A Model of Brand Positioning: Product-Portfolio View

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Abstract

Brand positioning is an essential but understudied economic activity. Beyond the real functional differences, the positioning of a specific brand can have profound effects on the purchase decisions of consumers. We provide a micro-foundation for why and how brand positioning can deliver credible information to consumers from a product-portfolio view and consumer search framework. Moreover, we offer a rationale for a firm’s specific brand positioning choice, such as “niche” or “mainstream” positioning. Consumers form the perception of brand positioning from various interactions with all different products of the same brand. To capture this intricacy of brand positioning, we conceptualize it as the average location of all products of the same brand on the horizontal line. Consumers search for product matches and are guided by the brand positioning. We show that a niche brand naturally conveys more information than a mainstream brand. A firm with a mainstream brand has incentives to opportunistically dilute its brand by including a wide range of products. A niche brand may arise even in a monopolistic market because it serves as a commitment tool for no dilution.

Keywords: brand, positioning, product portfolio, consumer search, niche and mainstream brand

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1 Introduction

“She may not like what we’ve made that following season, but she’ll give us the first look because she is a customer of ours, and it’s our job to make sure we know what she’ll like, and give her some of that.”

- Stuart Weitzman, the founder of Stuart Weitzman

Brand positioning is one of the most critical and fundamental concepts in marketing and firm strategy. How a firm chooses to position itself in the marketplace significantly affects its competitiveness and performance; for example, a carefully crafted brand positioning can substantially increase the demand for its products. While the results of honing a firm’s unique positioning in the market place may seem obvious, without understanding the underlying micro-mechanism of the consumer decision process, one may take brand positioning for granted. Despite its importance and practical relevance, little rigorous analysis has been executed to investigate the mechanism of brand positioning about how it affects consumer choice and market outcomes. What exactly is brand positioning, why is it so important, and how does it affect consumer choice?

Marketing literature defines brand positioning as the way consumers perceive the brand \( \text{(Kotler et al. (2014))} \), and it refers to the overall view about a brand that consumers have, which is often formed by a unique bundle of associations within the minds of target customers \( \text{(Avery and Gupta (2014))} \). In other words, brand positioning is the place a firm wants to own in the product space, and consumers form the perception of brand positioning from various interactions with several different products of the same brand, which all together identify and refine a firm’s distinctiveness.

For example, a consumer who owns Gucci handbags and shoes will have an overall brand image of Gucci, its sensual and fashion luxury images from all the experience that she has had with all these products and this image of the brand affect her subsequent
decisions for purchasing new apparel in the next season. Similarly, consumers may have a
different brand image about Hermes, which is another popular luxury brand, which owns
slightly different brand positioning of timeless elegance and classic luxury positioning. Both
Gucci and Hermes keep their branding positionings or identities running through all of their
products. Thus, Gucci and Hermes have effectively differentiated themselves from other
luxury brands. However, they still have effective brand positioning that resonates with
their target customers who may prefer its distinctiveness, whether it is fashion or classic
timelessness. Consumers who are interested in fashion luxury will choose to search Gucci
first for her next spring collection without knowing the individual product’s exact design or
style, while the consumer who would prefer class style may choose to search Hermes first.

Thus, brand positioning helps firms to provide and articulate critical information about
the characteristics of their products, which makes it easier for consumers to choose where
to purchase a product without searching through multiple brands to find the right one
for them. It is impossible to communicate to consumers what they can expect from a
brand without clear positioning. All in all, brand positioning can serve as an efficient
communication mechanism to invite consumer searches.

The purpose of this research is twofold. We first conceptualize the fundamental concept
of marketing by examining brand positioning in a product portfolio and consumer search
framework. Second, after providing a formal modeling structure on the concept of brand
positioning, we explore several different equilibria and examine the firm’s optimal brand
positioning strategy. This equilibrium analysis can shed insight into when it is optimal for
a firm to position as mainstream versus niche, and its economic tradeoffs.

A key feature that our model focuses on is that consumers form the perception of the
brand positioning from various interactions with all different products of the same brand.
To capture this intricacy of brand positioning, we conceptualize it as the average location
of all products of the same brand on the Hotelling line. Consumers are uncertain about
each product location of the brand but are aware of the average location of the brand’s products: brand position. Based on this information, consumers can decide whether to search for a brand (or visit a firm). In this sense, the brand positioning may convey sufficient information and guide the consumer’s visit decision. If a consumer decides to search for a brand, she incurs a search cost. We also focus on the role of search frictions in the market. We allow that there are two types of consumers: regular consumers who incur search costs and shoppers who costlessly search for a brand. Once a consumer visits a brand, she can inspect multiple products under the same brand and finds out whether a product matches her needs and decides whether to purchase a product. We model that a consumer can find the right product with a probability, which is a function of the distance from her location to the product location on the Hotelling line. Also, brands can decide their product locations along the Hotelling line simultaneously.

Under this model setup, a brand is characterized by two parts: The average location of its two products and the spread of two products, which is determined by the distance between two products. Firms then first choose their brand positioning to induce a consumer to search or visit the brand, who are still uncertain about the exact locations of two products, and her match with them. After observing the brand positioning, consumers can form rational expectations about the spread of two products and decides whether to visit. When a consumer visits a store, she observes the locations of two products, and the match with them will be realized. Then, she decides whether or not to purchase a product. If none of the two products matches her needs, she ends up with only incurring search costs to visit a brand without buying a product. The brand positioning choice, therefore, serves as a communication role for inducing consumer search by reducing the uncertainty of product characteristics. However, in the presence of search friction, this positioning information alone may not be sufficient to convince consumers to visit a brand.

When deciding whether to search a brand, consumers consider both her expected utility
based on the brand positioning and her search cost. If a brand positioning is close to the center of the Hotelling line, this brand can potentially appeal to the majority of consumers, and it may attract a greater portion of consumers to visit the store. However, such a central brand positioning may convey less “product” information. It is so because various product positions or “spread can support the central location of brand positioning.” For example, the same central positioning can have either two vastly different products or two very similar products around the central location of the Hotelling line. If two products are vastly different, then the probability that consumers located around the center of the Hotelling line find the right match with either product is low. Therefore, it may not be attractive for consumers to incur search costs to visit the brand. On the other hand, if the positioning is close to the endpoint of the Hotelling line, the brand could potentially appeal to a smaller portion of consumers. However, consumers around its position may have a higher expected utility. It is so because a consumer can correctly infer the positions of the two products must be close to that location by definition of the brand positioning; if the average is exactly at 1/2, for example, the two products must also be exactly 1/2, too. Therefore, a brand with a narrow appeal (i.e., located at the end of the Hotelling line) can convey more information about products to consumers, which increases the expected utility of consumers. This is the main tradeoff of brand positioning. In particular, the expectation of the spread can drastically change the consumer’s search decision. Consumers can correctly anticipate this tradeoff of the brand and expect the right degree of the spread in equilibrium.

