^*_A(\alpha_L, B_H) &= n - \Delta K, \quad k^*_C(\alpha_L, B_H) = \Delta K, \\
k^*_A(\alpha_L, B_L) &= \frac{B_L}{B_H} (n - \Delta K), \quad k^*_C(\alpha_L, B_L) = n - \frac{B_L}{B_H} (n - \Delta K).
\end{align*}
\]

All those who care highly about their symbolic benefits would refrain from buying what are available to them. In effect, these consumers consume fewer number of goods, authentic and counterfeit goods combined, than the rest in order to maximize their symbolic benefits. We have

\[
\begin{align*}
k^*_A(\alpha_H, B_H) &= n - \Delta K, \quad k^*_C(\alpha_H, B_H) = 0, \\
k^*_A(\alpha_H, B_L) &= \frac{B_L}{B_H} (n - \Delta K), \quad k^*_C(\alpha_H, B_L) = \left(1 - \frac{B_L}{B_H}\right) (n - \Delta K),
\end{align*}
\]

where \( \Delta K = \frac{1}{q_C} \left( \frac{\alpha_L}{1 - \alpha_L} \right) \left( \frac{\rho(\alpha_H, B_H)}{\rho(\alpha_H, B_H) + \rho(\alpha_H, B_L)} - \frac{\rho(\alpha_L, B_H)}{\rho(\alpha_L, B_L) + \rho(\alpha_L, B_H)} \right) \). The luxury brand sets its price
\[ p^*_A = \min \left\{ \frac{B_H}{n-\Delta K}, q_A - q_C \right\}. \]

Proposition 4 thus confirms our main model is robust to the extension of consumer types with qualifying conditions as expected.

4.2 Counterfeit Goods with Positive Prices

In our benchmark model, we assume that the counterfeits are supplied at zero price. This assumption makes the counterfeit goods a more attractive alternative than what it is in reality. Because of this assumption, the high-types need to sacrifice more functional benefits to signal their types, as the counterfeits are freely available to the low-types. Moreover, a zero price for counterfeits also eliminates the possibility that the low-types may not be able to afford all \( n \) products, so that the high-types can overspend to stand out. Corresponding to the minimalist luxury equilibrium in our benchmark model, we define as an “excessive luxury” equilibrium, one where the high-types purchase more, counterfeit and authentic goods combined, than the low-types do, or

\[
k^*_A(\alpha_H, B_H) + k^*_C(\alpha_H, B_H) > k^*_A(\alpha_L, B_L) + k^*_C(\alpha_L, B_L).
\]

This is the equilibrium where the high-types engage in excessive consumption so that the low-types cannot afford to mimic. Similar to the benchmark model, we also rule out the trivial case where low-types can always afford all available authentic products by assuming that \( n (q_A - q_C + p_C) > B_L \).

In addition, we focus on a viable market in which \( p_A > p_C \). Assuming that the price of a counterfeit good is not too high, i.e., \( \left( \sqrt{4 \left( \frac{q_A}{q_C} - 1 \right) + p_C + \sqrt{p_C}} \right)^2 \leq 4 (q_A - q_C) \left( \frac{q_A}{q_C} \right)^2 \frac{p(\alpha_H, B_H)}{p(\alpha_L, B_L)} \) ensures this.

Proposition 5. (Minimalist and Excessive Luxury Equilibria) When \( p_C > 0 \), we have

(i) the “minimalist luxury” equilibrium when \( p_C \leq \frac{B_L}{n} \), specifically with

\[
k^*_A(\alpha_H, B_H) = n - \left( \frac{\alpha_L}{1 - \alpha_L} \right) \frac{1}{q_C - p_C \left( \frac{q_A - q_C}{p_A - p_C} \right)}, \quad k^*_C(\alpha_H, B_H) = 0,
\]

\[
k^*_A(\alpha_L, B_L) = \frac{B_L - np_C}{p_A - p_C} \quad \text{and} \quad k^*_C(\alpha_L, B_L) = \frac{np_M^A - B_L}{p_A - p_C},
\]
prices, minimalist luxury equilibrium is more likely to emerge. As counterfeits are getting increasingly more accessible at cheaper penalties, and had to push their prices higher, the market may support an excessive rather than a large

Thus, the high-types can signal their status without sacrificing any functional benefits. This finding generates an insight that if the counterfeiters were exposed to higher production costs, or higher penalties, and had to push their prices higher, the market may support an excessive rather than a minimalist luxury equilibrium. As counterfeits are getting increasingly more accessible at cheaper prices, minimalist luxury equilibrium is more likely to emerge.

\[
P_A^M = \begin{cases} 
\bar{p} & \text{if } B_H < (p_C + q_A - q_C) \left( n - \frac{\alpha_L}{1-\alpha_L} \cdot \frac{1}{q_C - p_C} \right), \\
p_C + q_A - q_C & \text{if } B_H \geq (p_C + q_A - q_C) \left( n - \frac{\alpha_L}{1-\alpha_L} \cdot \frac{1}{q_C - p_C} \right), 
\end{cases}
\]

with \( \bar{p} = \frac{B_H}{k_A(\alpha_H, B_H)} = \left( nq_A - \frac{\alpha_L}{1-\alpha_L} \right)p_C + B_Hq_C + \sqrt{\left( \left( nq_A - \frac{\alpha_L}{1-\alpha_L} \right)p_C + B_Hq_C \right)^2 - 4B_Hp_Cq_A(nq_A - \frac{\alpha_L}{1-\alpha_L})} \).

(ii) the “excessive luxury” equilibrium when \( p_C > \frac{B_L}{n} \), specifically with

\[
k_A^*(\alpha_H, B_H) = n, k^*_C(\alpha_H, B_H) = 0,
\]

\[
k_A^*(\alpha_L, B_L) = \begin{cases} 
0 & \text{if } \frac{\rho(\alpha_L, B_L)}{\rho(\alpha_H, B_H)} B_L + n \left( \frac{q_A}{q_C} \right) p_C < B_H, \\
\frac{B_L}{p_A^E} & \text{if } B_H \leq \frac{\rho(\alpha_L, B_L)}{\rho(\alpha_H, B_H)} B_L + n \left( \frac{q_A}{q_C} \right) p_C, 
\end{cases}
\]

\[
k_C^*(\alpha_L, B_L) = \begin{cases} 
\frac{B_L}{p_C} & \text{if } \frac{\rho(\alpha_L, B_L)}{\rho(\alpha_H, B_H)} B_L + n \left( \frac{q_A}{q_C} \right) p_C < B_H, \\
0 & \text{if } B_H \leq \frac{\rho(\alpha_L, B_L)}{\rho(\alpha_H, B_H)} B_L + n \left( \frac{q_A}{q_C} \right) p_C, 
\end{cases}
\]

\[
P_A^E = \begin{cases} 
\min \left\{ \frac{B_H}{n}, p_C + q_A - q_C \right\} & \text{if } \frac{\rho(\alpha_L, B_L)}{\rho(\alpha_H, B_H)} B_L + n \left( \frac{q_A}{q_C} \right) p_C < B_H, \\
\min \left\{ \frac{B_H}{n}, \left( \frac{q_A}{q_C} \right) p_C \right\} & \text{if } B_H \leq \frac{\rho(\alpha_L, B_L)}{\rho(\alpha_H, B_H)} B_L + n \left( \frac{q_A}{q_C} \right) p_C, 
\end{cases}
\]

where \( p_A^M \) and \( p_A^E \) are the optimal prices of the respective equilibrium.

See the Appendix for the proof of Proposition 5. As we can see from this proposition, when \( p_C = 0 \), we recover all the expressions given in the benchmark model. In addition, \( p_C \) can be significantly above zero, depending on \( B_L/n \), and still support the minimalist luxury equilibrium. Only when \( p_C \) is sufficiently large does the excessive luxury equilibrium emerge. A sufficiently large \( p_C \) will make acquiring counterfeits very costly for the low-types, and the high-types can stand out by outspending the low-types, leveraging the difference in budget between the two types. Thus, the high-types can signal their status without sacrificing any functional benefits. This finding generates an insight that if the counterfeiters were exposed to higher production costs, or higher penalties, and had to push their prices higher, the market may support an excessive rather than a minimalist luxury equilibrium. As counterfeits are getting increasingly more accessible at cheaper prices, minimalist luxury equilibrium is more likely to emerge.
5 Conclusion

In this paper, we developed a theoretical model à la Spence (1973) investigating how the wealthy may signal their wealth through their purchases of luxury goods in the presence of high-quality, low-price counterfeits. Although quite parsimonious, our model allows us to look into the rationale of how refraining from luxury consumption can signal wealth. Our analysis shows that the classic Veblen type of equilibrium for the luxury goods markets where the wealthy engage in excessive consumption to signal their wealth can break down in the presence of high-quality, low-price counterfeits. Overspending ceases to be an effective way to break out of a pack when two conditions are present. First, when high-quality counterfeits are present, authentic and fake goods are hard to distinguish. Second, those high-quality counterfeits are available at low price so that the wealthy cannot out-purchase the rest in the total number of luxury or counterfeit goods. Under these two conditions, the most effective way for the wealthy to signal their wealth is, on the contrary, to purposefully refrain from luxury consumption and to stand out by sacrificing more functional benefits than what the rest are willing to do. This restraint is effective since the wealthy can sacrifice their functional benefits for the gains in their symbolic benefits, a trade-off that the rest, with less wealth and less appreciation for symbolic benefits, is simply not willing to make.

From this perspective we have developed, it is rather understandable that the trend of minimalist luxury can emerge and the wealthy engaging in “material purge” can become “a signifier of the global elite” (Chayka, 2016). Also from this perspective, it is no longer paradoxical that “The richer you are, the less you have” (Chayka, 2016).

This minimalist luxury equilibrium is not solely a theoretical curiosity, but it helps to generate three managerially relevant insights. First, we see that the problem with counterfeits is not merely a problem with counterfeit users. Our analysis shows that the wealthy, the ones who may never consider buying counterfeits, could also reduce their purchases of authentic luxury goods in an effort to distinguish themselves. Second, this reduction or behavioral distortion on the part of the wealthy is critically dependent on the existence of incomplete information in the market, as well as on the presence of counterfeits. Such distortion can be minimized if a luxury brand could enhance consumer preferences for functional utilities through either product designs or promotions. Finally, the right way to accommodate the trend of minimalist luxury is not to stem the sales decline through lower prices, but to solidify the exclusivity of a luxury brand by charging higher prices and making lower sales.
Although the minimalist luxury equilibrium is exciting to uncover and the mechanism through which it occurs is, we believe, robust, it is important to point out that our conclusions are limited by our model simplicity. For instance, we assume that prices for authentic goods are all identical and so are the prices for corresponding counterfeit goods. This is a limiting but innocuous assumption. Our analysis has shown that whether the minimalist luxury equilibrium occurs principally depends on whether the wealthy can outspend the rest to signal their wealth. Prices matter because given the budgets of high-types vs. low-types, they determine whether the high-types can actually buy a significantly larger number of goods than the low-types. In other words, it is the sum of all the prices in a portfolio that high-types vs. low-types pay that determines the emergence of minimalist luxury equilibrium. Therefore, future research can verify that our mechanism will survive even if there is a distribution of prices respectively for authentic and counterfeit goods. A fruitful future investigation can also be conducted by considering alternative formulations of symbolic and functional benefits from luxury brands. On the consumer side, much work also remains to empirically test how much consumers care about different kinds of benefits and how they may choose to pursue a minimalist vs. excessive strategy to signal their wealth.

