Monte Carlo, Monte Carlo....Who is the Best Schedule of Them All?

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Introduction

Project managers work in a world of uncertainty. Executing elements of the product development lifecycle take time and risk always needs to be managed. Software development projects, hardware development projects, process change projects are all fraught with risks and challenges. And these challenges and risks impact through all the project’s stages:

- Requirements Risks – Have the stakeholders adequately defined the projects requirements? Are there key performance parameters that cannot be traded away if the project gets into trouble or needs to rebaseline?
- Development Risks – Is there a key piece of development that is required for success? If the development doesn’t fully meet its objectives, will the project’s success be in question?
- Test Risks – Verifying that the system has met requirements can be risky; has the software regression plan fully exercised the code? In a physical system, does the final acceptance test accurately measure performance?
- Deployment Risks – Is the system used in the field differently than how it was intended to be used, causing it to be operated outside of its design envelope?
- Retirement Risks – As the system reaches a point where it needs to be taken offline due to obsolescence or other life limiting issue, are there challenges with system retirement that should be addressed?

Project management attempts to address these uncertainties and risks, principally through identification of the risk and development of mitigation strategies applying resources, whether it be financial (cost), temporal (schedule) or technical (performance requirements). Implementation of the mitigation strategies and risk tolerance are balanced against the cost of implementation. Yet for all the uncertainty involved in the project, company or customer management continuously drives the team for certainty. Planning for implementation or commercialization requires a critical level of certainty. For example, marketing plans for Apple’s introduction of new iPhone products are tied to key dates of announcement and availability. When risks become issues and impact either of those dates, Apple management faces the negative publicity with missed schedule dates.

How can the project manager provide confidence to management that team will be able to hit key milestones in the context of risk and uncertainty involved in every project? How does a project manager allocate what are typically scarce resources to the project to meet stakeholders’ execution requirements? How does a project manager build confidence with stakeholders when project execution difficulties force hard discussions about resource reallocation? Discussing a Microsoft Project schedule during a project review with key durations, critical path tasks, slack and milestones shown is one way to build confidence in schedule execution. But how does a project manager use deeper insight of the schedule to guide the application of project resources?

Tools are available to the project manager to assist in these endeavors. Various companies provide tools to execute schedule risk assessments as an add-on to Microsoft Project™. These tools integrate and assess critical path implications as well as overall schedule forecasting, providing an integrated analytical approach at the fingertips of the project manager. The challenge for the project manager is to understand the deeper insight that the schedule risk assessment provides; one such analytical technique applied to schedule risk assessments uses Monte Carlo techniques, and will be discussed further in this paper.
Project Manager’s Schedule

The project schedule is certainly one of the most important, if not the most important tool that the project manager has to manage the project. It is certainly the one of the easiest tools the project outsider can use to judge the project health and status. Success criteria are generally obvious; either milestones are met or missed.

The PMBOK, sixth edition defines project scheduling as “a detailed plan that represents how and when the project will deliver the products, services and results defined in the project scope and serves as a tool for communication, managing stakeholders’ expectations, and as a basis for performance reporting1.” Development of the list and ordering of tasks are generally straightforward activities. Identifying the durations for tasks is also typically straightforward. Task durations are usually based on past experience or expectation of the time required for the task or typical tasks. The schedule can be evaluated for critical path, available slack, and other metrics to support overall project management.

For a schedule example used through the rest of the paper, consider the development of a new widget represented by the following schedule in Figure 1. Widget A’s requirements are defined and flowed to subsystems, with three major subsystems developed and then system requirements are verified and validated to deliver Widget A.

Schedule Risk Assessment

For the purpose of this paper, evaluation of the schedule risk does not involve a review of the core schedule logic; in reality there may be risk with the ordering of tasks. The assumption for our purpose is that there is little risk associated with task ordering. Our focus will be on the risk of the schedule duration.

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Work package managers build schedules by defining activities and forecasting durations for those activities. Typical planning usually incorporates some level of slack or margin to accommodate for risks. Even with this approach, analysis of the schedule and critical path rely on a deterministic view of the durations. What this means is that any snapshot in time of the schedule generally relies on future tasks of the schedule being deterministic (or certain) representation of project. Completed tasks are in-fact deterministic and known, as their duration can be clearly measured. Any future task durations, however, should be analyzed with a stochastic lens; the likelihood of task durations taking the exact planned duration is small, especially with complex development work. Monte Carlo technique provides a “random” look at the project schedule and allows the program manager a unique perspective on schedule sensitivity and risk.

The true power of Monte Carlo techniques applied to schedules allows the project manager to evaluate key and critical inflection points with the goal of understanding schedule dependencies. Once schedule dependencies are revealed the project manager can better assess the strategic application of resources (time, people, money) or scope negotiation to guarantee project success.

Monte Carlo and Schedules

At their core, schedules are mathematical relationships containing the ordering and duration of tasks. Therefore, mathematical techniques can be applied for analysis. Monte Carlo simulation is one technique that can be applied to schedule durations to develop a view of possible outcomes. A detailed explanation of Monte Carlo simulation is provided in Appendix A– Monte Carlo Simulation Background and Application.