Moreover, there exists a critical issue of the holdup problem arising from the presence of shoppers who do not incur a search cost. Suppose that there are only regular consumers who need to incur search costs to visit a brand. Even if a consumer can anticipate the optimal spread of the firm, they have sunk the search cost when they visit the store. Thus, a consumer may still be locked in by the brand even if it deviates from the expected level of spread. Brands may have incentives to do so by locating two products further away.
The greater the spread is, it can cover more shoppers who would have visited the store anyway since they do not incur any search cost. This is the classic holdup problem due to the existence of shoppers. Anticipating the brand’s such opportunistic behavior, consumers update their beliefs about each product location, which, in turn, lowers her expected utility that comes from finding a good match product. In particular, when there is a significantly large number of shoppers in the market, regular consumers may update their beliefs that a brand will spread two products so widely that their expected utility becomes lower than their search costs. Hence, none of the regular consumers chooses to visit a brand, and there will be only shoppers in the market. Thus, brand positioning information alone may be insufficient to convince regular consumers to search for or visit the brand, which can be costly. Brands may sometimes need to credibly communicate its product spread, notably smaller spread so that it can increase consumer’s expected utility from visiting a brand.

We focus on the credibility of such a communication role in brand positioning. We build a micro-model for how and why brand positioning can deliver credible information to consumers and provide a rationale for a firm’s specific brand positioning choice such as “niche” or “mainstream” positioning.

We first show that any brand position, such as at the center of the Hotelling line (mainstream brand positioning to appeal to the majority of consumers) or at the endpoints of the Hotelling line (niche brand positioning to appeal only to a small portion of consumers), can be supported in equilibrium. Our main results show that the mainstream positioning brand has more incentive to spread its product locations, which lead to brand dilution while the niche positioning brand has less incentive to do so. As the brand positioning becomes more niche, the spread of individual products tends to become smaller as the range of possible spread gets smaller. This reduced spread of a specific brand positioning can convey more product information. Hence, the niche brand positioning serves as the commitment mechanism that allows a brand to communicate credible information about its spread to
consumers. Finally, we identify the conditions under which brands tend to adopt a niche as opposed to mainstream positioning strategy. We find that search friction tends to lead to the proliferation of niche brands. In particular, unless there exists too many shoppers or too few shoppers (in other words, the search friction in the market is in the middle range), brands find it better off with a niche brand positioning, which appeals to only a smaller range of consumer preferences, but a higher chance of matching their preferences. Thus, the presence of shoppers can change equilibrium dynamics and firm’s profit dramatically.

This article is organized as follow: we first discuss the related literature in Section 2 and the formal model is developed in Section 3. We analyze the model and characterize two different equilibria in Section 4 and present our main results in Section 5. Conclusions are drawn in Section 6.

2 Literature review

The importance of brand positioning is well-recognized in academics and practice (Aaker 1996; Kotler et al. 2014), and our study contributes to several streams of research about brand positioning in marketing and economics. First, there is a large body of literature studying the concept of brand positioning from the psychological perspective focusing on the process of consumer’s mind. In the existing literature, brand positioning is defined as the way a firm wants consumers to perceive, think and feel about its brand versus competitive entries (Trout and Ries 1986; Davis et al. 2000). Along this line of research, many papers point out that positioning is a process of emphasizing the brand’s distinctive and motivating attributes in the light of competition (Jan Alsem and Kostelijk 2008) and establishing both the point of unique difference and the point of parity association with the category (Keller et al. 2011). We follow the definition of the traditional approach and provide a more formal modeling structure to understand the strategic use of brand
positioning.

Also, several papers study the issues on estimates of consumer perceptions of brands along attributes to inform marketing strategy (John et al. 2006; Lehmann et al. 2008). Most importantly, such estimates are used as a primary input for generating perceptual maps, which have been widely used by managers (Shocker and Srinivasan 1979; Green et al. 1989; Hauser and Koppelman 1979; Johnson and Hudson 1996; Steenkamp et al. 1994; Bijmolt and van de Velden 2012; Kaul and Rao 1995).

Second, this research is related to the literature which investigates the issue of branding and brand positioning from an economic perspective. Especially in the context of umbrella branding, researchers have examined whether firms can credibly convey information about quality through branding (Sappington and Wernerfelt 1985; Wernerfelt 1988, 1991; Zhang 2015; Neeman et al. 2019; Klein et al. 2019; Yu 2019). In this paper, we study whether the firm can transmit horizontal information about its products through branding decisions. Kuksov et al. (2013) posit that brand positioning or image is determined by the type of consumers who consume the brand’s product. In the current paper, a brand affects which type of consumers will buy its products by choosing which type of products it will make available in the market. In that sense, the brand decides its positioning through its target customers, which are endogenously constructed through its product portfolio.

Also, the current paper is closely related to the stream of literature on product positioning and its effects on the equilibrium outcomes. In particular, many researchers study static positioning in markets with strategic interactions with competition (Hauser and Shugan 2008; Moorthy 1988; Kuksov 2004; Lauga and Ofek 2011). Recently, more papers analyze the dynamic aspects of product positioning, namely repositioning of the product over time (Sweeting 2013; Jeziorski 2014). While most of the repositioning studies are empirical,

1Also, see Bronnenberg et al. (2018) for an excellent review of the economics of brand and branding. Recent studies have focused on measuring the brand value in a static setting (Goldfarb et al. 2009), and dynamic environment (Borkovsky et al. 2017).
there are a few analytical studies on this topic (Villas-Boas 2018; Cong and Zhou 2019). Villas-Boas (2018) analyzed a monopolist’s optimal repositioning strategy under changing consumer preferences and Cong and Zhou (2019) focused on the role of commitment for repositioning under competition. Our paper differs from the existing literature on product positioning by highlighting the difference between brand positioning and individual positioning.

Finally, our paper is also related to several papers on consumer search theory. Zhou (2014) and Rhodes (2014) study consumer search over firms, each carrying multiple products. Consumers in our setting also search for multiple products at a firm’s store. Still, different from previous studies, their search decisions are guided by their prior observation of the firm’s brand positioning. In this sense, our paper is related to the literature of ordered search (Armstrong 2017), where some observable characteristics of the options guide consumers’ search decisions. Armstrong et al. (2009) shows the value of being the first shopping destination or prominent place in the context of consumer search. In our setting, the firm can use its brand positioning to influence consumer search decisions. Previous studies have examined other instruments firms can use to guide consumer search, for example, advertisement (Anderson and Renault 2006; Mayzlin and Shin 2011; Lu and Shin 2018), product design (Kuksov 2004; Bar-Isaac et al. 2012), price (Chen and He 2011; Armstrong and Zhou 2011; Choi et al. 2018; Zhou 2011), service (Shin 2007; Janssen and Ke 2020), and targeted advertising (Shin and Yu 2019).