References


Appendix

Proof of Lemma 1:

Recall that there are only two types of consumers in the benchmark model, the high-types \((\alpha_H, B_H)\) and low-types \((\alpha_L, B_L)\). Since consumer types are observable, the high-types obtain a symbolic benefit of 1 and the low-types zero regardless of their consumption choice. In other words, consumption is not needed for signaling. Therefore, both types of consumers purchase luxury goods only for their functional benefits. Since counterfeit goods are not available, consumers can only choose to buy authentic goods or nothing. Given that functional benefits increase linearly in the quantity purchased, both types of consumers would buy as many products as they can afford as long as the price of the luxury good is lower than the marginal benefit it provides, i.e., if \(p_A \leq q_A\). Otherwise, if \(p_A > q_A\), no one buys any product. Formally, we have
\[ k^*_A (\alpha_H, B_H) = \begin{cases} \min \left\{ \frac{n}{p_A}, B_H \right\} & p_A \leq q_A \\ 0 & p_A > q_A \end{cases} \]

and

\[ k^*_A (\alpha_L, B_L) = \begin{cases} \min \left\{ \frac{n}{p_A}, B_L \right\} & p_A \leq q_A \\ 0 & p_A > q_A \end{cases} \]  \( (1) \)

The price \( p^*_A \) that maximizes a luxury brand’s profit is thus given by

\[ p^*_A \in \arg \max_{p_A} \{ \rho (\alpha_H, B_H) \times k^*_A (\alpha_H, B_H) \times p_A + \rho (\alpha_L, B_L) \times k^*_A (\alpha_L, B_L) \times p_A \}. \]

Since \( B_L < B_H \) by assumption, it is straightforward to see that the luxury brand obtains equal profit with any price \( p^*_A \in \left[ \min \left\{ \frac{B_H}{n}, q_A \right\}, q_A \right] \). Nevertheless, consumers can afford fewer luxury products and are thus worse off when the price of a good is higher. Therefore, the Pareto dominant profit-maximizing equilibrium is when the luxury brand chooses the lowest price that maximizes its profit, i.e., \( p^*_A = \min \left\{ \frac{B_H}{n}, q_A \right\} \). Plugging the optimal price into the demand function (1), we obtain the equilibrium consumption of high-types and low-types, respectively,

\[ k^*_A (\alpha_H, B_H) = n \text{ and } k^*_A (\alpha_L, B_L) = \max \left\{ \left( \frac{B_L}{B_H} \right) n, \frac{B_L}{q_A} \right\}. \]

Therefore, the luxury brand obtains a profit of \( \pi^* \), where

\[ \pi^* = \rho (\alpha_H, B_H) \min \left\{ B_H, nq_A \right\} + \rho (\alpha_L, B_L) B_L \]

**Proof of Lemma 2:**

Unlike in Lemma 1, the symbolic benefit of a luxury good now depends on how many luxury goods a consumer owns when wealth is unobservable. We prove this lemma in two steps: first, we derive the Pareto dominant separating equilibrium in two cases respectively: (i) \( p_A \leq q_A \) and (ii) \( p_A > q_A \). Second, we show that case (i) is profit-maximizing if and only if \( B_H \leq \frac{\rho (\alpha_L, B_L)}{\rho (\alpha_H, B_H)} B_L + nq_A \), or \( B_L \geq \left( \frac{\alpha_L}{1 - \alpha_L} \right) \rho (\alpha_H, B_H) \). Otherwise, case (ii) is profit-maximizing.

**Case (i):** \( p_A \leq q_A \)

We show that the equilibrium we have derived in Lemma 1 is a separating equilibrium and it Pareto dominates any other separating equilibria where \( p_A \leq q_A \). First, note that the high-types and the low-types are naturally separated in the equilibrium derived in Lemma 1. This is
because the high-types buy all \( n \) available products, while the low-types cannot afford to mimic the high-types as \( B_L < np^*_A = \min\{B_H, nq_A\} \). Second, recall that the luxury brand obtains a profit of \( \pi^* = \rho(\alpha_H, B_H) \min\{B_H, nq_A\} + \rho(\alpha_L, B_L) B_L \) in that equilibrium. This is clearly the maximum rent a luxury brand can possibly extract from both types of consumers when \( p_A \leq q_A \). Therefore, any equilibrium where high-types consume fewer than \( n \) products is Pareto dominated by the separating equilibrium we have derived in Lemma 1.

**Case (ii):** \( p_A > q_A \)

First, note that when the low-types are separated from the high-types, they obtain zero symbolic benefits and strictly negative net functional benefit \( (q_A - p_A < 0) \). Hence the low-types do not purchase any goods under a Case-(ii) separating equilibrium, i.e., \( k^*_A(\alpha_L, B_L) = 0 \). By contrast, the high-types are willing to endure the loss due to high prices for additional symbolic benefits, by purchasing \( k^*_A(\alpha_H, B_H) > 0 \) luxury goods as long as the following individual rationality constraint holds:

\[
(1 - \alpha_H)(q_A - p_A) k^*_A(\alpha_H, B_H) + \alpha_H \cdot 1 \geq 0. \tag{2}
\]

To prevent the low-types from mimicking, the high-types’ consumption also satisfies the following \( IC \) condition:

\[
(1 - \alpha_L)(q_A - p_A) k^*_A(\alpha_H, B_H) + \alpha_L \cdot 1 \leq 0. \tag{3}
\]

In other words, we have

\[
\frac{\alpha_L}{1 - \alpha_L} \left( \frac{1}{p_A - q_A} \right) \leq k^*_A(\alpha_H, B_H) \leq \frac{\alpha_H}{1 - \alpha_H} \left( \frac{1}{p_A - q_A} \right).
\]

At any given price \( p_A (> q_A) \), the high-types purchase the minimum amount of luxury goods required for separation in a Pareto-dominant separating equilibrium, that is, \( k^*_A(\alpha_H, B_H) = \frac{\alpha_L}{1 - \alpha_L} \left( \frac{1}{p_A - q_A} \right) \).

Note that there exists such an equilibrium only if the high-types can afford this portfolio \( k^*_A(\alpha_H, B_H) p_A \leq B_H \), and it does exceed the maximum number of products available \( (k^*_A(\alpha_H, B_H) \leq n) \).

Since the low-types do not buy anything in this equilibrium, the profit-maximizing problem for the luxury brand is given by
The profit function $\pi$ strictly declines in $p_A$ and thus profit is maximized at the lowest possible $p_A$ that satisfies both conditions (5) and (6). In other words, the optimal price is given by

$$p^*_A = \begin{cases} q_A \left( \frac{B_H}{B_H - \frac{\alpha_L}{1 - \alpha_L}} \right) & \text{if } B_H < nq_A + \frac{\alpha_L}{1 - \alpha_L} \\ \left( \frac{\alpha_L}{1 - \alpha_L} \right) \frac{1}{n} + q_A & \text{if } nq_A < B_H \leq \frac{\alpha_L}{1 - \alpha_L} + nq_A \end{cases}$$

Plugging the optimal price into the demand and profit function, we obtain the equilibrium consumption of each high-type and the profitability of a brand, given respectively by

$$k^*_A (\alpha_H, B_H) = \min \left\{ \frac{B_H - \frac{\alpha_L}{1 - \alpha_L}}{q_A}, n \right\}$$

and

$$\pi^* = \rho (\alpha_H, B_H) \min \left\{ B_H, \frac{\alpha_L}{1 - \alpha_L} + nq_A \right\}.$$
Proof of Lemma 3:

Similar to the case in Lemma 1, consumption choice does not affect one’s symbolic benefits when consumer types are observable. Therefore, both types of consumers purchase luxury goods only for their functional benefits. We can thus rewrite the consumers’ maximization problem as follows:

\[
\max_{k_A, k_C} \left\{ (1 - \alpha) \left( q_A k_A + q_C k_C + (B - p_A k_A) \right) + \alpha \cdot I(B = B_H) \right\},
\]

\[
s.t. p_A k_A \leq B, \\
\qquad k_A + k_C \leq n,
\]

Since there are counterfeit goods available, and the price of counterfeit goods is normalized to zero, buying less than \( n \) products is a dominated strategy because consuming counterfeits yields positive benefits (\( q_C > 0 \)) at zero price. As a result, consumers choose to buy only counterfeit goods if the net benefit of authentic goods is less than that of counterfeits, i.e., if \( q_A - p_A < q_C \). Otherwise, if \( q_A - p_A \geq q_C \), they purchase either only authentic goods or a mixture of both authentic and counterfeit goods, depending on their budget. Consumers choose the former if they can afford all available authentic goods, and the latter if they can not. Formally, the consumption of high-types and low-types are given respectively by

\[
k^*_A (\alpha_H, B_H) = \begin{cases} 
\min \left\{ n, \frac{B_H}{p_A} \right\} & p_A \leq q_A - q_C \\
0 & p_A > q_A - q_C 
\end{cases}, \quad k^*_C (\alpha_H, B_H) = \begin{cases} 
\min \left\{ n, \frac{B_H}{p_A} \right\} & p_A \leq q_A - q_C \\
n & p_A > q_A - q_C 
\end{cases}
\]

and

\[
k^*_A (\alpha_L, B_L) = \begin{cases} 
\min \left\{ n, \frac{B_L}{p_A} \right\} & p_A \leq q_A - q_C \\
0 & p_A > q_A - q_C 
\end{cases}, \quad k^*_C (\alpha_L, B_L) = \begin{cases} 
\min \left\{ n, \frac{B_L}{p_A} \right\} & p_A \leq q_A - q_C \\
n & p_A > q_A - q_C 
\end{cases}
\]
The prices $p^*_A$ that maximize a luxury brand’s profit is thus given by

$$p^*_A \in \arg \max_{p_A} \{ k^*_A(\alpha_H, B_H) \rho(\alpha_H, B_H) + k^*_A(\alpha_L, B_L) \rho(\alpha_L, B_L) \}.$$  

We know that $B_L < B_H$ by assumption, hence it is straightforward to see that the luxury brand obtains equal profit with any price $p^*_A \in \left[ \min \left\{ \frac{B_H}{n}, q_A - q_C \right\}, q_A - q_C \right]$. Nevertheless, consumers can afford fewer luxury products and are thus worse off when the price of a good is higher. Therefore, the Pareto dominant profit-maximizing equilibrium is when the luxury brand chooses the lowest price that maximizes its profit, i.e., $p^*_A = \min \left\{ \frac{B_H}{n}, q_A - q_C \right\}$.