Schedule development and reporting using a limited perspective on the task durations can mischaracterize the future outcome. Utilizing Monte Carlo technique can avoid this myopic view by applying a random draw based on expected distributions around task durations. The draws allow the project manager to sample many different project outcomes and build an expected response profile for the final duration of the project. With this insight, the project manager has the ability to more effectively communicate expected project outcome to stakeholders and provide them a sense of risk profile.

So how does this work? Let’s go back to the sample schedule in Figure 1 to illustrate the use of Monte Carlo. In the baseline schedule, tasks in the schedule are deterministic events. But what if the tasks are treated as stochastic events?

In how many development projects does the testing phase take longer than planned? Testing can reveal issues with the development that require corrective action to meet requirements. Once the cycle of corrective action and retest begins, there can be several loops before confirming that the changes are implemented correctly. These loops eat away at time and can result in a slipped schedule. We can use a probabilistic approach to this corrective action loop for the first Monte Carlo example.

In this case, Task 6, Prototype Testing of Subsystem A’s current duration is 11 days. However, 11 days duration does not incorporate any corrective action loops that might occur. One way to represent the randomness of a corrective action loop is to use a triangular distribution for Task 6’s duration. One possible representation is the triangular distribution shown in Figure 2.
The triangular distribution of Figure 2 is interpreted that the duration of the task will have the highest probability of being 11 days, but can extend to as much as 22 days, with a declining probability of occurrence from day 11 to day 22. We can simulate the outcome of the project with 100,000 samples, reviewing the total duration to complete Subsystem A and the overall project outcome. These results are shown in Figure 3 and Figure 4.
Several interesting insights can be understood from the data. First, Figure 3 shows that the overall development duration of Subsystem 1 varies, depending on how long the testing takes. The duration can vary between a minimum of 36 days to a maximum of 47 days, corresponding to the fact that the testing duration is initially planned for 11 days but could extend to 22 days. Second, given the triangular distribution of the testing duration, the plot in Figure 3 shows that with about 90% confidence, Subsystem A should complete development within 44 days. Third, Figure 4 shows that the overall schedule is not impacted as the duration of 120 days remains the outcome after 100,000 simulations. Why is that? While the change in duration of Task 6 impacts the amount of time of Subsystem A’s development, the delta in Subsystem A’s development time does not impact the critical path. Had the uncertainty around Task 6 been significantly longer to impact the critical path, the overall schedule would have been impacted.

This initial example is interesting, but doesn’t show the true power of Monte Carlo for project managers. Let’s stick with the same example schedule from Figure 1. For this example, Task 6 will not have a distribution applied, but Task 9 (Subsystem B, Proof of Concept) will be a gamma distribution, with a location of 28, a scale of 3 and shape of 3 (Figure 5). This type of distribution represents a situation where there is a probability that the task will complete early from baseline (baseline assumed at 30 days, the originally planned finish date), but the majority of the probability distribution function states that the task will take longer than 30 days. Additionally, Task 15 (Subsystem 3, Verification Testing) is defined by a triangular distribution with a minimum and mean of 15 days and a maximum of 40 days (Figure 6). For Task 15, it won’t finish any earlier than 15 days, and could take as long as 40 days.
When this option is simulated with 100,000 random numbers based on the two distributions defined, the end date of the project varies between July 17 and September 4. But the valuable question that management usually asks is, “What’s the chance that the project will hit July 19th (the original baseline) for the delivery date?” Figure 7 provides the answer and shows that the project has a 98% chance of finishing later than July 19. In fact, the Monte Carlo model provides percentiles for the dates of completion in Table 1. Interpreting the table, for instance, would show that the 50/50 point is July 25. This means that there is a 50% chance that the project will finish before July 25, but it will not finish earlier than July 17. Conversely, there is a 50% chance of the project completing after July 25, but no later than September 4. The extrema of the percentiles define the bounds of the project’s finish dates (earliest and latest). And the percentiles themselves also define the likelihood of completing by a certain date.
Critical path changes can also be assessed through the Monte Carlo simulation. For the baseline schedule, the critical path runs through the Development of Subsystem 2. However, the Monte Carlo takes random draws on the durations of tasks 9 and 15 and there are instances where the critical path shifts to the Development of Subsystem 3, because task 15’s completion date is after the simulated completion date of the Development of Subsystem 2. The project manager would be best served in understanding the possibility of this change so that resources might be allocated to address the change or avoid the change, if that is so desired. In our example, the difference of the completion dates of the development of subsystems 2 and 3 from each Monte Carlo draw documents the individual simulated outcome; collecting those results into a histogram will identify the likelihood of a change in critical path. Figure 8 shows the model output of the difference of the completion dates of subsystem 2 and 3. When positive, the critical path runs through the Development of Subsystem 2 (baseline critical path); when negative, the critical path runs through the Development of Subsystem 3. Figure 8 illustrates that there is a 14% chance (100% - 85.92%) of a critical path change where the critical path would run through Development of Subsystem 3 instead of through the Development of Subsystem 2.
What does a project manager do with this data? There are several things that can be done. Obviously risk mitigation planning becomes enhanced with Monte Carlo data. In the case of our sample schedule, what if the project cannot extend beyond July 23? Using the data in Figure 7, there is a 30% chance of meeting or beating the July 23 date. What can a project manager do to increase the probability from 30% to something higher than 30%? Maybe an increase in resources (people, assets) is warranted. Maybe a scope change to reduce the content of work required for the critical path. What is clearly valuable from the Monte Carlo result is that the Project Manager is now armed with analytical insight and can have a very meaningful and focused discussion with the team and management stakeholders to arrive at a mutually agreed approach.