3 Model

Consider a consumer market that is represented by a Hotelling line in $[-1, 1]$. Consumers are distributed uniformly the Hotelling line in $[-1, 1]$. A monopolistic firm sells two products to consumers, and sets the locations of the two products, $x_1$ and $x_2$, on the Hotelling
line in $[-1/2,1/2]$. Therefore, we impose that the support for the firm’s product positions is a subset of the consumer market. As shown below, this assumption greatly simplifies the equilibrium analysis by eliminating truncation at endpoints.

We model that a consumer can find the right product with a probability, which is a function of the distance from her location to the product location on the Hotelling line. A consumer located at $x \in [0,1]$ gets utility $u_i(x)$ from buying product $i \in \{1,2\}$, where,

$$u_i(x) = \begin{cases} 
1, & \text{with probability } \theta - t|x - x_i|, \\
0, & \text{otherwise.}
\end{cases}$$

Notice that the consumer’s expected utility, $E[u_i] = \theta - t|x - x_i|$, which takes the same form as a standard Hotelling model. However, in our model, the consumer receives $u_i(x) = 1$ if the product $i$ is a match, otherwise the product is not a match and the consumer receives $u_i(x) = 0$. Thus, even a consumer who is exactly located where product $i$ is may not receive $u_i(x) = 1$. That is, when $x = x_i$, we have that $u_i(x) = 1$ with probability $\theta$. As the product locates further away from the consumer, a match is less likely.

For example, a consumer who is fashion-oriented and thus, one may consider her as having the same preference on the Hotelling line with the fashion luxury brand Gucci. However, Gucci’s current collection may not necessarily satisfy her specific needs at the moment. She may search for a long coat but Gucci have all different types of coats except long women’s coat. $\theta$ captures all unforeseen contingencies that may affect the match between consumer’s specific need and the product beyond the typical preference match.

To ensure that $\theta - t|x - x_i| \in [0,1]$ for any $x, x_i \in [-1/2,1/2]$, we impose the restriction that $0 < \frac{3t}{2} \leq \theta \leq 1$.\footnote{The farthest possible distance between a consumer and a product is $\frac{3t}{2}$, so we need $\theta - \frac{3t}{2} > 0$. Also, the consumer can be located precisely at an either product, so $\theta \leq 1$.} We also assume that $u_1(x)$ and $u_2(x)$ are independent. Hence, a consumer may find the match with a product which is located further from her location
than the product which is closer to her. The stochastic nature of the matching utility, together with this independence assumption, ensures that consumers prefer both products similarly located to her own position (or preference) so that she has a higher chance of finding the right match product. Even if the first product does not match, she has another chance to get matched from the other product. There is a deterministic outside option \( u_0 \in (0, 1) \) so that consumers prefer a matched product to the outside option and prefer the outside option to an unmatched product.

### 3.1 Brand positioning

To capture the idea that consumers perceive the brand positioning from various interactions with several different products of the same brand, we define the brand positioning of the firm, \( B \) as the average locations of products under the same brand name. A brand can carries \( n \) products, and designs its product by choosing the location of each of its products. In our case, we assume that a brand carries two products \( (n = 2) \), \( x_1 \) and \( x_2 \). That is,

\[
B = \frac{\sum_{k=1}^{n} x_k}{n} = \frac{x_1 + x_2}{2}.
\]

Consumers do not observe the location of each product of the the brand, and instead knows an average location of a firm’s products, brand positioning. This reflects the fact that brand image or positioning requires all the interactions of a consumer has had with all different products under the same brand – for example, consumers have a brand image that Gucci is a fashion luxury without knowing individual product’s exact design or style.

After observing the brand positioning \( B \), consumers decide whether to visit the brand at a search cost \( s \geq 0 \) and make a purchasing decision. There are two pieces of information that is relevant for consumer’s decision; average location of the products \( B \) and the spread of two products \( \Delta \). From the definition of brand positioning (Equation [1]), we can rewrite \( x_1 = B - \Delta \) and \( x_2 = B + \Delta \), where \( \Delta \equiv (x_2 - x_1)/2 \) represents the spread of the locations.
of the two products under the same brand. Without loss of generality, we assume that \( \Delta \geq 0 \), or equivalently, \( x_1 \leq x_2 \). Consumers consider both the brand positioning (which is observable) and the product spread (which is unobservable) to make their search decision.

3.2 Consumer type: regular consumers vs shoppers

There are two types of consumers in the market. An \( \alpha \) fraction of consumers are regular consumers who have a positive search cost \( s > 0 \). The remaining \( 1 - \alpha \) fraction of consumers are shoppers with zero search cost. Regular consumers observe the firm’s brand \( B \) before visiting it, but they do not observe the exact locations of his two products. That is, they do not observe the spread \( \Delta \) and have a rational expectation of \( \Delta \) as \( \Delta^* \), which coincides with the firm’s choice of \( \Delta \) in equilibrium. Further denote the regular consumers’ equilibrium expectations of the locations of the two products as \( x_1^* = B - \Delta^* \) and \( x_2^* = B + \Delta^* \), respectively. Moreover, regular consumers located at \( x \) do not observe the match values \( u_1(x) \) and \( u_2(x) \) a priori. Based on their expectation of \( x_1^* \) and \( x_2^* \), they form a rational expectation of the probability distribution of \( u_1(x) \) and \( u_2(x) \). By paying the search cost \( s \) to visit the store (or brand)\(^3\), the regular consumers discover all the information relevant to their purchase decisions—the actual spread \( \Delta \) as well as the match values \( u_1(x) \) and \( u_2(x) \). The shoppers have zero search cost, so they observe everything freely.

To summarize, we constructed the model with a monopolistic brand with two products. Consumers know their preferences, but are uncertain about each product location under the same brand prior to visiting a store (or searching a brand) because they only observe the average location of its products. In this setup, the sequence of the game is as following: First, the brand decides the locations of his two products, \( x_1 \) and \( x_2 \). Second, regular consumers observe the brand \( B = (x_1 + x_2)/2 \) and decide whether to pay the search cost and visit the firm. Upon the visit, they discover the relevant information and decide which

\(^3\)We use the terms store and brand interchangeably. Once can consider the situation where a consumer visit a brand shop for searching for a product.
product to buy or to take the outside option. Shoppers always visit and make purchase decisions after observing the matching value with both products.

3.3 Main tradeoff and intuition

Under our setup, a brand is characterized by two parts: (1) the average location of its two products $B$, and (2) the distance from this location to each product $\Delta$. If $B$ is close to 0, then the average location is close to the center of the Hotelling-line where a greater mass of consumers is concentrated. Therefore, we interpret a brand with $B$ close to 0 as a mainstream brand which can potentially appeal to the majority of consumers. On the other hand, if $B$ is close to an either end $\pm 1/2$, then the brand could only appeal to a smaller mass of consumers away from the center of the Hotelling-line. Therefore, we interpret a brand with $B$ close to $\pm 1/2$ as a niche brand.