Substituting out the optimal price $p^*_A$ in (7) and (8), we obtain the following equilibrium choice of high-types and low-types, respectively,

$$k^*_A(\alpha_H, B_H) = \min \left\{ n, \frac{B_H}{q_A - q_C} \right\}, k^*_C(\alpha_H, B_H) = 0 \quad (9)$$

and

$$k^*_A(\alpha_L, B_L) = \max \left\{ \left( \frac{B_L}{B_H} \right) n, \frac{B_L}{q_A - q_C} \right\}, k^*_C(\alpha_L, B_L) = \min \left\{ \left( 1 - \frac{B_L}{B_H} \right) n, n - \frac{B_L}{q_A - q_C} \right\} \quad (10)$$

Proof of Proposition 1:

When high-quality, low-price counterfeits are available, the public observes the size of one’s total consumption but cannot distinguish a fake from a real product. Therefore a separating equilibrium requires that the high-types buy a total of $k^*(\alpha_H, B_H) \equiv k^*_A(\alpha_H, B_H) + k^*_C(\alpha_H, B_H)$ goods whereas the low-types buy a total of $k^*(\alpha_L, B_L) \equiv k^*_A(\alpha_L, B_L) + k^*_C(\alpha_L, B_L)$ goods, such that:

$$k^*(\alpha_L, B_L) \neq k^*(\alpha_H, B_H).$$

We label the set of equilibria as “minimalist luxury” if $k^*(\alpha_H, B_H) < k^*(\alpha_L, B_L)$ and “excessive luxury” if $k^*(\alpha_H, B_H) > k^*(\alpha_L, B_L)$.

We prove this Proposition in four steps: first, we show that there exists no equilibrium of “excessive luxury.” Second, we show that the market for authentic goods is not viable when $p_A \in (q_A - q_C, +\infty)$. Third, we show that for any $p_A \in (0, q_A - q_C]$, there exists a unique separating
equilibrium of “minimalist luxury” that satisfies Intuitive Criterion. Fourth, we derive the profit-maximizing “minimalist luxury” equilibrium that is Pareto dominant.

We first prove that there exists no separating PBE of “excessive luxury.” We can prove this by contradiction. Suppose the public believes that a consumer is a high-type if the consumer purchase no less than \( \hat{k} \) goods in total or otherwise believes that she is a low-type. Formally, suppose \( \mu \) is given by

\[
\mu(k) = \begin{cases} 
1 & \text{if } k \geq \hat{k} \\
0 & \text{otherwise}
\end{cases}
\]

In this separating equilibrium, the high-types obtain a symbolic benefit of 1 whereas the low-types obtain a symbolic benefit of 0, which requires \( k_A^* (\alpha_H, B_H) + k_C^* (\alpha_H, B_H) \geq \hat{k} > k_A^* (\alpha_L, B_L) + k_C^* (\alpha_L, B_L) \). Since the price of counterfeits is zero, the low-types would be strictly better off by deviating to purchasing \( k'_C \) counterfeits and \( k'_A \) authentic goods, such that \( k'_C + k'_A = \hat{k} \), where \( k'_C > k_C^* (\alpha_L, B_L) \) and \( k'_A = k_A^* (\alpha_L, B_L) \). This deviation is profitable in that it not only yields higher symbolic benefits but higher functional benefits as well, contradicting the notion of an equilibrium. When the price of counterfeits is zero, purchasing more becomes an ineffective strategy for the high-types to separate themselves from the low-types.

Then if \( p_A > q_A - q_C \), the demand for authentic luxury goods is zero. Recall that the consumption of numeraire goods (\( X \)) is represented by one’s savings from purchasing luxury goods. We can thus rewrite the consumers’ maximization problem by substituting out \( X \) in the utility function:

\[
\max_{k_A, k_C} \left\{ (1 - \alpha) \left( q_A k_A + q_C k_C + (B - p_A k_A) \right) + \alpha \mu(k = k_A + k_C) (B = B_H) \right\},
\]

s.t. \( p_A k_A \leq B \),

\( k_A + k_C \leq n, \) \hspace{1cm} (11)

A counterfeit good is not only cheaper (zero-priced), but it also provides strictly higher functional benefits net of price than an authentic good does (\( q_C > q_A - p_A \)). To maximize their utility, both the high-types and the low-types strictly prefer counterfeit goods. In that case, the market for authentic goods is not viable.

Next we show that for any price \( p_A \in (0, q_A - q_C] \), there exists a separating PBE of “minimalist
luxury” where the public believes that a consumer is of high-type as long as she purchases fewer than \( \tilde{k} \) luxury items, translating into the following belief:

\[
\mu(k) = \begin{cases} 
1 & \text{if } k \leq \tilde{k} \\
0 & \text{otherwise}
\end{cases}
\]

In this case, consumers are faced with the trade-off of increasing symbolic benefits and losing functional benefits. In a separating equilibrium, the high-types have stronger incentives to maximize their symbolic benefits, whereas the low-types who care relatively more about functional benefits prefer to maximize functional benefits at the expense of their symbolic benefits. Therefore high-types purchase a total of \( k^*_A(\alpha_H, B_H) + k^*_C(\alpha_H, B_H) = \tilde{k} \) items and obtain a symbolic benefits of 1. Specifically,

\[
k^*_A(\alpha_H, B_H) = \min \left\{ \tilde{k}, \frac{B_H}{p_A} \right\}, \quad k^*_C(\alpha_H, B_H) = \max \left\{ 0, \tilde{k} - \frac{B_H}{p_A} \right\}.
\]

While the low-types may not afford to buy all authentic products, they can buy as many counterfeits as they want at a zero price. Therefore the quantity constraint (12) is binding for the \( L \)-types. In other words, the low-types purchase a total of \( n \) products. Specifically,

\[
k^*_A(\alpha_L, B_L) = \min \left\{ n, \frac{B_L}{p_A} \right\}, \quad k^*_C(\alpha_L, B_L) = \max \left\{ 0, n - \frac{B_L}{p_A} \right\}.
\]

For these consumption quantities to support a separating equilibrium, the low-types should have no incentive to mimic the high-types while high-types should have no incentive to purchase more than \( \tilde{k} \) items. In other words, net of price, the low-types’ gain in symbolic benefits should not exceed their loss in functional benefits from mimicking, whereas the high-types’ loss in symbolic benefits should be larger than their gain in functional benefits from deviating. We examine the IC constraints for both the low-types and the high-types in five distinct cases, conditional on the price of a luxury brand.

**Case 1:** \( p_A \in (0, \frac{B_L}{n}] \)

In this case, both the high-types and the low-types can afford all \( n \) authentic products. Thus the IC constraints are characterized by:
\( \alpha_L (1 - 0) \leq (1 - \alpha_L) \left( n - \tilde{k} \right) (q_A - p_A), \)
\( \alpha_H (1 - 0) \geq (1 - \alpha_H) \left( n - \tilde{k} \right) (q_A - p_A). \)

Or equivalently,
\[
\begin{align*}
    n - \left( \frac{\alpha_H}{1 - \alpha_H} \right) \frac{1}{q_A - p_A} \leq \tilde{k} \leq n - \left( \frac{\alpha_L}{1 - \alpha_L} \right) \frac{1}{q_A - p_A}.
\end{align*}
\]  

(15)

Case 2: \( p_A \in \left( \frac{B_L}{n}, \frac{B_L}{\tilde{k}} \right) \)

In this case, the high-types can afford all \( n \) authentic products, whereas the low-types cannot. The low-types can, however, afford \( \tilde{k} \) authentic goods. Thus the IC constraints are characterized by:
\[
\begin{align*}
    \alpha_L (1 - 0) &\leq (1 - \alpha_L) \left[ \left( n - \frac{B_L}{p_A} \right) q_C + \left( \frac{B_L}{p_A} - \tilde{k} \right) (q_A - p_A) \right], \\
    \alpha_H (1 - 0) &\geq (1 - \alpha_H) \left( n - \tilde{k} \right) (q_A - p_A).
\end{align*}
\]

Or equivalently,
\[
\begin{align*}
    n - \left( \frac{\alpha_H}{1 - \alpha_H} \right) \frac{1}{q_A - p_A} - \frac{B_L}{p_A} \leq \tilde{k} \leq n - \left( \frac{\alpha_L}{1 - \alpha_L} \right) \frac{1}{q_A - p_A} - \frac{B_L}{p_A}. \\
\end{align*}
\]  

(16)

Case 3: \( p_A \in \left[ \frac{B_L}{\tilde{k}}, \frac{B_H}{n} \right] \)

In this case, the high-types can still afford all \( n \) authentic goods, but the low-types cannot afford \( \tilde{k} \) authentic goods. Thus the IC constraints are characterized by:
\[
\begin{align*}
    \alpha_L (1 - 0) &\leq (1 - \alpha_L) \left( n - \tilde{k} \right) q_C, \\
    \alpha_H (1 - 0) &\geq (1 - \alpha_H) \left( n - \tilde{k} \right) (q_A - p_A).
\end{align*}
\]

Or equivalently,
\[ n - \left( \frac{\alpha_H}{1 - \alpha_H} \right) \frac{1}{q_A - p_A} \leq \tilde{k} \leq n - \left( \frac{\alpha_L}{1 - \alpha_L} \right) \frac{1}{q_C} \tag{17} \]

Case 4: \( p_A \in \left[ \frac{B_H}{n}, \frac{B_H}{\tilde{n}} \right] \)

In this case, the high-types cannot afford \( n \) authentic goods, but they can afford \( \tilde{k} \) authentic goods. The low-types cannot afford \( \tilde{k} \) authentic goods. Thus the \( IC \) constraints are characterized by:

\[
\alpha_L (1 - 0) \leq (1 - \alpha_L) \left( n - \tilde{k} \right) q_C,
\]

\[
\alpha_H (1 - 0) \geq (1 - \alpha_H) \left[ \left( n - \frac{B_H}{p_A} \right) q_C + \left( \frac{B_H}{p_A} - \tilde{k} \right) (q_A - p_A) \right].
\]