**Conclusion**

Imagine being empowered with Monte Carlo output when discussing the project schedule forecast with managers and leaders within your organization. Insight into deeper dependencies of the scheduled elements, such as potential changes in the critical path, provides the project manager with key understanding and enhances the conversation with the team and management. While Monte Carlo techniques are not new, their application to project management challenges are typically not widely executed.

Using an innovative approach such as Monte Carlo simulation provides the project manager the ability to identify and convey exactly where the risk in the schedule exists as well as the impact to the completion dates and critical path changes. Just as the Gantt chart illustrates the project schedule, a Monte Carlo analysis on the schedule will focus the conversation on the expected outcomes and support examination of mitigation strategies to address the risks within the schedule.
Appendix A– Monte Carlo Simulation Background and Application

Monte Carlo method is the use of “random or pseudorandom numbers, for solution of a model.” The earliest problems solved using the Monte Carlo method include Buffon’s needle problem and the Boltzmann equation. In 1908, the statistician Student used Monte Carlo method to define the t-distribution. Major development and use of the Monte Carlo method took place during World War II, when von Neumann and Ulam implemented the method on work with the atomic bomb, including investigations in to random neutron diffusion in fissile material.

While Monte Carlo methods are quite powerful in solving a wide range of complex problems, the pure application of random numbers to solve a model specific to project managers is of interest in this paper. But what does this the use of random numbers to solve a problem mean?

Let us consider the rolling of two normal dice, each with six faces showing numbers one through six. Normal probability theory would forecast that the likelihood of rolling a number one through six on each dice is equal and is one-sixth (1/6). Therefore the combination of any given outcome of the two dice tolling together is 1/36 (1/6 x 1/6). Certain sums of the die rolls have higher probabilities of occurrence because there are more combinations of those outcomes possible, such as seven, for example. Table 1 shows the possible dice outcomes yielding a seven.

<table>
<thead>
<tr>
<th>Die 1 Roll</th>
<th>Die 2 Roll</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
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<td>7</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>7</td>
</tr>
</tbody>
</table>

If each of these outcomes is equally likely and independent from each other, the probability of seven being rolled is 1/36 x 6 = 16.667%. Seven also happens to be the most likely outcome (highest probability) from rolling two dice.

Now consider an experiment, where a pair of dice is rolled 10 times. The results from 10 die rolls are as follows: 7, 4, 5, 2, 7, 8, 6, 2, 6.

Creating a histogram (Figure 9) of the outcomes, we see that six appeared more often than all other numbers, two and seven appeared twice, other numbers appeared once or did not appear at all. If we know nothing else about our system of dice and how it operates, we might look at this observations and draw the conclusion that 6 has a better likelihood of happening than all the other outcomes. And we would be drawing the wrong conclusion from the observed data!

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2 Simulation and the Monte Carlo Method, Rubinstein, John Wiley and Sons, 1981
3 Monte Carlo Methods, Hammersley and Handscomb, Methuen & Company, 1964
This is where Monte Carlo techniques and the use of random numbers and replications can help us solve this model and better understand the true outcome. We can develop a mathematical model of the roll of the dice, and simulate the outcome over and over with the use of a computer. This simulation can be conducted hundreds, thousands or hundreds of thousands of times to provide confidence in the outcome. Using a Monte Carlo simulation, the result looks much different, as shown in Figure 10.

![Dice Experiment Outcomes](image)

**Figure 9 - Histogram of Dice Experiment**

Figure 10 represents the outcome of a Monte Carlo model of the dice experiment using a software tool called Crystal Ball®. The model shows that over 100,000 dice rolls, the expected outcome is that seven will show up 16.6% of the time. The number two will show up less than 3% of the time. The same is true for twelve; it will occur less than 3% of the time.
The Monte Carlo model takes a random draw of the dice roll and sums the results and replicates this 100,000 times (user selected) in a matter of seconds. The power of this that the manual experiment of rolling dice and recording over and over would take significant time to execute 1,000 times, let alone 100,000 times. However, with the use of a computer and an accurate representation of the randomness of the inputs, in this case the random behavior of the roll of a die as represented by the probabilistic outcome of each roll, can yield a much larger “experiment” and provide a higher confidence as to the overall outcome. So why was there such a difference between the experimental result and Monte Carlo simulation? Clearly, a small sample size skews the results and can cause incorrect conclusions to be drawn. Using a larger sampling and random number draws paints a more accurate picture of the expected outcome.