As consumers have heterogenous demand, the brand has an incentive to locate its products somewhat distant in order to capture more demand. Moreover, this would lead to a brand positioning closer to the center, $B = 0$, a mainstream brand positioning. On the other hand, consumers may expect that $B = 0$ provides noisy information about true locations of the brand’s products because there are many combinations that can generate $B = 0$ as long as $x_i = -x_2$, i.e. $x_1 = -\epsilon$ and $x_2 = \epsilon$ for any $\epsilon \in [0, 1]$. In this case, the brand position $B = 0$ does not necessarily provide more information about the style of any product that consumers can find in the store. Then, this lack of informational value can hurt the brand by discouraging consumer search.

By comparison, a niche brand conveys more product information to consumers than a mainstream brand. Particularly, by observing $B = 1/2$, consumers can perfectly infer the positions of the two products as $x_1 = x_2 = 1/2$; similarly, by observing $B = -1/2$, consumers can perfectly infer the positions of the two products as $x_1 = x_2 = -1/2$. More generally any brand position $B$ can be supported by $x_1 = B - \Delta$ and $x_2 = B + \Delta$ for
\( \Delta \in [0, \min\{1/2 + B, 1/2 - B\}] \). As \( B \) goes from 0 to \( \pm 1/2 \), the range of possible \( \Delta \) gets smaller, and a niche brand is always a strong brand that provide more product information with \( \Delta = 0 \). By claiming a niche brand positioning, the brand is able to communicate credibly that all of its products are indeed consistent with the brand image. Knowing that, consumers who are close to the brand positioning will visit the brand. Thus, in the presence of consumer search costs, the brand can convince consumers to incur search costs for the increased information value.

The main tradeoffs that a brand faces in choosing its positioning are coming from the average location which can appeal to greater set of consumers (i.e., the center of the line) versus the expected spread of two products. The tradeoff indicates that the brand may want to position itself at the center with smaller spreads of the products to induce consumer search. However, because the brand faces consumers who only observe \( B \), this decision may not be so credible if the position is close to the center, where the brand can deviate in opposite directions. Therefore, such a product-line decision may not be credible if the brand is positioned close to the center.

We will explore this tradeoff in the subsequent equilibrium analysis. We will show depending on the degree of search frictions in the market, the brand may want to choose a niche brand sometimes and a mainstream brand other times.

### 4 Equilibrium Analysis

We start our analysis from characterizing consumer demand. After that, we analyze two benchmark cases with (1) only with shoppers and (2) only with regular consumers. These cases can be obtained by setting \( \alpha = 0 \) and \( \alpha = 1 \), respectively in the general model. These benchmark models help us to isolate the true effect of search friction on the optimal brand positioning, which we explore in Section 5.
4.1 Consumer Demand

Given the brand’s choice of brand positioning $B$, we calculate the mass of regular consumers who visit. Shoppers have zero search cost, so they will always visit.

By visiting the brand, a regular consumer at location $x$ pays the search cost $s$, and discovers the match utility with both products, $u_1(x)$ and $u_2(x)$. Her expected utility from the brand is,

$$E[u(x)] = E[\max\{u_0, u_1(x), u_2(x)\}]$$

$$= 1 - (1 - u_0) (1 - \theta + t|x - x_1^*|) (1 - \theta + t|x - x_2^*|)$$

$$= 1 - (1 - u_0) (1 - \theta + t|x - B + \Delta^*|) (1 - \theta + t|x - B - \Delta^*|). \quad (2)$$

The regular consumer will visit the brand if and only if

$$E[\text{search}] \geq E[\text{no search}] \iff E[u(x)] \geq u_0 + s. \quad (3)$$

This is equivalent to $x \in D(B, \Delta^*)$, where,

$$D(B, \Delta^*) = \left\{ \begin{array}{ll}
(i) & [B - \Gamma_1(\Delta^*), B + \Gamma_1(\Delta^*)] \text{ if } \Delta^* \leq \Delta_1, \\
(ii) & [B - \Gamma_1(\Delta^*), B - \Gamma_2(\Delta^*)] \cup [B + \Gamma_2(\Delta^*), B + \Gamma_1(\Delta^*)] \text{ if } \Delta_1 < \Delta^* \leq \Delta_2, \\
(iii) & \emptyset, \text{ otherwise if } \Delta^* > \Delta_2 \\
\end{array} \right. \quad (4)$$

where $\Gamma_1(\Delta^*) \equiv \sqrt{\Delta^* + \frac{1-\sigma}{\sigma}} - \frac{1-\theta}{t}$, $\Gamma_2(\Delta^*) \equiv \sqrt{(\Delta^* + \frac{1-\theta}{t})^2 - \frac{1-\sigma}{\sigma}}$. Furthermore, we denote $\sigma = \frac{\sigma}{1-u_0} \in (0, 1 - (1 - \theta)^2)$, which measures the relative magnitude of the search cost.

Equation (3) shows that the demand from the regular consumers can be characterized by three different cases: (1) demand comes from one interval on the Hotelling line, (2) two disjoint intervals on the Hotelling line, or (3) none, depending on the amount of $\Delta^*$. 

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Figure 1: Regular consumers who visit (a) a strong brand and (b) a weak brand

Figure 1 illustrates the characteristics of this demand system. The regular consumers who visit the brand depend on the strength of the brand positioning, in particular whether it is strong (a small $\Delta^*$), or weak (a large $\Delta^*$). Regular consumers of mass $\alpha$ and shoppers of mass $1 - \alpha$ are distributed uniformly on the Hotelling line $[-1, 1]$. The two products are located at $B + \Delta$ and $B - \Delta$ inside the interval $[-\frac{1}{2}, \frac{1}{2}]$, which is the dark grey part of the horizontal axis. Graphs plot the expected utility of each regular consumer from the

Figure 2: No regular consumer visits the brand if the brand is very weak ($\Delta^* > \Delta_2$)
brand. So, a regular consumer will visit the brand if and only if $E[u(x)] \geq u_0 + s$. The set of regular consumers who will visit the brand is represented by light grey lines above the horizontal axis.

First, if $\Delta^*$ is sufficiently small, i.e., $\Delta^* \leq \Delta_1 \equiv \sqrt{1 - \sigma}/t - (1 - \theta)/t$, the brand’s two products are close to each other on the Hotelling line. Therefore, every consumer in between the two products are willing visit the brand by incurring a search cost. This implies that, as shown in Figure 1–(a), the regular consumers who visit the brand comes from one interval centered around $B$, $[B - \Gamma_1(\Delta^*), B + \Gamma_1(\Delta^*)]$.