Or equivalently,

\[
\frac{B_H}{p_A} + \left( n - \frac{B_H}{p_A} \right) \left( \frac{q_C}{q_A - p_A} \right) - \left( \frac{\alpha_H}{1 - \alpha_H} \right) \frac{1}{q_A - p_A} \leq \tilde{k} \leq n - \left( \frac{\alpha_L}{1 - \alpha_L} \right) \frac{1}{q_C} \tag{18} \]

Case 5: \( p_A \in \left( \frac{B_H}{\tilde{n}}, q_A - q_C \right] \)

In this case, neither the high-types nor the low-types can afford \( \tilde{k} \) authentic goods. Thus the \( IC \) constraints are characterized by:

\[
\alpha_L (1 - 0) \leq (1 - \alpha_L) \left( n - \tilde{k} \right) q_C,
\]

\[
\alpha_H (1 - 0) \geq (1 - \alpha_H) \left( n - \tilde{k} \right) q_C.
\]

Or equivalently,

\[
n - \left( \frac{\alpha_H}{1 - \alpha_H} \right) \frac{1}{q_C} \leq \tilde{k} \leq n - \left( \frac{\alpha_L}{1 - \alpha_L} \right) \frac{1}{q_C} \tag{19} \]

Note that the solution set for \( \tilde{k} \) is non-empty in all of the above 5 cases as long as

\[
\left( \frac{\alpha_H}{1 - \alpha_H} \right) \frac{1}{q_A - p_A} \geq \left( \frac{\alpha_L}{1 - \alpha_L} \right) \frac{1}{q_C}. \tag{20} \]
Given the assumption on $\alpha_H$ and $\alpha_L$ that \( \left( \frac{\alpha_L}{1 - \alpha_L} \right) / \left( \frac{\alpha_H}{1 - \alpha_H} \right) \leq \frac{q_C}{q_A} < \frac{q_C}{q_A - p_A} \), the inequality (20) always holds for any $p_A \in (0, q_A - q_C]$. Let $\tilde{k}$ be the maximum amount of luxury goods (i.e. minimal distortion) required for the high-types to separate from the low-types, that is,

\[
\tilde{k} = \begin{cases} 
 n - \left( \frac{\alpha_L}{1 - \alpha_L} \right) \frac{1}{q_A - p_A} & \text{if } p_A \leq \frac{B_L}{n} \\
 \frac{B_L}{p_A} + \frac{1}{q_A - p_A} \left( n - \frac{B_L}{p_A} \right) q_C - \left( \frac{\alpha_L}{1 - \alpha_L} \right) & \text{if } \frac{B_L}{n} < p_A \leq \frac{B_L}{n - \left( \frac{\alpha_L}{1 - \alpha_L} \right) q_C} \\
 n - \left( \frac{\alpha_L}{1 - \alpha_L} \right) \frac{1}{q_A - p_A} & \text{if } \frac{B_L}{n - \left( \frac{\alpha_L}{1 - \alpha_L} \right) q_C} < p_A \leq q_A - q_C
\end{cases}
\]

The beliefs in luxury equilibrium are consistent with the optimal strategies chosen by both the high-types and the low-types because high-types purchase fewer goods in total compared to the low-types:

\[
k^*_A (\alpha_H, B_H) + k^*_C (\alpha_H, B_H) = \tilde{k} < n = k^*_A (\alpha_L, B_L) + k^*_C (\alpha_L, B_L).
\]

Hence this strategy profile associated with belief $\mu$ is a PBE.

Lastly, we prove that this separating equilibrium of “minimalist luxury” is the unique equilibrium for a given $p_A$ that satisfies Intuitive Criterion and is Pareto dominant. We can do this in three steps:

**Step 1** - Show that any other separating equilibria of minimalist luxury is Pareto dominated. This is straightforward since $\tilde{k}$ is the maximum amount of luxury goods required for the high-types to separate from the low-types. In any other separating equilibria where the high-types consume less than $\tilde{k}$ goods, they obtain lower functional benefits while the low-types’ utility remains the same.

**Step 2** - Show that any pooling equilibrium where consumers purchase less than $n$ items is Pareto dominated. This holds because in any pooling equilibrium, Bayes rule implies that $\mu(B_H|k^*) = \rho(\alpha_H, B_H)$. Hence the overall symbolic benefit of luxury products is $\mathbb{P}(B = B_H|k) = \rho(\alpha_H, B_H)$ for both high-types and low-types. Moreover, since counterfeit goods provide positive functional benefits at a zero price, buying less than $n$ items is strictly dominated.

**Step 3** - Show that the pooling equilibrium where consumers purchase exactly $n$ items fails the
Intuitive Criterion. For any $p_A \in (0, q_A - q_C]$, there exists a $k' < n$ such that

\[
(1 - \alpha_L) \left( \frac{B_L}{p_A} (q_A - p_A) + \left( n - \frac{B_L}{p_A} \right) (q_C - p_C) \right) + \alpha_L \rho(\alpha_H, B_H) > (1 - \alpha_L) \left( \frac{B_L}{p_A} (q_A - p_A) + \left( k' - \frac{B_L}{p_A} \right) (q_C - p_C) \right) + \alpha_L \cdot 1, \tag{21}
\]

and

\[
(1 - \alpha_H) n (q_A - p_A) + \alpha_H \rho(\alpha_H, B_H) < (1 - \alpha_H) k' (q_A - p_A) + \alpha_H \cdot 1. \tag{22}
\]

The left hand sides of inequalities (21) and (22) are the equilibrium payoffs for the low-types and the high-types, respectively; whereas the right hand sides are the maximum payoffs that the low-types and the high-types could obtain by consuming $k'$ products given that the public holds rational beliefs, in this case $\mu(B_H | k') = 1$. Therefore, $k'$ is equilibrium dominated for the low-types and not for the high-types. Once we restrict the public’s beliefs to $\mu(B_H | k') = 1$, the minimum payoff that the high-types obtain is larger than the equilibrium payoff. Hence this pooling equilibrium fails Intuitive Criterion.

Given the consumer choice discussed above, a luxury brand chooses a price $p_A \in (0, q_A - q_C]$ to maximize its profit $\pi$, given by

\[
\pi = p_A \left[ \rho(\alpha_H, B_H) k_A^* (\alpha_H, B_H) + \rho(\alpha_L, B_L) k_A^* (\alpha_L, B_L) \right] = \rho(\alpha_H, B_H) p_A \min \left\{ k, \frac{B_H}{p_A} \right\} + \rho(\alpha_L, B_L) p_A \min \left\{ n, \frac{B_L}{p_A} \right\}, \tag{23}
\]

where

\[k' \leq n - \frac{\alpha_L (1 - \rho(\alpha_H, B_H))}{1 - \alpha_L} \cdot \frac{1}{q_C}\]

\[k' \geq n - \frac{\alpha_H (1 - \rho(\alpha_H, B_H))}{1 - \alpha_H} \cdot \frac{1}{q_A - p_A}\]

It is easy to see that the solution set is non-empty as long as the IC constraints for both the low-types (28) and the high-types (29) are satisfied.
\[
\tilde{k} = \begin{cases} 
\frac{B_L}{p_A} + \frac{1}{q_A - p_A} \left[ (n - \frac{B_l}{p_A}) q_c - \frac{\alpha_L}{1 - \alpha_L} \right] & \text{if } p_A \in (0, \frac{B_L}{n}) \\
\frac{\alpha_L}{1 - \alpha_L} \frac{1}{q_C} & \text{if } p_A \in (\frac{B_L}{n}, \frac{B_L}{n - (\frac{B_L}{1 - \alpha_L}) \frac{1}{q_C}}) \\
n - \frac{\alpha_L}{1 - \alpha_L} \frac{1}{q_C} & \text{if } p_A \in (\frac{B_L}{n - (\frac{B_L}{1 - \alpha_L}) \frac{1}{q_C}}, q_A - q_C) 
\end{cases}
\]

Plugging \( \tilde{k} \) into the profit function in (23) yields the following piece-wise function:

\[
\pi = \begin{cases} 
\rho (\alpha_H, B_H) p_A \left[ n - \left( \frac{\alpha_L}{1 - \alpha_L} \right) \frac{1}{q_A - p_A} \right] + \rho (\alpha_L, B_L) n p_A & \text{if } p_A \in (0, \frac{B_L}{n}) \\
\rho (\alpha_H, B_H) \left( \frac{p_A}{q_A - p_A} \right) \left[ (n - \frac{B_l}{p_A}) q_c - \frac{\alpha_L}{1 - \alpha_L} \right] + B_L & \text{if } p_A \in (\frac{B_L}{n}, \frac{B_L}{n - (\frac{B_L}{1 - \alpha_L}) \frac{1}{q_C}}) \\
\rho (\alpha_H, B_H) p_A \left[ n - \left( \frac{\alpha_L}{1 - \alpha_L} \right) \frac{1}{q_C} \right] + \rho (\alpha_L, B_L) B_L & \text{if } p_A \in (\frac{B_L}{n - (\frac{B_L}{1 - \alpha_L}) \frac{1}{q_C}}, \min \left\{ q_A - q_C, \frac{B_H}{n - (\frac{\alpha_L}{1 - \alpha_L}) \frac{1}{q_C}} \right\}) \\
\rho (\alpha_H, B_H) B_H + \rho (\alpha_L, B_L) B_L & \text{if } p_A \in (\min \left\{ q_A - q_C, \frac{B_H}{n - (\frac{\alpha_L}{1 - \alpha_L}) \frac{1}{q_C}} \right\}, q_A - q_C] 
\end{cases}
\]

We show that \( p_A^* = \min \left\{ q_A - q_C, \frac{B_H}{n - (\frac{\alpha_L}{1 - \alpha_L}) \frac{1}{q_C}} \right\} \) is the Pareto dominant profit-maximizing price. To see this, note that the profit function strictly increases in \( p_A \) when \( p_A \leq p_A^* \) and then remains constant when \( p_A > p_A^* \). Note that the luxury brand obtains identical profit when \( p_A \in [p_A^*, q_A - q_C] \), while consumers obtain the highest utility when \( p_A = p_A^* \) because they can only afford fewer goods when \( p_A > p_A^* \). Therefore, the Pareto dominant equilibrium is the one associated with the lowest price that supports a profit-maximizing equilibrium, i.e., \( p_A = p_A^* \). Plugging the expression for \( p_A^* \) into equations (13) and (14), we obtain the equilibrium purchase quantities of the high-types and the low-types, given respectively by

\[
k_A^* (\alpha_H, B_H) = n - \left( \frac{\alpha_L}{1 - \alpha_L} \right) \frac{1}{q_C},
\]

\[
k_C^* (\alpha_H, B_H) = 0,
\]

and
In this equilibrium, the luxury brand obtains a profit of 

\[
\pi^* = \begin{cases} 
\rho(\alpha_H, B_H) B_H + \rho(\alpha_L, B_L) B_L & \text{if } B_H < (q_A - q_C) \left( n - \frac{\alpha_L}{1 - \alpha_L} \cdot \frac{1}{q_C} \right) \\
\rho(\alpha_H, B_H) \left[ n (q_A - q_C) - \left( \frac{\alpha_L}{1 - \alpha_L} \right) \frac{q_A - q_C}{q_C} \right] + \rho(\alpha_L, B_L) B_L & \text{if } B_H \geq (q_A - q_C) \left( n - \frac{\alpha_L}{1 - \alpha_L} \cdot \frac{1}{q_C} \right) 
\end{cases}
\]

Proofs of Proposition 2 and 3:
The proofs directly follow from the proofs of Lemma 2-3 and Proposition 1. We obtain \(\Delta k^O_A\) and \(\Delta p^O_A\) from taking the difference between the equilibrium quantities and prices given in Lemma 3 and Proposition 1. Similarly, we obtain \(\Delta k^C_A\) and \(\Delta p^C_A\) from taking the difference between the equilibrium quantities and prices we have derived in Lemma 3 and Proposition 1.

Proof of Proposition 4:
The “minimalist luxury” survives as a partial-pooling equilibrium if there are four types of consumers. To see this, note that any incentive compatible strategy that may separate the type-(\(\alpha_H, B_H\)) and the type-(\(\alpha_L, B_L\)) consumers would induce type (\(\alpha_H, B_L\)) to imitate the former by using counterfeits; thus pooling the consumption choices of the wealthy consumer (\(\alpha_H, B_H\)) with those who lack wealth but seek social status (\(\alpha_H, B_L\)). Hence there exist only partial pooling equilibria where all those who care highly about symbolic benefits purchase an identical number of items, denoted by \(k^* (\alpha_H)\)

\[
k^*_A (\alpha_H, B_H) + k^*_C (\alpha_H, B_H) = k^*_A (\alpha_H, B_L) + k^*_C (\alpha_H, B_L) \equiv k^* (\alpha_H)
\]

and all those who care little about symbolic benefits purchase the same number of items, denoted by \(k^* (\alpha_L)\):
\[ k_A^*(\alpha_L, B_L) + k_C^*(\alpha_L, B_L) = k_A^*(\alpha_L, B_B) + k_C^*(\alpha_L, B_H) \equiv k^*(\alpha_L). \]

First, we prove that if the proportion of consumers who care more about the functional benefits of the product is higher among the low-types than the wealthy (i.e., \( \frac{\rho(\alpha_L, B_H)}{\rho(\alpha_H, B_H)} \leq \frac{\rho(\alpha_L, B_L)}{\rho(\alpha_H, B_L)} \)) then a partial pooling equilibria where \( k^*(\alpha_L) > k^*(\alpha_H) \) exists with the following belief:

\[
\mu(k) = \begin{cases} 
\frac{\rho(\alpha_H, B_H)}{\rho(\alpha_H, B_H) + \rho(\alpha_L, B_L)} & \text{if } k \leq k^*(\alpha_H) \\
\frac{\rho(\alpha_L, B_H)}{\rho(\alpha_L, B_H) + \rho(\alpha_L, B_L)} & \text{o/w}
\end{cases}
\]

Re-arrange the condition \( \frac{\rho(\alpha_L, B_L)}{\rho(\alpha_L, B_H)} < \frac{\rho(\alpha_L, B_L)}{\rho(\alpha_H, B_H)} \) by multiplying both sides with \( \rho(\alpha_H, B_H)/\rho(\alpha_L, B_L) \), we obtain

\[
\frac{\rho(\alpha_L, B_H)}{\rho(\alpha_L, B_L) + \rho(\alpha_L, B_H)} < \frac{\rho(\alpha_H, B_H)}{\rho(\alpha_H, B_H) + \rho(\alpha_L, B_L)}.
\]

The expression above indicates that consuming more than \( k^*(\alpha_H) \) units would be associated with lower symbolic benefits. In this case, consumers face a trade-off between symbolic and functional benefits. In a partial pooling equilibrium, all those who care highly about the symbolic benefits have stronger incentives to signal their type, whereas those who care little about it prefer to gain high functional benefits at the expense of symbolic benefits. Therefore those who care little about status can either buy all available products if they are wealthy (\( k_A^*(\alpha_L, B_H) = n, k_C^*(\alpha_L, B_H) = 0 \)) or they can exhaust their budget to own a combination of luxury goods and counterfeits if they are not wealthy (\( k_A^*(\alpha_L, B_L) = \frac{B_L}{B_A}, k_C^*(\alpha_L, B_L) = n - \frac{B_L}{B_A} \)). All those who care little about status purchase a maximum of \( k^*(\alpha_L) = n \) units in total.

Second, we prove that if \( k^*(\alpha_H) \) satisfy certain conditions, those who care highly about symbolic benefits are willing to sacrifice functional benefits by purchasing fewer items for the sake of signaling status. This implies that IC constraint (24) for type \( (\alpha_H, B_H) \) consumer and (25) for type \( (\alpha_H, B_L) \) consumer must hold:
\[ \alpha_H \cdot \left( \frac{\rho(\alpha_H, B_H)}{\rho(\alpha_H, B_H) + \rho(\alpha_H, B_L)} - \frac{\rho(\alpha_L, B_H)}{\rho(\alpha_L, B_L) + \rho(\alpha_L, B_H)} \right) \geq (1 - \alpha_H) \left( n - k^* (\alpha_H) \right) (q_A - p_A) \]  

(24)

and

\[ \alpha_H \cdot \left( \frac{\rho(\alpha_H, B_H)}{\rho(\alpha_H, B_H) + \rho(\alpha_H, B_L)} - \frac{\rho(\alpha_L, B_H)}{\rho(\alpha_L, B_L) + \rho(\alpha_L, B_H)} \right) \geq (1 - \alpha_H) \left( n - k^* (\alpha_H) \right) q_C \]  

(25)

which places a lower bound on \( k^* (\alpha_H) \):

\[ k^* (\alpha_H) \geq n - \frac{1}{q_A - p_A} \left( \frac{\alpha_H}{1 - \alpha_H} \right) \left( \frac{\rho(\alpha_H, B_H)}{\rho(\alpha_H, B_H) + \rho(\alpha_H, B_L)} - \frac{\rho(\alpha_L, B_H)}{\rho(\alpha_L, B_L) + \rho(\alpha_L, B_H)} \right) \]  

Then we show that when the gain in status cannot compensate for the loss in function utility, those who care little about symbolic benefits do not want to mimic other types. That is, the following IC constraint in (26) for type- \((\alpha_L, B_L)\) and in (27) for type- \((\alpha_L, B_H)\) consumer must hold:

\[ \alpha_L \left( \frac{\rho(\alpha_H, B_H)}{\rho(\alpha_H, B_H) + \rho(\alpha_H, B_L)} - \frac{\rho(\alpha_L, B_H)}{\rho(\alpha_L, B_L) + \rho(\alpha_L, B_H)} \right) \leq (1 - \alpha_L) \left( n - k^* (\alpha_H) \right) q_C \]  

(26)

and

\[ \alpha_L \left( \frac{\rho(\alpha_H, B_H)}{\rho(\alpha_H, B_H) + \rho(\alpha_H, B_L)} - \frac{\rho(\alpha_L, B_H)}{\rho(\alpha_L, B_L) + \rho(\alpha_L, B_H)} \right) \leq (1 - \alpha_L) \left( n - k^* (\alpha_H) \right) (q_A - p_A) \]  

(27)

Hence (26) and (27) place an upper bound on \( k^* (\alpha_H) \), given by

\[ k^* (\alpha_H) \leq n - \frac{1}{q_C} \left( \frac{\alpha_L}{1 - \alpha_L} \right) \left( \frac{\rho(\alpha_H, B_H)}{\rho(\alpha_H, B_H) + \rho(\alpha_H, B_L)} - \frac{\rho(\alpha_L, B_H)}{\rho(\alpha_L, B_L) + \rho(\alpha_L, B_H)} \right). \]

By assumption on \( \alpha_H \) and \( \alpha_L \) that \( \left( \frac{\alpha_H}{1 - \alpha_L} \right) / \left( \frac{\alpha_L}{1 - \alpha_H} \right) \leq \frac{q_A}{q_C} \), we can see that the solution set is non-empty for \( k^* (\alpha_H) \) to satisfy the IC constraints for all four types.
Then we select the equilibrium where $k^*_A(\alpha_H, B_H)$ is equal to its upper bound, which is the only equilibrium outcome that satisfies the Intuitive Criterion and is not Pareto dominated. In this equilibrium, all those who care highly about symbolic benefits purchase a total number of $k^*_A(\alpha_H)$ items. Specifically, those who care little about status choose to buy only authentic products if they are wealthy:

$$k^*_A(\alpha_L, B_H) = \min \left\{ \frac{B_H}{p_A}, n - \Delta K \right\}, k^*_C(\alpha_L, B_H) = \max \left\{ 0, n - \Delta K - \frac{B_H}{p_A} \right\}$$

or they exhaust their budget to own a combination of authentic goods and counterfeits if they are not wealthy:

$$k^*_A(\alpha_L, B_L) = \min \left\{ \frac{B_L}{p_A}, n - \Delta K \right\}, k^*_C(\alpha_L, B_L) = \max \left\{ 0, n - \Delta K - \frac{B_L}{p_A} \right\}$$

where $\Delta K = \frac{1}{q_C} \left( \frac{\alpha_L}{1 - \alpha_L} \right) \left( \frac{\rho(\alpha_H, B_H)}{\rho(\alpha_H, B_H) + \rho(\alpha_L, B_L)} - \frac{\rho(\alpha_L, B_H)}{\rho(\alpha_L, B_H) + \rho(\alpha_L, B_H)} \right)$.