Second, if $\Delta^*$ becomes larger, i.e., $\Delta_1 \equiv \sqrt{1 - \sigma}/t - (1 - \theta)/t < \Delta^* \leq \Delta_2 \equiv [(1 - \sigma) - (1 - \theta)^2]/[2t(1 - \theta)]$, then the brand’s two products are far apart. Then, a consumer in between the two products finds it not worthwhile to visit the brand because neither product is expected to provide a good match. Therefore, as shown in Figure 1–(b), regular consumers who visit the brand comes from two disjoint intervals that are, roughly speaking, located around $B - \Delta^*$ and $B + \Delta^*$, respectively. Finally, when $\Delta^* > \Delta_2$, as illustrated in Figure 2, the two products are so distant from each other that no regular consumer will visit the brand.

We impose some restrictions on the search cost to simplify the equilibrium analysis and focus on the most interesting case. First, if the search cost is too high, no regular consumers will visit the brand even if the brand provides the highest possible benefits by locating its two products at the same location. To avoid this trivial case, we require that a regular consumers is willing to visit the brand if she is at the same location with the two products.

**Assumption 1** $\sigma < 1 - (1 - \theta)^2$.

This assumption says, roughly, that there always consumer search if the product matches a consumer preference.

Moreover, we want to simplify the analysis by requiring “no truncation” of demand for regular consumers at endpoints. If demand truncation happens, it leads to unnecessary
complication to analyze an abundance of corner solution cases, which makes the analysis extremely tedious without adding insight. Particularly, we require that the range of regular consumers that visit, $D(B, \Delta^*) \in (-1, 1)$ for any $B$. This is satisfied if and only if $D(1/2, 0) \in (-1, 1)$, which is satisfied if the consumer located at 1 does not visit the brand if its both products are located at $\frac{1}{2}$.

**Assumption 2** $\sigma \geq 1 - (1 - \theta + t/2)^2$.

This assumption ensures that in equilibrium the demand for the niche product (which is located at the end point of the Hotelling line) is smooth.

We now consider the brand’s decision on product positions $x_1$ and $x_2$, or equivalently $B$ and $\Delta$. We first take the brand $B$ as given at some level, and first consider the brand’s decision on the spread $\Delta$. After solving the optimal $\Delta(B)$ for a given $B$, we will compare the brand’s profit across different values of $B$. The brand’s objective is to maximize profit. Because the prices are exogenously fixed, the brand’s de facto objective is to maximize demand.

### 4.2 Benchmark: Shoppers only vs. regular consumers only

We first begin our analysis by examining two benchmark cases where only one type of consumers exist in the market. This benchmark analysis helps us to identify the underlying forces for the optimal brand positioning decision.

#### 4.2.1 Benchmark model 1: shoppers only ($\alpha = 0$)

As the first benchmark, we consider the case with only shoppers, which is by setting $\alpha = 0$ in the main model. In this case, all consumers in the market do not incur search cost to visit the brand, and thus, both the brand positioning $B$ and the spread of products $\Delta$ are observable to consumers since all consumers can immediately inspect both products without incurring any cost.
In this situation where both information is observable to consumers, we find that the brand always chooses a mainstream positioning. The following proposition formally shows this point.

**Proposition 1** If $\alpha = 0$, there is a unique equilibrium with

$$B^* = 0,$$

$$\Delta^*(\alpha = 0) = \max \left\{ \frac{t + \theta - 1}{2t}, 0 \right\}.$$  

(5)

This is quite intuitive since both information is observable and it becomes a standard Hotelling line and product positioning game. Next, we will analyze the other extreme case with only regular consumers, i.e., $\alpha = 1$.

4.2.2 Benchmark model 2: regular consumers only ($\alpha = 1$)

What is different about this case from the previous case with only shoppers is that consumers choose to visit or not based on their expectations about $\Delta^*$. Only after visiting the brand, they observe the true $\Delta$. Therefore, the demand is a function of $B$, $\Delta^*$, and $\Delta$. In equilibrium, the brand must find it optimal to choose the spread which coincides with the consumers’ expected spread, i.e., $\Delta = \Delta^*$.

As characterized in Equation (4), depending on $\Delta^*$, the set of regular consumers who visit the brand, $D(B, \Delta^*)$, can be one interval, two disjoint intervals, or an empty-set.

$$D(B, \Delta^*, \Delta) = \frac{1}{2} \int_{D(B, \Delta^*)} \left[ 1 - (1 - \theta + t|B + \Delta - x|)(1 - \theta + t|B - \Delta - x|) \right] dx.$$

Then, in equilibrium, a brand chooses the brand positioning $B$ and the product spread $\Delta$ to maximize the demand, and it must satisfies $\frac{\partial D(B, \Delta^*, \Delta^*)}{\partial \Delta} = 0$. Then, we have the following result in Proposition 2.
Proposition 2 If $\alpha = 1$, regular consumers come from one interval in any equilibrium, which can be characterized as follows:

$$B^* \in \begin{cases} 
\left[ - (1 - \Delta_1), (1 - \Delta_1) \right] & \text{if } 1 - 4(1 - \theta)^2 < \sigma < 1 - (1 - \theta)^2, \\
\left[ - (1 - \Delta_3), (1 - \Delta_3) \right] & \text{if } 1 - \left( 1 - \theta + \frac{t}{2} \right)^2 < \sigma \leq 1 - 4(1 - \theta)^2.
\end{cases}$$

(6)

$$\Delta^*(\alpha = 1) = \begin{cases} 
0 & \text{if } 1 - 4(1 - \theta)^2 < \sigma < 1 - (1 - \theta)^2, \\
\frac{\Delta_3}{2} - \frac{1 - \theta}{2t} & \text{if } 1 - \left( 1 - \theta + \frac{t}{2} \right)^2 < \sigma \leq 1 - 4(1 - \theta)^2.
\end{cases}$$

(7)

where $\Delta_3 \equiv \frac{2\sqrt{3(1-\sigma) + 4(1-\theta)^2 - 5(1-\theta)}}{3t}$

Similar to the case of only shoppers in Proposition 1, the equilibrium with only regular consumers features mainstream brand with $B^*$ around zero. In fact, $B^* = 0$ is always an equilibrium even in the case with only regular consumers.

It is worthwhile to notice that the equilibrium spread always satisfies that $\Delta^*(\alpha = 1) \leq \Delta_1 \equiv \sqrt{1 - \sigma/t - (1 - \theta)/t}$, and therefore the regular consumers come from one interval on the Hotelling line according to equation (4). It is interesting to see that $\Delta^*(\alpha = 1)$ decreases with $\sigma$ and thus decreases with the search cost $s$. Therefore, a higher search cost leads to a stronger mainstream brand. $\frac{\partial \Delta^*(\alpha = 1)}{\partial s} < 0$.

Proposition 3 In equilibrium, the optimal spread of the product ($\Delta^*(\alpha = 1)$) decreases with $\sigma$ and search cost $s$: $\frac{\partial \Delta^*(\alpha = 1)}{\partial \sigma} < 0$ and $\frac{\partial \Delta^*(\alpha = 1)}{\partial s} < 0$.