Given the consumer choice discussed above, a luxury brand chooses a price to maximize the total expenditure by all four types in the market, as follows,

$$\max_{p_A \in [0, q_A - q_C]} \pi = \rho(\alpha_H, B_H) k^*_A(\alpha_H, B_H) p_A + \rho(\alpha_L, B_H) k^*_A(\alpha_L, B_H) p_A + \rho(\alpha_H, B_L) k^*_A(\alpha_H, B_L) p_A + \rho(\alpha_L, B_L) k^*_A(\alpha_L, B_L) p_A + \rho(\alpha_H, B_H) \min \{ p_A(n - \Delta K), B_H \} + \rho(\alpha_L, B_H) \min \{ np_A, B_H \} + \rho(\alpha_H, B_L) \min \{ p_A(n - \Delta K), B_L \} + \rho(\alpha_L, B_L) \min \{ np_A, B_L \}$$

Expanding the min operator, we obtain the following piece-wise profit function:

$$\pi = \begin{cases} 
  p_A(n - \Delta K) \left[ \rho(\alpha_H, B_H) + \rho(\alpha_H, B_L) \right] + np_A \left[ \rho(\alpha_L, B_H) + \rho(\alpha_L, B_L) \right] & p_A \in [0, \frac{B_H}{n}) \\
  p_A(n - \Delta K) \left[ \rho(\alpha_H, B_H) + \rho(\alpha_H, B_L) \right] + np_A \rho(\alpha_L, B_H) + \rho(\alpha_L, B_L) B_L & p_A \in [\frac{B_H}{n}, \frac{B_H}{n - \Delta K}) \\
  p_A(n - \Delta K) \rho(\alpha_H, B_H) + np_A \rho(\alpha_L, B_H) + \rho(\alpha_L, B_L) B_L & p_A \in [\frac{B_H}{n - \Delta K}, \frac{B_H}{n}) \\
  p_A(n - \Delta K) \rho(\alpha_H, B_H) + np_A \rho(\alpha_L, B_H) + \rho(\alpha_L, B_L) B_H + \rho(\alpha_L, B_L) B_L & p_A \in [\frac{B_H}{n}, p_0) \\
  \left[ \rho(\alpha_H, B_H) + \rho(\alpha_L, B_H) \right] B_H + \rho(\alpha_L, B_L) B_L & p_A \in [p_0, q_A - q_C] 
\end{cases}$$
where \( p_0 \equiv \min \left\{ \frac{B_H}{n - \Delta K}, q_A - q_C \right\} \). Note that the profit strictly increases in \( p_A \in [0, p_0] \) and then remains constant. Therefore, the Pareto dominant profit-maximizing price is given by

\[
p^*_A = p_0 = \min \left\{ \frac{B_H}{n - \Delta K}, q_A - q_C \right\},
\]

where \( \Delta K = \frac{1}{q_C} \left( \frac{\alpha_L}{1 - \alpha_L} \right) \left( \frac{\rho(\alpha_H, B_H)}{\rho(\alpha_H, B_H) + \rho(\alpha_H, B_L)} - \frac{\rho(\alpha_L, B_H)}{\rho(\alpha_L, B_H) + \rho(\alpha_L, B_L)} \right). \]

Proof of Proposition 5:

Recall that we define the set of equilibria as “minimalist luxury” if

\[
k^*_A(\alpha_H, B_H) + k^*_C(\alpha_H, B_H) < k^*_A(\alpha_L, B_L) + k^*_C(\alpha_L, B_L),
\]

and as “excessive luxury” if

\[
k^*_A(\alpha_H, B_H) + k^*_C(\alpha_H, B_H) > k^*_A(\alpha_L, B_L) + k^*_C(\alpha_L, B_L).
\]

We prove this Proposition in three steps: first, we show that the market for authentic goods is not viable when \( p_A \in (p_C + q_A - q_C, +\infty) \). Second, we show that for any \( p_A \in (0, p_C + q_A - q_C] \), there exists only a separating equilibrium of “minimalist luxury” if \( 0 < p_C \leq \frac{B_L}{n} \). There exists an “excessive luxury” equilibrium that Pareto dominates any other equilibria if \( p_C > \frac{B_L}{n} \). Third, we derive the profit-maximizing equilibrium in both cases.

We first show that if \( p_A > p_C + q_A - q_C \), the demand for authentic goods is zero. This is because the counterfeit and the authentic goods are perfect substitutes in providing symbolic utility (as they are virtually indistinguishable), and a counterfeit good is not only cheaper, but it also provides strictly higher functional benefits net of price \((q_C - p_C)\) than an authentic good do \((q_A - p_A)\). To maximize their utility, both the high-types and low-types strictly prefer to purchase counterfeit goods. Therefore, the market for authentic goods is not viable in this case.

1. “Minimalist luxury” equilibrium \((0 < p_C \leq \frac{B_L}{n})\)

Next we prove that for any \( p_A \leq p_C + q_A - q_C \), there exists only a separating equilibrium of “minimalist luxury” type if \( 0 < p_C \leq \frac{B_L}{n} \). We begin with showing by contradiction that there exists no “excessive luxury” equilibrium in this case. Suppose there exists an “excessive luxury”
equilibrium where a consumer is perceived to be a high-type iff the consumer purchases no less than \( \hat{k} \) goods in total, and otherwise is perceived to be a low-type. In this separating equilibrium, the high-types obtain a symbolic benefit of 1 whereas the low-types obtain 0, which requires that 
\[
k^*_A(\alpha_H, B_H) + k^*_C(\alpha_H, B_H) \geq \hat{k} > k^*_A(\alpha_L, B_L) + k^*_C(\alpha_L, B_L).
\]
The low-types would be strictly better off by deviating to purchasing \( k'_C \) counterfeit and \( k'_A \) authentic goods, such that \( k'_C + k'_A = \hat{k} \), where 
\[
p_C k'_C + p_A k'_A = p_C k^*_C(\alpha_L, B_L) + p_A k^*_A(\alpha_L, B_L).
\]
Since \( B_L \geq n p_C \), this deviation is feasible and profitable for the low-types as it not only yields higher symbolic benefits, but also higher functional benefits (net of price), contradicting the notion of an equilibrium. To see this, we examine the following two cases separately.

Case 1: \( p_A \in (p_C, \left( \frac{q_A}{q_C} \right) p_C) \):

From \( \frac{p_A}{p_C} < \frac{q_A}{q_C} \) and 
\[
p_C k'_C + p_A k'_A = p_C k^*_C(\alpha_L, B_L) + p_A k^*_A(\alpha_L, B_L),
\]
we can obtain the following condition
\[
\frac{k'_A - k^*_A(\alpha_L, B_L)}{k^*_C(\alpha_L, B_L) - k'_C} = \frac{p_C}{p_A} > \frac{q_C}{q_A},
\]
and thus we have 
\[
q_A \left( k'_A - k^*_A(\alpha_L, B_L) \right) > q_C \left( k^*_C(\alpha_L, B_L) - k'_C \right)
\]
as long as \( k'_C < k^*_C(\alpha_L, B_L) \). Therefore we can show that \( (k'_A, k'_C) \) yields higher functional benefits (net of price) than \( (k^*_A(\alpha_L, B_L), k^*_C(\alpha_L, B_L)) \) because
\[
k'_A (q_A - p_A) + k'_C (q_C - p_C) > k'_A (q_A - p_A) + k'_C (q_C - p_C) - \left[ q_A \left( k'_A - k^*_A(\alpha_L, B_L) \right) - q_C \left( k^*_C(\alpha_L, B_L) - k'_C \right) \right] > 0
\]
\[
= q_A \left( k'_A - (k'_A - k^*_A(\alpha_L, B_L)) \right) + q_C \left( k'_C + (k^*_C(\alpha_L, B_L) - k'_C) \right) - p_C k'_C - p_A k'_A
\]
\[
= q_A k^*_A(\alpha_L, B_L) + q_C k^*_C(\alpha_L, B_L) - p_C k^*_C(\alpha_L, B_L) - p_A k^*_A(\alpha_L, B_L)
\]
\[
= k^*_A(\alpha_L, B_L) (q_A - p_A) + k^*_C(\alpha_L, B_L) (q_C - p_C).
\]

Case 2: \( p_A \in \left[ \left( \frac{q_A}{q_C} \right) p_C, p_C + q_A - q_C \right] \):

From \( \frac{p_A}{p_C} \geq \frac{q_A}{q_C} \) and 
\[
p_C k'_C + p_A k'_A = p_C k^*_C(\alpha_L, B_L) + p_A k^*_A(\alpha_L, B_L),
\]
we can obtain the following condition
\[
\frac{k'_C - k^*_C(\alpha_L, B_L)}{k^*_A(\alpha_L, B_L) - k'_A} = \frac{p_A}{p_C} \geq \frac{q_A}{q_C},
\]

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which implies that \( q_C (k_C^* (\alpha_L, B_L) - k'_C) \geq q_A (k'_A - k_A^* (\alpha_L, B_L)) \) as long as \( k'_A < k_A^* (\alpha_L, B_L) \) and \( k'_C > k_C^* (\alpha_L, B_L) \). Therefore we can show that \((k'_A, k'_C)\) yields higher functional benefits (net of price) than \((k_A^* (\alpha_L, B_L), k_C^* (\alpha_L, B_L))\), because

\[
k'_A (q_A - p_A) + k'_C (q_C - p_C) \geq k'_A (q_A - p_A) + k'_C (q_C - p_C) - \left[ q_C (k'_C - k_C^* (\alpha_L, B_L)) - q_A (k'_A - k_A^* (\alpha_L, B_L)) \right]_{> 0}
\]

\[
= q_A (k'_A + (k_A^* (\alpha_L, B_L) - k'_A)) + q_C (k'_C - (k'_C - k_C^* (\alpha_L, B_L))) - p_C k'_C - p_A k'_A
\]

\[
= q_A k_A^* (\alpha_L, B_L) + q_C k_C^* (\alpha_L, B_L) - p_C k_C^* (\alpha_L, B_L) - p_A k_A^* (\alpha_L, B_L)
\]

\[
= k_A^* (\alpha_L, B_L) (q_A - p_A) + k_C^* (\alpha_L, B_L) (q_C - p_C),
\]

Next we show that there exists a "minimum luxury" equilibrium where a consumer is perceived as the high-type as long as she purchases fewer than \( \tilde{k} \) luxury items, translating into the following belief:

\[
\mu(B_H|k) = \begin{cases} 
1 & \text{if } k \leq \tilde{k} \\
0 & \text{o/w}
\end{cases}.
\]

In this separating equilibrium, the high-types have a stronger incentive to maximize their symbolic benefits by purchasing a total of \( k_A^* (\alpha_H, B_H) = \tilde{k} \) items and obtain a symbolic utility of 1. To maximize their utility, the high-types purchase only authentic goods if they can afford all \( \tilde{k} \) of them. Alternatively, if they are budget-constrained and cannot afford \( \tilde{k} \) authentic goods, they will assemble \( \tilde{k} \) goods through purchasing a combination of authentic goods and counterfeits. Specifically,

\[
k_A^* (\alpha_H, B_H) = \min \left\{ \frac{B_H - \tilde{k} p_C}{p_A - p_C}, \tilde{k} \right\}, k_C^* (\alpha_H, B_H) = \max \left\{ 0, \frac{\tilde{k} p_A - B_H}{p_A - p_C} \right\}.
\]

By contrast, the low-types who care relatively more about functional benefits prefer to maximize functional benefits at the expense of their symbolic benefits. Recall that in this case, the low-types can afford all \( n \) available counterfeits \((np_C \leq B_L)\). Since an authentic product provides higher net functional benefits than a counterfeit goods does \( (i.e., q_A - p_A > q_C - p_C)\), the low-types assemble \( n \) products by purchasing as many authentic goods as they can afford. Specifically,

\[
k_A^* (\alpha_L, B_L) = \min \left\{ n, \frac{B_L - np_C}{p_A - p_C} \right\}, k_C^* (\alpha_L, B_L) = \max \left\{ 0, \frac{np_A - B_L}{p_A - p_C} \right\}.
\]
For these consumption quantities to hold in a separating equilibrium, the low-types should have no incentives to mimic the high-types by refraining from their consumption. In other words, the low-types’ gain in symbolic benefits should not exceed their loss in functional benefits from mimicking, as the following IC constraint shows

\[ \alpha_L (1 - 0) \leq (1 - \alpha_L) \left( \frac{p_A \left( n - \tilde{k} \right) (q_C - p_C) - p_C \left( n - \tilde{k} \right) (q_A - p_A)}{p_A - p_C} \right) \]

which can be rewritten as

\[ \tilde{k} \leq n - \frac{\alpha_L}{1 - \alpha_L} \cdot \frac{1}{q_C - p_C} \left( \frac{q_A - q_C}{p_A - p_C} \right) \] (28)

Similarly, the high-types should not have an incentive to purchase more than \( \tilde{k} \) items. In order to ensure the Individuality Rationality of the high-types, their loss in symbolic benefits should be larger than their gain in functional benefits from purchasing more than \( \tilde{k} \) items, which is characterized by the following set of constraints:

\[
\begin{cases}
\alpha_H (1 - 0) \geq (1 - \alpha_H) \left( n - \tilde{k} \right) (q_A - p_A) & \text{if } B_H > np_A \\
\alpha_H (1 - 0) \geq (1 - \alpha_H) \left( \frac{B_H - np_C}{p_A - p_C} - \tilde{k} \right) (q_A - p_A) + \left( \frac{np_A - B_H}{p_A - p_C} \right) (q_C - p_C) & np_A > B_H \geq \tilde{k} p_A . \\
\alpha_H (1 - 0) \geq (1 - \alpha_H) \left( \frac{p_A (n - \tilde{k})(q_C - p_C) - p_C (n - \tilde{k})(q_A - p_A)}{p_A - p_C} \right) & B_H < \tilde{k} p_A
\end{cases}
\]

We can rewrite these expressions to simply them as follows:

\[
\tilde{k} \geq \begin{cases}
\begin{aligned}
& n - \left( \frac{\alpha_H}{1 - \alpha_H} \right) \frac{1}{q_A - p_A} & \text{if } B_H > np_A \\
& \frac{B_H - np_C}{p_A - p_C} + \left( \frac{np_A - B_H}{p_A - p_C} \right) (q_C - p_C) - \left( \frac{\alpha_H}{1 - \alpha_H} \right) \frac{1}{q_A - p_A} & \text{if } np_A > B_H \geq \tilde{k} p_A . \\
& n - \frac{\alpha_H}{1 - \alpha_H} \cdot \frac{1}{q_C - p_C} \left( \frac{q_A - q_C}{p_A - p_C} \right) & \text{if } B_H < \tilde{k} p_A
\end{aligned}
\end{cases}
\] (29)

A sufficient condition for (29) to hold in all three cases is

\[ \tilde{k} \geq n - \left( \frac{\alpha_H}{1 - \alpha_H} \right) \frac{1}{q_A - p_A} . \] (30)
Given our assumption on $\alpha_H$ and $\alpha_L$ that \( \left( \frac{\alpha_H}{1 - \alpha_H} \right) / \left( \frac{\alpha_L}{1 - \alpha_L} \right) \geq \frac{q_A}{q_C} \), we can show that the solution set for $\tilde{k}$ is non-empty as long as $p_A \geq \frac{pc}{qc} \left( \frac{qa-qc}{pa-pc} \right)$, or equivalently, $p_A \geq \frac{1}{2} \left( \sqrt{4pc \left( \frac{qa}{qc} - 1 \right) + p_c^2 + p_c} \right)$.

This is equivalent to showing that \( \left( \frac{\alpha_H}{1 - \alpha_H} \right) / \frac{qa-pA}{qa-pc} \geq \left( \frac{\alpha_L}{1 - \alpha_L} \right) / \frac{qa-pC}{qc-pc} \). Note that

\[
p_A \geq \frac{pc}{qc} \left( \frac{qa-qc}{pa-pc} \right) \implies -pc \left( \frac{qa-qc}{pa-pc} \right) \geq -p_A \left( \frac{qc}{qa} \right) \implies -\left( \frac{qa}{qc} \right) pc \left( \frac{qa-qc}{pa-pc} \right) \geq -p_A \implies qa - \left( \frac{qa}{qc} \right) pc \left( \frac{qa-qc}{pa-pc} \right) \geq qa - p_A \implies \frac{qa}{qc} \geq \frac{qa-pA}{qc-pc} \left( \frac{qa-qc}{pa-pc} \right)
\]

Therefore we have

\[
\left( \frac{\alpha_H}{1 - \alpha_H} \right) / \left( \frac{\alpha_L}{1 - \alpha_L} \right) \geq \frac{qa}{qc} \geq \frac{qa-pA}{qc-pc} \left( \frac{qa-qc}{pa-pc} \right).
\]

Let $\tilde{k} = n - \frac{\alpha_L}{1 - \alpha_L} \left( \frac{1}{qc-pc} \left( \frac{qa-qc}{pa-pc} \right) \right)$ be the maximum amount of luxury goods (i.e., minimal sacrifice by the high-types) required for separation. This belief is consistent with the equilibrium outcome because the high-types purchase fewer goods in total compared to the low-types:

\[
k_A^* (\alpha_H, B_H) + k_C^* (\alpha_H, B_H) = \tilde{k} < n = k_A^* (\alpha_L, B_L) + k_C^* (\alpha_L, B_L).
\]

Hence this strategy profile associated with belief $\mu$ is a PBE.

Lastly, we prove that this separating equilibrium of “minimalist luxury” is the unique equilibrium that satisfies Intuitive criterion and is Pareto dominant. We can do this in three steps:

**Step 1** - Show that any other separating equilibria of minimalist luxury is Pareto dominated. This is straightforward since the wealthy can purchase no more than $\tilde{k}$ items when equation 28 holds.

**Step 2** - Show that any pooling equilibrium where consumers purchase less than $n$ items are Pareto dominated. This is easy to show because in any pooling equilibrium, Bayes rule implies that $\mu(B_H|k^*) = \rho(\alpha_H, B_H)$. Hence the overall symbolic benefit of luxury products is $P(B = B_H|k) =$
\(\rho(\alpha_H, B_H)\) for both high-types and low-types. Moreover, the low-types can afford the entire product line of counterfeits and thus buying less than \(n\) items is strictly dominated for the low-types.

**Step 3 -** Show that the pooling equilibrium where consumers purchase exactly \(n\) items fails the Intuitive Criterion. There exists a \(k' < n\) such that

\[
(1 - \alpha_L) \left( \frac{B_L - np_C}{p_A - p_C} \right) (q_A - p_A) + \left( \frac{np_A - B_L}{p_A - p_C} \right) (q_C - p_C) + \alpha_L \rho(\alpha_H, B_H) > (1 - \alpha_L) \left( \frac{B_L - k'p_C}{p_A - p_C} \right) (q_A - p_A) + \left( \frac{k'p_A - B_L}{p_A - p_C} \right) (q_C - p_C) + \alpha_L \cdot 1, \tag{31}
\]

and

\[
(1 - \alpha_H) n (q_A - p_A) + \alpha_H \rho(\alpha_H, B_H) < (1 - \alpha_H) k' (q_A - p_A) + \alpha_H \cdot 1. \tag{32}
\]

The left hand sides of inequality (31) and (32) are the equilibrium payoffs for the low-types and high-types, respectively, whereas the right hand sides are the maximum payoff that low-types and high-types could achieve by consuming \(k'\) given that the public hold reasonable beliefs, in this case \(\mu(B_H|k') = 1\). Therefore, \(k'\) is equilibrium dominated for low-types and not for high-types. Once we restrict the public’s beliefs to \(\mu(B_H|k') = 1\), the minimum payoff that high-types can get is bigger than the equilibrium payoff and hence this pooling equilibrium fails intuitive criterion.

Given the consumer choice in a viable market, a luxury brand chooses a price \(p_A (\leq p_C + q_A - q_C)\) to maximize its profit \(\pi\), given by

\[
\pi = p_A \left[ \rho(\alpha_H, B_H) k^*_A (\alpha_H, B_H) + \rho(\alpha_L, B_L) k^*_A (\alpha_L, B_L) \right] = \begin{cases} ho(\alpha_H, B_H) p_A k_A + \rho(\alpha_L, B_L) p_A \left( \frac{B_L - np_C}{p_A - p_C} \right) & \text{if } B_H \geq \tilde{k}p_A, \\ \rho(\alpha_H, B_H) p_A \left( \frac{B_H - k'p_C}{p_A - p_C} \right) + \rho(\alpha_L, B_L) p_A \left( \frac{B_L - np_C}{p_A - p_C} \right) & \text{if } B_H < \tilde{k}p_A, \end{cases}
\]

\(^4\)We can rearrange and rewrite inequality (31) and (32) as follows:

\[
k' \leq n - \frac{\alpha_L (1 - \rho(\alpha_H, B_H))}{1 - \alpha_L}, \quad \frac{1}{q_C - p_C \left( \frac{q_A - q_C}{p_A - p_C} \right)}
\]

\[
k' \geq n - \frac{\alpha_H (1 - \rho(\alpha_H, B_H))}{1 - \alpha_H}, \quad \frac{1}{q_A - p_A}
\]

It is easy to see that the solution set is non-empty as long as IC constraints for both low-types (28) and high-types (29) are satisfied.
where \( \hat{k} = n - \frac{\alpha L}{1 - \alpha L} \left( \frac{1}{q_C - p_C} \frac{q_A - q_C}{p_A - p_C} \right) \).