The two benchmarks of $\alpha = 0$ and $\alpha = 1$ cases demonstrate interesting opposing forces generated by shoppers and regular consumers. For the regular consumers who have to pay a search cost to visit the brand, the brand needs to provide enough benefits by locating two products sufficiently close. Then, regular consumers located close to both products are willing to visit the brand. This is apparent from the fact that $\Delta^*$ for $\alpha = 1$ case is relatively small, in particular less than $\Delta_1 \equiv \sqrt{1 - \sigma/t - (1 - \theta)/t}$. On the other hand,
for shoppers who do not need to pay any search cost, the brand wants to spread out its products sufficiently in order to provide a decent probability of a match for more shoppers who are distributed along the Hotelling line. Therefore, as shown in the following corollary, the equilibrium spread is greater for the case with only shoppers \((\alpha = 0)\) than the case with only regular consumers \((\alpha = 1)\).

**Corollary 1** The optimal spread for the case for only regular consumers \((\alpha = 1)\) is smaller than the case for only shoppers \((\alpha = 0)\): \(\Delta^*(\alpha = 1) < \Delta^*(\alpha = 0)\)

Therefore, if both shoppers and regular consumers co-exist in the market, the brand would face a trade-off from both segments in choosing the optimal spread of its brand. This is the main tradeoff the brand faces in the presence of search friction in the market.

## 5 Main results: Equilibrium Product Positioning

In this section, we analyze the main model with \(\alpha \in (0,1)\). In order to encourage some regular consumers to visit its store, the brand would want to locate their products close to each other by choosing a small \(\Delta^*\). However, once regular consumers (who cannot observe the spread of the product directly, but only anticipate the spread) visit the brand, the brand has an incentive to deviate by increasing the spread in order to better serve the other segment of consumers, shoppers. This is the classic hold-up problem. The regular consumers are rational and can anticipate the brand’s opportunistic behavior and less likely to visit the brand. This can be costly for the brand because it can discourage regular consumers’ visit to the brand. Importantly, we will show that the brand may discipline itself by choosing its brand location \(B\) close the an either end of the Hotelling line. In other words, the brand may endogenously choose a niche brand as a commitment device to preserve a strong brand positioning (or the small spread), and thereby serve both shoppers and regular consumers.
5.1 A mainstream brand (non-binding case)

Recall that in a benchmark with only regular consumers ($\alpha = 1$), the set of regular consumers who visit the brand is one interval. There is no equilibrium where regular consumers of two disjoint intervals visit the brand. We find the same to be true in the main model as long as $\alpha$ is sufficiently large. So, in equilibrium, either no regular consumers, or regular consumers of one interval will visit the brand. In the former case, the brand only serves shoppers, and therefore Equation (5) characterizes the equilibrium. In the latter case, the brand chooses $(B^*, \Delta^*)$ to maximize the demand as follows:

$$(B^*, \Delta^*) = \arg \max_{-\frac{1}{2} \leq B \leq \frac{1}{2}} \max_{0 \leq \Delta \leq \frac{1}{2} - |B|} D(B, \Delta, \Delta^*),$$

where

$$D(B, \Delta, \Delta^*) = \frac{1 - \alpha}{2} \int_{-1}^{1} [1 - (1 - \theta + t|B + \Delta - x|)(1 - \theta + t|B - \Delta - x|)] dx + \frac{\alpha}{2} \int_{B - \Gamma(\Delta^*)}^{B + \Gamma(\Delta^*)} [1 - (1 - \theta + t|B + \Delta - x|)(1 - \theta + t|B - \Delta - x|)] dx. \quad (8)$$

We first consider a mainstream brand where $B^* + \Delta^* < \frac{1}{2}$ such that the upper bound of $\Delta^*$ is not binding. Then, by solving the unconstrained optimization, we have that,

$B^* = 0,$

$$\Delta^* = \max \left\{ \frac{\alpha \sqrt{[(1 + \alpha)(1 - \theta) - (1 - \alpha)t]^2 + (4 - \alpha^2)(1 - \sigma) - 2[(1 + \alpha)(1 - \theta) - (1 - \alpha)t]}{(4 - \alpha^2)t}, 0 \right\}. \quad (9)$$

If $\Delta^* \leq \Delta_1 \equiv \sqrt{1 - \sigma}/t - (1 - \theta)/t$, then $(B^*, \Delta^*)$ is an equilibrium. Otherwise, this is not an equilibrium because the brand cannot attract regular consumers to its brand. The next proposition formally states this result.
Figure 3: A mainstream brand with weak brand positioning (a large $\Delta^*$)

Proposition 4 (Mainstream Brand) Suppose $B^* + \Delta^* < \frac{1}{2}$, i.e., a mainstream brand. (i) If $\alpha$ is sufficiently large, i.e., $\alpha > \overline{\alpha}$, then $(B^*, \Delta^*)$ characterized in Equation (9) is an equilibrium and regular consumers of one interval visit the brand. (ii) If $\sigma < \frac{3\theta - \theta^2}{2}$, there is no equilibrium in which regular consumers of two disjoint intervals visit the brand. (iii) If $\alpha \leq \overline{\alpha}$ and $\sigma < \frac{3\theta - \theta^2}{2}$, then there is no equilibrium in which a mainstream is visited by any regular consumers, and therefore $(B^*, \Delta^*)$ in Equation (5) is the unique equilibrium.

This proposition characterizes equilibrium as the composition of the customers change. Regular consumers and shoppers impose opposite effects on the brand’s optimal spread, $\Delta^*$. For regular consumers who have to pay a search cost to visit the brand, the brand wants to keep its products close to each other to provide sufficient probability of match to those who will choose to visit. On the other hand, for shoppers, the brand wants to keep their products sufficiently distant in order to provide a decent probability of match for all shoppers distributed on the Hotelling line. So, $\Delta^*$ is decreasing in $\alpha$, the fraction of regular consumers. The first case where there are many regular consumers ($\alpha > \overline{\alpha}$) is similar to the benchmark model 2 with only regular consumers. So, the optimal average location is $B = 0$ and a single interval of regular consumers will visit the brand, similar to Figure 1-(a).

However, in the third case with a small fraction of regular shoppers ($\alpha < \overline{\alpha}$) and a small search cost ($\sigma < \frac{3\theta - \theta^2}{2}$), then the brand would want to spread out its products far apart
that in equilibrium no regular consumers will visit the brand. This case is illustrated in Figure 3. When $\Delta^*$ is too large, then the expected utility of regular consumers are lower than the effective search cost ($u_0 + \sigma$). Thus, no regular consumers will visit the brand. Then, the unique equilibrium is identical to the equilibrium in the benchmark model 1 with only shoppers.