Note that the profit function \( \pi \) strictly increases in \( p_A \) when \( B_H \geq \hat{k}p_A \), and then declines in \( p_A \) if \( B_H < \hat{k}p_A \) since \( \partial \hat{k} / \partial p_A > 0 \). Thus the profit-maximizing price \( p_A^M \) is given by

\[
p_A^M = \begin{cases} 
\bar{p} & \text{if } B_H < (p_C + q_A - q_C) \left( n - \frac{\alpha L}{1 - \alpha L} \cdot \frac{1}{q_C - p_C} \right), \\
p_C + q_A - q_C & \text{if } B_H \geq (p_C + q_A - q_C) \left( n - \frac{\alpha L}{1 - \alpha L} \cdot \frac{1}{q_C - p_C} \right),
\end{cases}
\]

where \( \bar{p} \equiv \frac{(nq_A - \frac{\alpha L}{1 - \alpha L})p_C + B_H q_C + \sqrt{[(nq_A - \frac{\alpha L}{1 - \alpha L})p_C + B_H q_C]^2 - 4B_H p_C q_A (nq_C - \frac{\alpha L}{1 - \alpha L})}} {2(nq_C - \frac{\alpha L}{1 - \alpha L})} \) such that \( \bar{p} \equiv \frac{p_A^M}{B_H} \). Consequently, the luxury brand obtains a profit of

Taking the F.O.C. of the profit function \( \pi (\hat{k}p_A \leq B_H) \) given as follows:

\[
\frac{\partial \pi (p_A \hat{k} \leq B_H)}{\partial p_A} = \rho (\alpha_H, B_H) \left( \hat{k} + \frac{\partial \hat{k}}{\partial p_A} \right) - \rho (\alpha_L, B_L) \left( \frac{p_C (B_L - np_C)}{(p_A - p_C)^2} \right)
\]

\[
= \rho (\alpha_H, B_H) \left( n - \frac{\alpha L}{1 - \alpha L} \cdot \frac{1}{q_C - 1} \left( 1 - \frac{p_C^2 q_A}{(q_C p_A - p_C q_A)^2} \right) \right) - \rho (\alpha_L, B_L) \left( \frac{p_C (B_L - np_C)}{(p_A - p_C)^2} \right)
\]

Let \( g_H \equiv n - \frac{\alpha_L}{1 - \alpha L} \cdot \frac{1}{q_C - 1} \left( 1 - \frac{p_C^2 q_A}{(q_C p_A - p_C q_A)^2} \right) \) and \( g_L \equiv \frac{p_C (B_L - np_C)}{(p_A - p_C)^2} \). It is easy to see that \( g_H > 0, g_L > 0, \partial g_H / \partial p_A < 0 \) and \( \partial g_L / \partial p_A < 0 \). Hence for any \( \frac{1}{2} \left( \sqrt{4p_C \left( \frac{q_A}{q_C} - 1 \right)} + p_C + p_C \right) \leq p_A \leq p_C + q_A - q_C \), we know that

\[
g_H (\cdot) \geq g_H (p_A = p_C + q_A - q_C) = n + \frac{\alpha L}{1 - \alpha L} \cdot \frac{1}{q_C - 1} \left( \frac{q_C}{p_C} - 1 \right)^2 \geq n
\]

\[
g_L (\cdot) \leq g_L \left( p_A = \frac{1}{2} \left( \sqrt{4p_C \left( \frac{q_A}{q_C} - 1 \right)} + p_C + p_C \right) \right) = \frac{p_C (B_L - np_C)}{(p_A - p_C)^2} \left( \frac{q_C}{q_A} \right)^2 \leq \frac{n}{(q_A - q_C)} \left( \frac{q_C}{q_A} \right)^2 \left( \sqrt{\frac{4}{q_C} p_C + p_C + 1} \right)^2
\]

where the last inequality follows the assumption that \( B_L \leq n (q_A - q_C) + p_C \). Given that \( \left( \sqrt{4 \left( \frac{q_A}{q_C} - 1 \right) + p_C + \sqrt{p_C}} \right)^2 \leq \frac{\rho (\alpha_H, B_H) (q_A - q_C)}{n (q_A - q_C)^2} \) by assumption, we can obtain the following:

\[
\frac{\partial \pi (p_A \hat{k} \leq B_H)}{\partial p_A} \geq \rho (\alpha_H, B_H) n - \rho (\alpha_L, B_L) \left( \frac{n}{q_A - q_C} \right) \left( \frac{q_C}{q_A} \right)^2 \left( \sqrt{\frac{4}{q_C} p_C + p_C + 1} \right)^2
\]

\[
\geq 0
\]
\[ \pi^* = \begin{cases} \rho(\alpha_H, B_H)B_H + \rho(\alpha_L, B_L) \frac{B_L - np c}{p_C - pc} & \text{if } B_H \leq B_0 \\ \rho(\alpha_H, B_H)B_H + \rho(\alpha_L, B_L) \frac{B_L - np c}{q_A - qC} & \text{if } B_H > B_0 \end{cases} \]

where \( B_0 \equiv (p_C + q_A - qC) \left( n - \frac{\alpha_L}{1 - \alpha_L} \cdot \frac{1}{qC - pC} \right) \).

2. “Excessive luxury” equilibrium \( (p_C > \frac{B_L}{n}) \)

We show that when \( p_C > \frac{B_L}{n} \), there always exists an “excessive luxury” equilibrium that is profit-maximizing and Pareto dominant. We proceed by studying two distinct cases separately. We can verify that there exists an “excessive luxury” equilibrium with \( p_A \leq \min \left\{ \frac{B_H}{n}, p_C + q_A - qC \right\} \), where the high-types purchase all \( n \) available authentic goods and the low-types exhaust their budget to purchase either counterfeit or authentic goods, depending the price of luxury goods. Specifically,

\[ k^*_A(\alpha_H, B_H) = n, k^*_C(\alpha_H, B_H) = 0, \]

and

\[ k^*_A(\alpha_L, B_L) = \begin{cases} 0 & p_A > \left( \frac{q_A}{qC} \right) p_C \\ \frac{B_L}{p_A} & p_A \leq \left( \frac{q_A}{qC} \right) p_C \end{cases}, 
 k^*_C(\alpha_L, B_L) = \begin{cases} \frac{B_L}{p_C} & p_A > \left( \frac{q_A}{qC} \right) p_C \\ 0 & p_A \leq \left( \frac{q_A}{qC} \right) p_C \end{cases}. \]

The belief \( \mu \) associated with this equilibrium is given by

\[ \mu(B_H|k) = \begin{cases} 1 & \text{if } k \geq n \\ 0 & \text{o/w} \end{cases} \]

which is consistent with the optimal strategies chosen by the consumers because

\[ k^*_A(\alpha_H, B_H) + k^*_C(\alpha_H, B_H) = n > k^*_A(\alpha_L, B_L) + k^*_C(\alpha_L, B_L). \]

Moreover, the strategies specified above are incentive-compatible. This is because the low-types cannot afford a total of \( n \) goods even by mixing with counterfeits \( (B_L < npC) \), and thus cannot mimic the high-types. Hence this strategy profile associated with belief \( \mu \) is a PBE. In the next two steps, we show that this PBE passes the Intuitive Criterion and is not Pareto dominated.

**Step 1** - Show that any other separating equilibria is Pareto dominated. In any separating equilibria, the high-types always gain a symbolic benefit of 1 and the low-types obtain zero sym-
bolic benefit. Therefore the only separating equilibrium that is not Pareto dominated is the one where both the high-types and low-types obtain the highest function benefits. This is exactly the equilibrium we have presented above.

**Step 2** - Show that any pooling equilibrium fails the Intuitive Criterion. Let $k^*$ denote the total number of luxury items purchased by high-types or low-types in a pooling equilibrium.

$$k^*_A (\alpha_H, B_H) + k^*_C (\alpha_H, B_H) = k^*_A (\alpha_L, B_L) + k^*_C (\alpha_L, B_L) \equiv k^*$$

Bayes rule implies that $\mu(B_H|k^*) = \rho(\alpha_H, B_H)$. Therefore the overall symbolic benefit of luxury products is $\mathbb{P}(B = B_H|k) = \rho(\alpha_H, B_H)$ for both high-types and low-types. Given that $\frac{B_H}{p_A} \geq n > \frac{B_L}{p_C}$, there exists a $k' > k^*$ such that

$$k' > \frac{B_L}{p_C} \quad (33)$$

and

$$(1 - \alpha_H) k^* (q_A - p^*_A) + \alpha_H \rho(\alpha_H, B_H) < (1 - \alpha_H) k' (q_A - p^*_A) + \alpha_H \cdot 1 \quad (34)$$

Inequality (33) implies that consuming $k'$ is infeasible for the low-types due to budget constraint. The left hand side of inequality (34) is the equilibrium payoff for the high-types whereas the right hand side is the maximum payoffs that the high-type could obtain by consuming $k'$ given that the public holds rational beliefs: in this case $\mu(B_H|k') = 1$. Therefore, $k'$ is equilibrium dominated for the low-types but not for the high-types. Once we restrict the public’s beliefs to $\mu(B_H|k') = 1$, the minimum payoff that high-types can get is larger than the equilibrium payoff. Hence this pooling equilibrium fails Intuitive Criterion.

Finally, we derive the profit-maximizing equilibrium by calculating the profit of the luxury brand $\pi$, given by:

$$\pi = \rho(\alpha_H, B_H) k^*_A (\alpha_H, B_H) p_A + \rho(\alpha_L, B_L) k^*_C (\alpha_H, B_H) p_A$$

$$= \begin{cases} \rho(\alpha_H, B_H) n p_A & p_A > \left(\frac{q_A}{q_C}\right) p_C \\ \rho(\alpha_H, B_H) n p_A + \rho(\alpha_L, B_L) B_L & p_A \leq \left(\frac{q_A}{q_C}\right) p_C \end{cases}$$
The luxury brand faces the trade-off of increasing its price to capture more rent from the high-types and of decreasing its price to cater to the low-types. The optimal price of the luxury products is thus determined by the relative wealth of the high-types and low-types. Formally,

\[
p^*_A = \begin{cases} 
\min \left\{ \frac{B_H}{n}, p_C + q_A - q_C \right\} & \text{if } \frac{\rho(\alpha_L, B_L)}{\rho(\alpha_H, B_H)} B_L + n \left( \frac{q_A}{q_C} \right) p_C < B_H \\
\min \left\{ \frac{B_H}{n}, \left( \frac{q_A}{q_C} \right) p_C \right\} & \text{if } B_H \leq \frac{\rho(\alpha_L, B_L)}{\rho(\alpha_H, B_H)} B_L + n \left( \frac{q_A}{q_C} \right) p_C 
\end{cases}
\]

As a result, the profit of the luxury brand is given by

\[
\pi^* = \begin{cases} 
\rho(\alpha_H, B_H) \min \left\{ B_H, n \left( p_C + q_A - q_C \right) \right\} & \text{if } \frac{\rho(\alpha_L, B_L)}{\rho(\alpha_H, B_H)} B_L + n \left( \frac{q_A}{q_C} \right) p_C < B_H \\
\rho(\alpha_H, B_H) \min \left\{ B_H, n \left( \frac{q_A}{q_C} \right) p_C \right\} + \rho(\alpha_L, B_L) B_L & \text{if } B_H \leq \frac{\rho(\alpha_L, B_L)}{\rho(\alpha_H, B_H)} B_L + n \left( \frac{q_A}{q_C} \right) p_C 
\end{cases}
\]