It follows that if $\alpha \leq \alpha$ and $\sigma < \frac{3\theta - \theta^2}{2}$, then the equilibrium demand for a mainstream brand is

$$D(B = 0) = \frac{1 - \alpha}{2} \int_{-1}^{1} \left[ 1 - (1 - \theta + t|\Delta - x|)(1 - \theta + t|\Delta + x|) \right] dx$$

$$= (1 - \alpha) \cdot \frac{t(3 + 18\theta - 9\theta^2) - 3t^2 - 15t^2(1 - \theta) - (1 - \theta)^3}{12t}.$$ \hspace{1cm} (10)

5.2 A niche brand (binding case)

If the brand chooses $B$ close to an end of the Hotelling-line (e.g., $\frac{1}{2}$ without loss of generality), then the brand may choose to locate one of its product at the end of the Hotelling-line, i.e., $B + \Delta = \frac{1}{2}$. This is a niche brand case. In this case, even if the brand wanted to increase the spread of its brand, it cannot do so while maintaining the same brand location $B$.

Figure 4 depicts the demand from regular consumers for a niche brand. The niche brand’s product on the right-hand side is located at $1/2$, the end of the dark grey part of the Hotelling line. So, even if the brand had an incentive to spread out its products further away from each other, it cannot do so without changing the average brand location $B$. So, the spread of the brand is kept relatively small, and therefore, the set of regular consumers who visit the niche brand will come from one interval $[B - \Gamma_1(\Delta^*), B + \Gamma_1(\Delta^*)]$, which is represented in the figure in the light grey.

An equilibrium of our interests is, therefore, $(B^*, \Delta^*) = (B^*, \frac{1}{2} - B^*)$. Given this strategy, regular consumers make their search decisions. In turn, the brand must maintain
its spread $\Delta^* = \frac{1}{2} - B^*$.

**Proposition 5 (Niche Brand)** *For a niche brand where $B^* + \Delta^* = \frac{1}{2}$, the unique equilibrium is $(B^*, \Delta^*) = (\frac{1}{2} - \Delta_1, \Delta_1)$.**

This proposition shows that a niche brand is able to serve some regular consumers because the fact that their average brand location is close to an end of the Hotelling line provides a commitment not to stretch its brand excessively. Therefore, being a niche positioning provides the brand with a commitment to locate its products close to one another. In turn, regular consumers close to the expected locations of the brand’s products will be close to each other, and choose to visit. Therefore, unlike a mainstream brand, a niche brand is able to serve some regular consumers, which could make it more profitable than a mainstream brand.

Here, we establish the result that both a mainstream brand positioning and a niche brand positioning can be supported as an equilibrium. In the next section, we identify the conditions when either brand positioning strategy can be an optimal for the brand.
5.3 Optimal brand positioning: mainstream vs. niche

If there are enough shoppers (i.e., $\alpha$ not too large), then a mainstream brand chooses to spread its product locations and serve only shoppers (Proposition 4). A niche brand, on the other hand, could serve both shoppers and regular consumers. For the niche brand’s benefit of serving regular consumers to be large, there has to be enough regular consumers (i.e., $\alpha$ not too small). Therefore, if the number of regular consumers fall in an intermediate range, then a niche brand could be more profitable than a mainstream brand.

Proposition 6 If $\alpha$ is in an intermediate range, i.e., $\alpha \in (\underline{\alpha}, \overline{\alpha})$, then the brand can obtain a greater profit as a niche brand than as a mainstream brand.

The proposition, which is the main result of the paper, demonstrates the opposing forces induced by two segments of consumers—regular consumers and shoppers—and how their interplay fundamentally influences the brand’s branding choice between a mainstream and a niche brand. Regular consumers only have limited information about the brand and must visit the brand by incurring a search cost in order to find out exactly how much they enjoy the brand’s products. For these consumers, the brand has an incentive to establish a strong brand positioning by placing its products close to each other, thereby conveying more information about the brand’s products. However, the presence of shoppers who can freely visit the brand tempts the brand to spread out its product locations, and thereby cater to the needs of shoppers with heterogeneous preferences more efficiently. The more shoppers there are, the stronger this temptation becomes. As the brand spreads out its products sufficiently far apart, the brand positioning becomes weaker. And, the amount of information about the brand’s products that can be conveyed to consumers will decreases. This will discourage regular consumers from visiting the brand, which leaves only the shoppers in the market. Therefore, the brand needs some kind of a commitment mechanism to overcome the temptation and serve both shoppers and regular consumers.
The proposition shows that a niche brand can provide an effective commitment device.

It must also be noted that a niche brand can be more profitable than a mainstream brand under a specific condition, i.e., if $\alpha$ is in an intermediate range. Otherwise, the mainstream brand is always better. This implies that even if niche brand can provide more information about the brand’s products, under a wide range of parameter space, the brand prefers a mainstream brand to a niche brand.

6 Conclusion

Brand is one of the most valuable assets that a firm possesses, and it can have a significant impact on the consumer’s purchase decisions. However, it still largely remains an under-researched area in economics and marketing. In particular, brand positioning helps firms to create market differentiation by providing or articulating critical information about the characteristics of its products. Thus, it facilitates consumer search and helps them to choose where to purchase a product without searching through multiple brands to find the right one for them.

In this paper, we conceptualize the role of brand positioning from a product portfolio and consumer search framework. First, we provide a micro-foundation for how and why brand positioning can deliver credible information to consumers and provide a rationale for firm’s specific brand positioning choice such as “niche” or “mainstream” positioning. Consumers are uncertain about each product location of the brand but are aware of the average location of the brand’s products: brand position. Based on this information, consumers can make decisions on whether to visit a brand (or firm) first, and brands can decide their individual product locations simultaneously. We identify the conditions under which brands tend to adopt a niche as opposed to mainstream positioning strategy and find that when search friction is in the intermediate range, it leads to the proliferation
of niche brands. Our main results show that the mainstream positioning brand has more incentive to spread its individual product locations, which lead to brand dilution while the niche positioning brand has less incentive to do so. Thus, the niche brand can serve as a commitment tool for a brand not to spread its individual product location to invite more consumers to visit the store.

We hope that our framework of modeling the brand positioning can serve as a workhorse model for further studies in the future. Also, our results can shed insight into the coexistence of difference brand positioning, and provide practical guidance about brand positioning strategy when and why a certain type of brand positioning is optimal.
Appendix: Proofs of results

Proof of Proposition 1

Given only shoppers, the firm’s demand is that, \( D(B, \Delta) = \frac{1}{2} \int_{-1}^{1} [1 - (1 - \theta + t|B + \Delta - x|) (1 - \theta + t|B - \Delta - x|)dx \). By taking first- and second-order derivatives of \( D \) with respect to \( B \) and \( \Delta \), we can solve the optimal \( B^* = 0 \) and \( \Delta^*(\alpha = 0) = \max \{ \frac{t+\theta-1}{2t}, 0 \} \).

Proof of Proposition 2

Suppose \( \Delta^* \) is large enough that regular consumers who visit the firm consists of two disjoint intervals. Then, we find that \( \Delta^* = \frac{1-\sigma-(1-\theta)^2}{2(1-\theta)} = \Delta_2 \). Then, only regular consumers located precisely at \( B - \Delta^* \) and \( B + \Delta^* \) will visit the firm. This results in a measure zero demand. Therefore, there cannot be an equilibrium (with a positive profit) in which regular consumers visit from two disjoint intervals. This leaves us with an equilibrium in which regular consumers of a single interval visit the brand. So, we solve for a first order condition \( \frac{\partial D(B, \Delta^*, \Delta^*)}{\partial \Delta^*} = 0 \) to identify the optimal \( \Delta^* \) given \( B \). And, given that there are only regular consumers without demand truncation, where exactly \( B \) is does not affect \( B \) as long as it is close enough to the center, 0. The case in which \( B \) is far away from the center is analyzed in the niche brand case. Then, equilibria are characterized as follows:

\[
B^* \in \begin{cases} 
(1 - (1 - \Delta_1), (1 - \Delta_1)] & \text{if } 1 - 4(1 - \theta)^2 < \sigma < 1 - (1 - \theta)^2, \\
(1 - (1 - \Delta_3), (1 - \Delta_3)] & \text{if } 1 - \left(1 - \theta + \frac{t}{2}\right)^2 < \sigma \leq 1 - 4(1 - \theta)^2.
\end{cases}
\]

\[
\Delta^*(\alpha = 1) = \begin{cases} 
0 & \text{if } 1 - 4(1 - \theta)^2 < \sigma < 1 - (1 - \theta)^2, \\
\Delta_3 - \frac{1 - \theta}{2t} & \text{if } 1 - \left(1 - \theta + \frac{t}{2}\right)^2 < \sigma \leq 1 - 4(1 - \theta)^2.
\end{cases}
\]
**Proof of Proposition 4**

For part (i), the first order condition is

$$\frac{\partial D}{\partial \Delta}(B, \Delta^*, \Delta^*) = 2t\Delta^*( - (1 + \alpha)(1 - \theta) + t(1 - 2\Delta - \alpha(1 - \sqrt{(\Delta^*)^2 + \frac{1 - \sigma}{t}}))) = 0,$$

where \(\Delta^*\) is a function of model parameters, including \(\alpha\). By differentiating both sides with respect to \(\alpha\) using the chain rule,

$$-(1 - \theta) - 2t \frac{\partial \Delta^*}{\partial \alpha} - t(1 - \sqrt{(\Delta^*)^2 + \frac{1 - \sigma}{t}}) + \alpha \cdot t((\Delta^*)^2 + \frac{1 - \sigma}{t})^{-1/2} \Delta^* \cdot \frac{\partial \Delta^*}{\partial \alpha} = 0,$$

and therefore

$$t(\alpha \Delta^*(\Delta^*)^2 + \frac{1 - \sigma}{t}) - \frac{1}{2} - 2) \cdot \frac{\partial \Delta^*}{\partial \alpha} = 1 - \theta + t(1 - \sqrt{(\Delta^*)^2 + \frac{1 - \sigma}{t}}).$$

The right-hand side is always positive, and on the left-hand side the term multiplied by \(\frac{\partial \Delta^*}{\partial \alpha}\) is negative because \(\alpha \cdot \frac{\Delta^*}{\sqrt{(\Delta^*)^2 + \frac{1 - \sigma}{t}}} < \alpha < 1\).

Therefore, for the first-order condition implies \(\frac{\partial \Delta^*}{\partial \alpha} < 0\). From the previous analysis on benchmark cases, for \(\alpha = 1\), \(\Delta^*_{\alpha=1} < \Delta_1 \equiv \sqrt{1 - \sigma/t - (1 - \theta)/t}\). For \(\alpha = 0\), \(\Delta^*_{\alpha=0} > \Delta_1\). Therefore, there exists \(\overline{\alpha}\) such that \(\Delta^* < \Delta_1\) if and only if \(\alpha > \overline{\alpha}\), and therefore \((B^*, \Delta^*)\) is an equilibrium.

For part (ii), first assume that regular consumers visit the firm from two disjoint intervals. Then, if \(\sigma < \frac{3\theta - \theta^2}{2}\), \(\frac{\partial D(B, \Delta^*, \Delta^*)}{\partial \Delta^*} < 0\) so that the firm has an incentive to continue to reduce the spread \(\Delta^*\), which breaks down the equilibrium.

Part (iii) immediately follows from part (i) and (ii).

**Proof of Proposition 5**

Suppose that \((B, \frac{1}{2} - B)\) is an equilibrium where regular consumers in one interval visit the firm. Then, we can show that the firm’s demand is decreasing in \(B\) such that the firm is better off by playing the average brand location closer to the center 0. Since \(\Delta_1 \equiv \sqrt{1 - \sigma/t - (1 - \theta)/t}\) is the largest possible spread to have regular consumers of one interval visit the firm, the choice of \(B\) closest possible to the center is \(B = \frac{1}{2} - \Delta_1\). If \(B > \frac{1}{2} - \Delta_1\), i.e., the average brand location is closer to 1/2, then the firm has an incentive to push \(B\) closer to the center. If \(B < \frac{1}{2} - \Delta_1\), then the spread \(\frac{1}{2} - B\) is too large. In particular, it violates the assumption that regular consumers who visit the firm comes from one interval.
Proof of Proposition 6

Profit (or demand) for a mainstream brand for \( \alpha < \bar{\alpha} \), \( D(B = 0, \Delta_{B=0}^*) \), is as shown in equation (3). And, for a niche brand with \( (B, \Delta^*) = (\frac{1}{2} - \Delta_1, \Delta_1) \), the demand is

\[
D(B, \Delta^*) = \frac{1 - \alpha}{12t} \left( 8((1 - \theta)(2 + 2\theta - \theta^2) - 3\sigma(1 - \theta) - 2\sqrt{1 - \sigma}(1 - \sigma)) - 3t^2(9 - 9\theta - 4\sqrt{1 - \sigma}) \\
- 12t(1 + 2\theta^2 - 4\theta + (1 + \theta)\sqrt{1 - \sigma}) - 7t^3 \right) \\
+ \frac{\alpha}{3t} \left( (6 - 4\theta + 2\theta^2 - 4(1 - \theta)\sqrt{1 - \sigma} - \sigma)\sqrt{1 - 2\theta + \theta^2 - 2(1 - \theta)\sqrt{1 - \sigma} + 2(2 - \sigma)} \\
+ 2 - 3\theta^2 + \theta^3 - 6(1 - \theta)\sigma - 4(1 - \sigma)\sqrt{1 - \sigma} \right). \tag{11}
\]

We can show that \( D(B = 0, \Delta_{B=0}^*) < D(B = \frac{1}{2} - \Delta_1, \Delta_B^*) \) holds if \( \alpha \) is not too small, i.e., \( \alpha > \bar{\alpha} \). Moreover, \( \bar{\alpha} > \alpha \), which completes the proof.  

\[\Box\]
References


